SOME ASPECTS OF ULTRA-RELATIVISTIC
HEAVY ION COLLISIONS

FRANÇOIS GELIS

CERN, PH-TH, 1211 Geneva 23, Switzerland

(Received January 8, 2008)

In this paper, I discuss some recent results obtained in Heavy Ion Collisions and what they tell us — or what questions they raise — about the physics of the system of quarks and gluons formed in these collisions.

PACS numbers: 25.75.–q, 25.75.Nq, 12.38.–t

1. Quark-gluon plasma and heavy ion collisions

The existence of a deconfined phase of nuclear matter was conjectured long ago on the basis of asymptotic freedom, and has now received ample support from QCD simulations on the lattice [1]. Some of these results are displayed in Fig. 1. In the left figure the potential between two static quarks as a function of the distance between them is displayed. The solid line is the zero temperature potential, that shows a linear rise at large distance — a sign of quark confinement. The dotted lines show the same potential at increasing temperatures. The main feature is that, while the short distance behavior

Fig. 1. Lattice results. Left: quark potential as a function of distance at various temperatures. Right: pressure as a function of temperature.

is not much affected, the linearly increasing tail eventually disappears. This indicates that above a certain temperature, it costs only a finite energy to separate the two quarks. Another evidence for a phase transition is shown in the right plot of Fig. 1, where one can see that the pressure rises very rapidly at a certain temperature (the value of which depends on the quark content of the theory), indicating a sudden increase in the number of degrees of freedom in the system. This is interpreted as a transition from hadronic bound states to a plasma of quarks and gluons.

This phase transition has occurred in the expansion of the early universe (left panel of Fig. 2), but unfortunately this has not left any visible relic in today’s observable sky. Another place to look for experimental evidence of color deconfinement is in the collisions of large nuclei at high energy (right panel of Fig. 2). The basic idea of these experiments is to deposit a large amount of energy in order to create matter with an energy density larger than the critical one, and to do so in an extended volume, i.e., large compared to the typical hadronic size, so that thermodynamical concepts have a chance to apply. Such collisions have been performed at the AGS (BNL), at the SPS (CERN), presently at the RHIC (BNL), and in the near future at the LHC.

The standard scenario for a high energy nucleus–nucleus collision involves several stages (see Fig. 3). At extremely short time scales, the very hard processes that account for the hard particles in the final state take place. It is believed that they can be calculated using the standard tools of perturbative QCD and collinear factorization. However, most of the particles that make the final state are in fact quite soft (99% of the multiplicity in a collision at RHIC is made of particles with $p_\perp \leq 2\text{GeV}$). The bulk of this particle production takes place slightly later ($t \sim 0.2\text{fm}$), and since it involves the low $x$ part of the nuclear wavefunction, i.e., large parton densities, it is expected to be amenable to a treatment in terms of classical fields. The Color Glass Condensate [2] effective theory is a framework in
which such calculations can be carried out in a systematic way. Eventually, this deconfined matter reaches a state of local thermal equilibrium, and can be described by hydrodynamics. Because it is in expansion, it cools down and reaches the critical temperature where hadrons are formed again. At a later stage, the density becomes too low to have an interaction rate high enough to sustain equilibrium, and the system freezes out. Experimentally, one is trying to infer properties of the early and intermediate stages of these collisions from the measured hadrons in the final state.

Fig. 3. Successive stages of the collision of two nuclei.

2. Small coupling wisdom

Until recently, the theoretical studies of the properties of the quark gluon plasma have been based on the assumption that the temperature is high enough to apply weak coupling techniques, thanks to asymptotic freedom (the average distance between two particles in the plasma goes like $T^{-1}$). When the coupling constant $g$ is small, there is a useful separation between various distance scales, each of them corresponding to different physical phenomena, and being described by a specific effective theory [3]. These scales are:

- $\ell \sim T^{-1}$. This is the mean distance between two particles in the plasma. Also, $T$ is the typical energy of a plasma particle, and it is therefore this scale that dominates the bulk of the pressure ($P \sim T^4$).
- $\ell \sim (gT)^{-1}$. This is the scale at which the main collective effects occur in the QGP. For instance, the Debye screening length is of the order of $(gT)^{-1}$, and plasma particles acquire a thermal mass of order $gT$ due to their interactions with the other particles.
- $\ell \sim (g^2T\ln(1/g))^{-1}$ is the mean free path between two collisions with a soft momentum transfer, i.e. a scattering angle of order $g$. This is the relevant scale for phenomena such as electromagnetic emission from the plasma.
- $\ell \sim (g^4T)^{-1}$ is the mean free path between two hard scatterings, i.e. with a scattering angle of order unity. This is the natural scale for the transport of momentum.
However, one also sees from this small coupling analysis that the scale that characterizes thermalization is the last one, \( (g^4T)^{-1} \). In other words, it is quite unnatural to have a fast thermalization in a weakly coupled system.

3. RHIC results

In seven years of operation, the RHIC has obtained many results relative to heavy ion collisions [4]. Let me only discuss two specific results, that are amongst the most important.

The first one is the phenomenon of jet quenching. In proton–proton collisions, one is used to final states being predominantly 2-jet events. Jets are very hard to measure directly in nuclear collisions due to the high final state multiplicity, but one can measure the azimuthal correlation function between pairs of particles (see the left panel of Fig. 4). In \( pp \) collisions, this correlation function is peaked both at \( \phi = 0 \) and at \( \phi = \pi \). The same is true in deuteron–nucleus collisions. However, in nucleus–nucleus collisions, one observes only a correlation around \( \phi = 0 \), and no correlation at \( \phi = \pi \).

This led to the idea that the matter formed in nuclear collisions is opaque to the propagation of hard partons (\( i.e. \) they fragment much more than in the vacuum and are undistinguishable from the bulk of the other particles when they come out on the other side). The correlation at \( \phi = 0 \) would be from jets produced near the surface of the medium, a configuration in which the partner jet in the opposite direction has a much larger length of matter to cross before escaping, hence its disappearance.

The second striking phenomenon observed at RHIC is elliptic flow. In peripheral collisions, the initial shape of the matter formed in the collision has an elliptic azimuthal shape (see Fig. 5). Since its pressure is zero at the outer surface, and maximal at the center, the pressure gradients are larger in the direction of the small axis of the ellipsis. This means that the particles
will acquire a larger flow velocity in this direction, and the net result of this is an anisotropy of the transverse momenta of the particles measured in the final state. This anisotropy is characterized by the measure of a number called $v_2$, defined by

$$\frac{dN}{d\phi} \sim v_2 \cos(2(\phi - \Phi_R)),$$

where $\Phi_R$ is the azimuthal direction of the reaction plane, i.e. the plane that contains the impact parameter of the collision. Experimental results for $v_2$ are displayed for various particles, as a function of their $p_T$, in the left panel of Fig. 5. The solid curves in this plot are the result of a simulation based on ideal hydrodynamics, in which one treats the matter produced in the collision as a fluid that has no viscosity. Let us recall here that no viscosity means that the fluid must be in equilibrium so that it is not the siege of dissipative phenomena. Moreover, it has been argued that the system must have been close to equilibrium from early times, in order to produce an elliptic flow of that magnitude (dissipative effects tend to reduce $v_2$), which is hard to explain in the conventional weak coupling approach.

![Fig. 5. Left: elliptic flow $v_2$ measured at RHIC, compared with hydrodynamical simulations. Right: cartoon of a peripheral $AA$ collision.](image)

4. AdS/CFT duality and the QGP

The parameter that controls viscous effects in hydrodynamics is the dimensionless ratio of the viscosity to entropy density, $\eta/s$. When evaluated in perturbative QCD, this ratio is $\eta/s \sim (g^4 \ln(1/g))^{-1}$ [5]. It is therefore large in the weak coupling limit, in apparent contradiction with the fact that non viscous hydrodynamics reproduces well RHIC data on $v_2$. $\eta/s$ cannot
be evaluated in the strong coupling limit of QCD. However, one can perform such strong coupling calculations in an $\mathcal{N} = 4$ super-symmetric Yang–Mills theory, thanks to the AdS/CFT correspondence.

