PROTON–NEUTRON RANDOM PHASE APPROXIMATION STUDIED BY THE LIPKIN–MESHKOV–GLICK MODEL IN THE SU(2)×SU(2) GROUP*

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(Received January 10, 2017)

We study the proton–neutron RPA with an extended Lipkin–Meshkov–Glick model. We pay attention to the effect of correlated ground state and the case when neutron and proton numbers are different. The effect of the correlated ground state is tested on the basis of quasi-boson approximation. We obtain the result that the RPA excitation energies and transition strengths are in a good agreement with the exact solution up to a certain strength of the particle–particle interaction. However, the transition strength shows deviations from the exact solution if we consider the case in which neutron and proton numbers are different even at a weak particle–particle interaction.

DOI:10.5506/APhysPolBSupp.10.131

1. Introduction

The random phase approximation (RPA) is one of the useful approaches to describe a collective motion of nuclei and helps us to understand the basic mechanism of nuclear excitations. Its application to charge exchange reaction is widely used in calculations of neutrino–nucleus reactions [1], β-decay [2] and isospin symmetry breaking [3]. The RPA is able to provide basic physical insights of nuclear excitations by its simple picture of coherent 1 particle–1 hole (1p1h) excitations on the one hand, it does not describe coupling to more complicated states, like phonon coupling as well as multi-particle multi-hole states on the other hand. Therefore, several approaches beyond the RPA have been also studied, for example, particle-vibration coupling [4], finite-rank separable approximation [5], second RPA [6, 7] and the Tamm–Dancoff-approximation (TDA) [8].

* Presented at the XXIII Nuclear Physics Workshop "Marie and Pierre Curie", Kazimierz Dolny, Poland, September 27–October 2, 2016.

To take into account the higher order correlation beyond the RPA, there is another approach which focuses on the ground state. When one derives the standard RPA, the RPA ground state, namely, the correlated ground state, is replaced with the Hartree–Fock (HF) one. This prescription omits a part of the multi-particle multi-hole effects. In this respect, several extensions of the RPA to include the correlation have been studied. Renormalized RPA [9], which considers renormalized single particle states invoked by a correlated nuclear ground state, is a leading example. While using the correlated ground state is more practical to describe the nuclear collective vibration than using the HF ground state, it is pointed out that the ground state correlation is not so significant in the case of the charge exchange reactions. This is because proton and neutron have different Fermi energies and occupy different shells. This might be true for heavy $N > Z$ nuclei. In fact, the proton–neutron RPA calculation for $N > Z$ nuclei shows almost the same result as the proton–neutron TDA calculation, which implies the ground state correlation is weak enough. However, we should keep in mind that the RPA for a specific transition does not take into account all the ground state correlation.

In order to check the validity of the uncorrelated ground state, it would be one of the reasonable ways to study the RPA with an exact solvable model before investigating a practical case. The Lipkin–Meshkov–Glick (LMG) model [10] can be a good tool for this end because it enables us to compare the model with the exact solution in a simple model space. It has been widely used so far to validate various kinds of models with interests [11–14]. To check the validity of the RPA in the case of charge exchange reactions, the LMG model on the SU(2)×SU(2) basis was studied by Stoica's group [15, 16]. According to their results, the RPA works well in case of a large nucleon number system, if the particle–particle interaction is relatively weak enough. They also considered the effect of correlated ground state up to the first order [17]. In this formalism, the ground and excited states of mother and daughter nuclei are first calculated with the RPA, and then the transition between them are considered. They compared the transition strengths calculated by the RPA on the basis of the correlated ground state with the exact solutions, and showed that the RPA works reasonably well. Then the next question is whether the same result can be obtained in the case of the proton–neutron RPA. It should be mentioned that reliability of the RPA and quasi-particle RPA (QRPA) as well as renormalized QRPA for charge-exchange reactions has been also investigated in several different ways [18–21].

In this work, we present the effect of the correlated ground state characterized by the phonon operator of the proton–neutron RPA with the LMG model on the SU(2)×SU(2) basis. What is different from Ref. [17] is that charge exchange phonon creation operators are used to construct the excited and the correlated ground states. We particularly pay attention to nuclei with different neutron and proton numbers. Our formalism is based on the work of Ref. [16], however they did not investigate the effect of the correlated ground state. As shown in the next section, we obtain the different result from $N = Z$ nuclei in the case of $N \neq Z$. A very similar work has been performed in the case of SO(5) group [21, 22], but the present work using SU(2)×SU(2) will give another insight into the effect of the correlated ground state.

This paper organizes as follows. Section 2 describes our formalism briefly. Section 3 shows the result and compares the RPA with the exact one and Sec. 4 gives summary of this paper.

2. Calculation

Our model is almost the same as the work of Ref. [16]. However, we would like to describe some key points briefly. We use the SU(2)×SU(2) group algebra characterized by $T_+^{(1)}, T_-^{(1)}, T_z^{(1)}, T_+^{(2)}, T_-^{(2)}, T_z^{(2)}$ defined in [16]. Let us consider two levels each for proton and neutron. As defined in Ref. [16], $p+(n+)$ and $p-(n-)$ are the symbols representing the higher and the lower levels of proton (neutron). The Hamiltonian considered in this work is

$$H = \epsilon(T_z(1) + T_z(2)) + V_{pn}\left(T_+^{(1)}T_+^{(2)} + T_-^{(2)}T_-^{(1)}\right) + W_{pn}\left(T_+^{(1)}T_-^{(1)} + T_+^{(2)}T_-^{(2)}\right), \tag{1}$$

where ϵ is the energy difference between the lower and higher levels of proton and neutron. The third and forth terms of Eq. (1) are the particle–particle and particle–hole interactions. To diagonalize the Hamiltonian, we consider the following basis as the set of the eigenvectors,

$$|\mu\rangle = \left|T^{(1)}, T_z^{(1)}\right\rangle \otimes \left|T^{(2)}, T_z^{(2)}\right\rangle , \tag{2}$$

where the index μ stands for $\mu = (T_z^{(1)}, T_z^{(2)})$. $T^{(1)} = N_n/2$ and $T^{(2)} = N_p/2$, where N_n and N_p are the neutron and proton numbers. The uncorrelated ground state is then given by $|0\rangle \equiv |T^{(1)}, -T^{(1)}\rangle \otimes |T^{(2)}, -T^{(2)}\rangle$. The Hamiltonian given in Eq. (1) can be exactly diagonalized by the linear combination,

$$|\Psi_i\rangle = \sum_\mu c_{\mu i}|\mu\rangle , \tag{3}$$

where i stands for eigenstates. The RPA formalism is also the same as in Ref. [16]. To take into account the correlated ground state, we follow the same prescription as [9, 11, 17]. Up to the first order, it is given by

$$|\mathrm{RPA}\rangle \sim N_0\left(1 - \frac{1}{2N}\sqrt{\frac{\epsilon - \Omega}{\epsilon + \Omega}}\Theta^\dagger\Theta^\dagger\right)|0\rangle , \tag{4}$$

where $N = N_n + N_p$, $\Omega > 0$ is the eigenvalue of the RPA equation, and N_0 is the normalization factor satisfying $\langle \mathrm{RPA}|\mathrm{RPA}\rangle = 1$. The second term of Eq. (4) takes into account 1 proton particle 1 neutron particle–1 proton hole 1 neutron hole $[\pi\nu(\pi\nu)^{-1}]$ configurations in addition to the 0 particle–0 hole configuration appearing in the first term[1]. In what follows, we refer to results using Eq. (4) as RPA(corr.). The transition strength for β^- transition in RPA is then given by

$$T_- = \left|\left\langle 1\left|M^+\right|\mathrm{RPA}\right\rangle\right|^2 \sim \left|\left\langle 1\left|M^+\right|0\right\rangle\right|^2 , \tag{5}$$

where $|1\rangle = \Gamma^\dagger|\mathrm{RPA}\rangle \sim \Gamma^\dagger|0\rangle$ and the transition operator $M^+ = \chi^+ \sum_{i,j,\sigma,\sigma'} a^\dagger_{p_i\sigma} a_{n_j\sigma'}$ is given in [16]. The phonon operator $\Gamma^\dagger$ is given by Eq. (8) of [16] in the case of the RPA, and denominator is replaced by $\sqrt{\langle \mathrm{RPA}|[\Theta^-, \Theta^+]|\mathrm{RPA}\rangle}$ in the case of RPA(corr.). The second and third equations of Eq. (5) correspond to that of the RPA(corr.) and RPA, respectively. Similarly, the transition strength for β^+ transition is given by

$$T_+ = |\langle 1|M^-|\mathrm{RPA}\rangle|^2 \sim |\langle 1|M^-|0\rangle|^2 . \tag{6}$$