The AdS/CFT correspondence states that this particular gauge theory is dual to a type IIB string theory on an $\text{AdS}_5 \times S^5$ background. Moreover, the correspondence between the parameters of the two theories is such that the limit $g^2 \ll 1, g^2 N_c \gg 1$ of the gauge theory corresponds to the limit where the string theory simplifies into classical super-gravity. Then, correlators in the gauge theory can be calculated from classical solutions in the gravity dual, with boundary conditions on the 4-dimensional boundary of the $\text{AdS}_5$ that depend on the specific problem one wants to study in the 4-dimensional gauge theory.

Using this correspondence, $\eta/s$ was calculated exactly in the strong coupling limit of an $\mathcal{N} = 4$ super-symmetric Yang–Mills theory, and it was found to be $\eta/s = 1/4\pi$ [6]. It was also argued that $\eta/s$ must have a lower bound of order unity because of the uncertainty principle ($\eta/s$ is of the order of the ratio of the mean free path to the De Broglie wavelength of the particles), and that the $\mathcal{N} = 4$ SUSY Yang–Mills theory in the strong coupling limit actually realizes the lower bound. Although one cannot use these techniques in QCD, it is believed that the strong coupling regime of QCD also exhibits a small value of $\eta/s$, but it is at the moment impossible to make this statement more rigorous than an educated guess.

5. Early time dynamics

As we have seen, invoking a strong gauge coupling may explain why the viscosity is so small, and also why the system thermalizes early. Another point of view exists in the community, which assumes that the early stages of high energy heavy ion collisions can be studied by perturbative techniques. First of all, one should recall that the bulk of particle production in heavy ion collisions is due to partons that carry a small momentum fraction $x$.
in the incoming nuclei. Because of the rise of the gluon distribution at small $x$, the density of such partons is high, leading to the phenomenon of saturation: the gluon phase-space density cannot grow larger than $1/\alpha_s$ due to the repulsive interactions among the gluons. Thus, the gluons at low $x$ must occupy higher $p_{\perp}$ modes: at a given $x$, all the modes up to $p_{\perp} = Q_{\text{sat}}(x)$ are occupied with a phase-space density of the order of $1/\alpha_s$. $Q_{\text{sat}}(x)$, the saturation momentum, grows like $x^{-0.3}$ at small $x$. It also depends on the nuclear atomic number like $A^{1/3}$, which means than nuclei have a larger saturation momentum than protons at a given $x$.

![Image](image.png)

Fig. 7. Left: saturation domain. Right: typical particle production process in the Color Glass Condensate approach.

In the saturated regime ($Q^2 \leq Q_{\text{sat}}^2(x)$ in the diagram on the left of Fig. 7), gluon recombination and multiple scatterings play a crucial role in the mechanisms of particle production (see the right panel of Fig. 7). The Color Glass Condensate [2] is an effective theory in which these effects are systematically taken into account, based on the separation of the degrees of freedom into color fields (that represent the low $x$ partons), and static color sources (that represent the large $x$ partons). In particular, the single inclusive spectrum of the gluons and quarks produced in the collision of two heavy ions have been computed at leading order [7] in this framework. Work is under way in order to evaluate the NLO corrections to the gluon spectrum, and to establish a factorization theorem for nucleus–nucleus collisions in the saturated regime [7]. Note that these calculations only give the distribution of produced particles at a short time after the impact (of the order of $\tau \sim Q_{\text{sat}}^{-1}$), which can then be used in order to model the initial conditions for the subsequent hydrodynamical evolution.

These studies of the initial conditions in heavy ion collisions have also led to an interesting development, which is expected to have connections with the issue of thermalization. It has been noted that the classical color field configurations encountered in the CGC formalism at leading order are unstable: small rapidity dependent perturbations grow exponentially in time.
Fig. 8. Left: field lines at leading order. Right: effect of the instability. (The horizontal axis represents the space-time rapidity, and the vertical planes the Lorentz contracted nuclei.)

and eventually become as large as the classical field itself [8]. It turns out that perturbations of this kind are generated by quantum fluctuations [9], and therefore occur naturally among the NLO corrections. These quantum fluctuations, amplified by the instability, would lead to extremely disordered configurations of strong fields (see Fig. 8). Particles moving in such a medium would also have a very short mean free path, and the viscosity would be close to the uncertainty lower bound [10]. This opens up an interesting alternative to the strongly coupled scenarios: the quasi-ideal fluidity inferred from RHIC data may be due to strong disordered color fields, even if the coupling is rather weak.

REFERENCES