3. Result

First of all, we discuss the case in which neutron and proton numbers are the same. Figure 1 shows the excitation energy of $N_n = N_p = 5$ (the left panel) and $N_n = N_p = 20$ (the right panel). We set the model parameter of the particle–hole interaction as $NW_{pn} = -0.2$. We also compare our result with the TDA which can be obtained in the RPA by setting the backward amplitude $Y = 0$. The RPA and RPA(corr.) results show a similar curve to the exact one at a small NV_{pn}. At $NV_{pn} \sim 1.8$ (critical point), both the RPA and RPA(corr.) collapse due to the phase transition, however, RPA(corr.) shows a larger critical point than the RPA. The RPA(corr.) result is closer to the exact one than RPA one, both for $N_n = N_p = 5$ and $N_n = N_p = 20$, but the difference between RPA and RPA(corr.) is smaller in the case of $N_n = N_p = 20$. Namely, the effect of the correlated ground state becomes not so significant for nuclei with larger number for wide range of NV_{pn}. The TDA, which shows the constant straight line as a function of NV_{pn}, deviates both from RPA and the exact solution above approximately $NV_{pn} = 0.2$. This result means that the ground state correlation resulted from the particle–particle interaction is important, as already mentioned in Ref. [16].

[1] Equation (4) also includes 2 proton particles–2 neutron holes and 2 neutron particles–2 proton holes configurations. However, they are not important because the Hamiltonian of Eq. (1) does not allow to form such a configuration in the ground state.

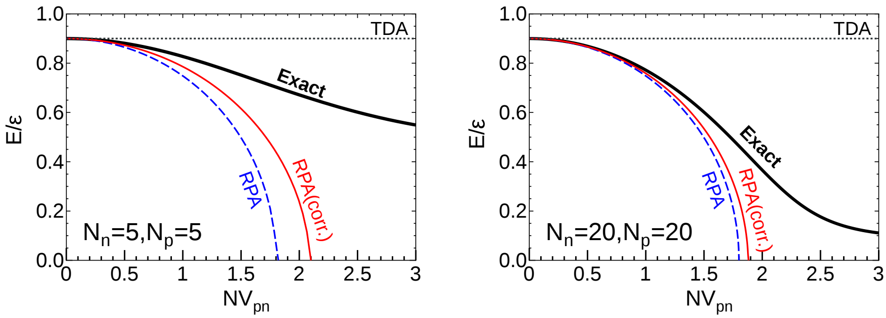

Fig. 1. Excitation energies in the case of $N_n = N_p = 5$ (left) and $N_n = N_p = 20$ (right) as a function of NV_{pn}. The thick solid, thin solid, dashed, and dotted lines are the results for the exact, RPA(corr.), RPA, and TDA.

Figure 2 shows the transition strengths of the system of $N_n = N_p = 5$ (the left panel) and $N_n = N_p = 20$ (the right panel). Both the RPA and RPA(corr.) show a similar result to the exact solution from $NV_{pn} = 0$ to ~ 1.0. Above $NV_{pn} = 1.0$, the RPA collapses rapidly due to the phase transition. The RPA(corr.) also collapses at a higher NV_{pn} than the RPA. The difference between them is, however, not as large as the excitation energies shown in Fig. 1. Namely, the effect of the correlated ground state is not significant both for small and large nuclei for a wide range of NV_{pn}. Again, the TDA shows a large deviation from the RPA and the exact solution, similar to the excitation energies.

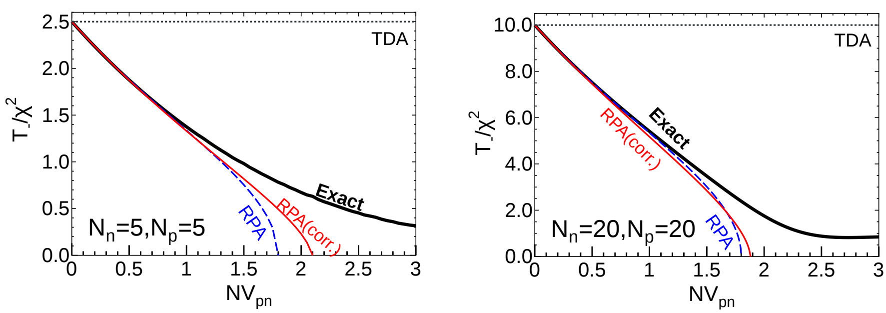

Fig. 2. The same as Fig. 1, but for transition strengths.

Next, we discuss the case of $N_n \neq N_p$. We keep $N_p = 20$ and vary the neutron number from $N_n = 24$ to 32. The results are shown in Fig. 3. The left and right panels illustrate the excitation energies and the transition strengths, respectively. The difference of the excitation energy between the RPA, RPA(corr.) and the exact solution does not change significantly even if we change the N_n. The variations of the critical points of the RPA and RPA(corr.) are also small between different N_n. However, the result of the transition strength shows a different tendency from the excitation energy. Looking at the right panels, the difference between the RPA and the exact solution becomes larger as N_n increases. The difference already starts at a small NV_{pn} in the case of $N_n = 32$. Let us remind that in the case of $N_n = N_p$, the RPA and RPA(corr.) showed a good agreement with the exact solution as seen in Fig. 2. The RPA(corr.) remedies the RPA result to some extent, but the difference from the exact one can be still seen.

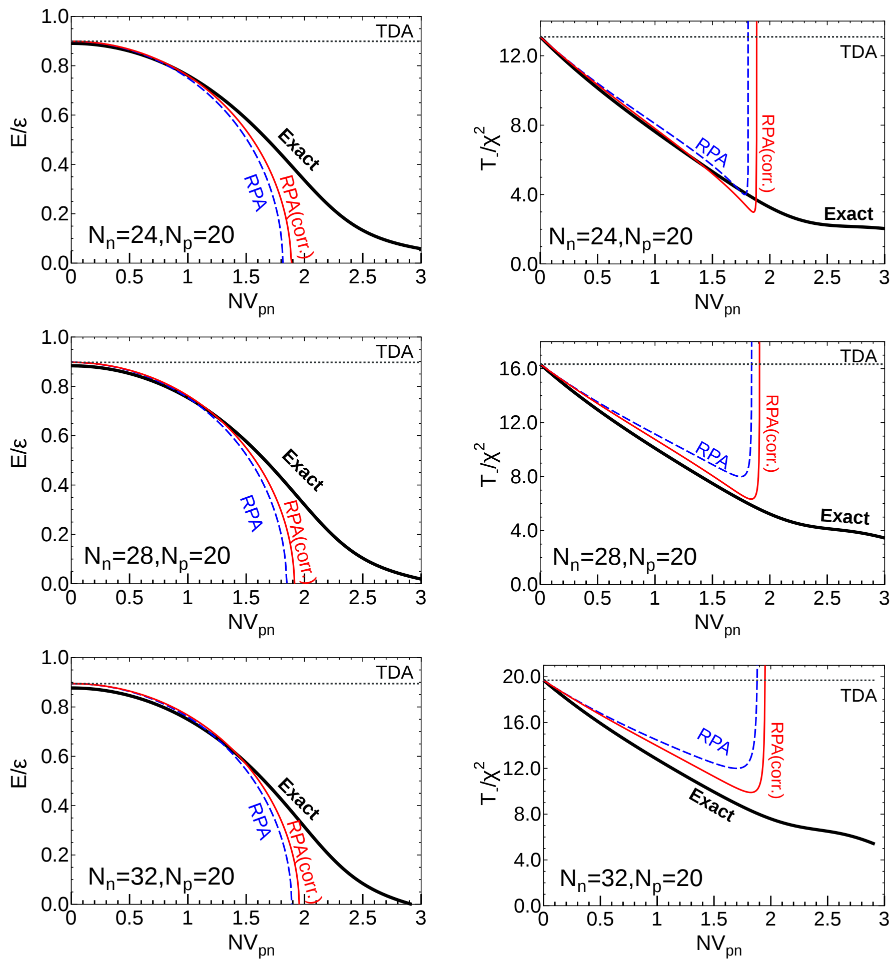

Fig. 3. Excitation energies (left panels) and transition strengths (right panels) in the case of $N_n = 24$ (top), $N_n = 28$ (middle) and $N_n = 32$ (bottom) as a function of NV_{pn}. The thick solid, thin solid, dashed, and dotted lines are the results for the exact, RPA(corr.), RPA, and TDA, respectively.

We have also investigated the non-energy weighted sum-rule of charge exchange reaction defined by $T_- - T_+$. The result is shown in Fig. 4. While the RPA perfectly satisfies the total sum-rule, which must be equal to $N_n - N_p$, up to the critical points, RPA(corr.) does not. The reason would be attributed to the fact that the phonon creation operator $\Gamma^\dagger$ does not consider the transition from the excited single particle states, as discussed in Ref. [11]. The exact solution also does not seem to satisfy the sum-rule. However, it satisfies the total sum-rule if we include the transition to other excited states besides the first one, which cannot be treated in the RPA in two-level model. It is clear that the difference between the RPA and the exact solution becomes large when we consider the $N_n \neq N_p$ case. Analyzing the exact solution, transition to 2 proton particles–1 proton hole 1 neutron hole $[\pi^2(\pi\nu)^{-1}]$ configurations from the ground state becomes important, which cannot be connected to M^+ operator from the correlated ground state given by Eq. (4). It is expected that the second RPA, which enables us to include such 2p2h configurations, can improve the result.

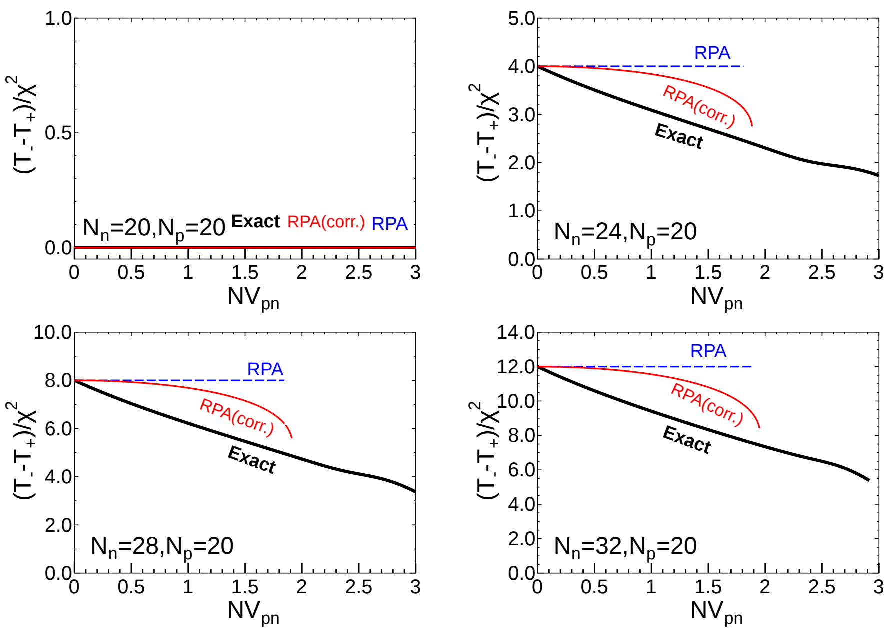

Fig. 4. The non-energy weighted sum-rule, $T_- - T_+$ for the first excited states. The thick solid, thin solid and dashed lines are the results for the exact, RPA(corr.), and RPA.

4. Summary

We investigated the validity of proton–neutron RPA with the LMG model in SU(2)×SU(2) group. In the case when neutron and proton numbers are the same, the RPA and RPA(corr.) works well both for small and large nuclei when the particle–particle interaction is weak. If the particle–particle interaction becomes strong, the RPA and RPA(corr.) results begin to deviate from the exact solution. On the other hand, the transition strengths are still reproduced well. This situation changes in the case when neutron and proton numbers are different. The excitation energies are reproduced reasonably up to $NV_{pn} \sim 1.5$, but the transition strengths are not. It turned out that the 2p2h configurations, which cannot be covered by the correlated ground state used in the present formalism, start to become important from a small NV_{pn} value. It is expected that the extension of the model to include such a 2p2h configuration can reduce the difference between the RPA and the exact solution. The work on this problem is now in progress.

This work was supported by JSPS KAKENHI grant No. 25400303.

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