Vol. 10 (2017) *Acta Physica Polonica B Proceedings Supplement* No 1

FRAGMENT MASS DISTRIBUTIONS IN LOW-ENERGY FISSION OF ^{236}Pu* **

K. Pomorski, B. Nerlo-Pomorska

Department of Theoretical Physics, Maria Curie Skłodowska University
Radziszewskiego 10, 20-031 Lublin, Poland

J. Bartel

Université de Strasbourg, CNRS IPHC UMR 7178, 67000 Strasbourg, France

C. Schmitt

GANIL, 14000 Caen, France

(Received January 10, 2017)

The fission-fragment mass distribution is evaluated in a quantum mechanical framework using mass asymmetry, neck and elongation as the relevant collective degrees of freedom. The potential energy surfaces (PES) are calculated within the macroscopic–microscopic model based on the Lublin–Strasbourg Drop (LSD), the Yukawa-folded (YF) single-particle potential and a monopole pairing force. The PES is presented and analysed in detail for the isotope ^{236}Pu, which reveals a deep asymmetric valley. The fission-fragment mass distribution is obtained from the eigenstates of a collective Hamiltonian computed within the Born–Oppenheimer approximation (BOA), applying the WKB approximation and introducing a neck-dependent fission probability. For spontaneous fission of ^{236}Pu, the calculated mass distribution is found in a good agreement with the data.

DOI:10.5506/APhysPolBSupp.10.183

* Presented at the XXIII Nuclear Physics Workshop "Marie and Pierre Curie", Kazimierz Dolny, Poland, September 27–October 2, 2016.

** This work has been partly supported by the Polish–French COPIN-IN2P3 collaboration agreement under project number 08-131 and by the Polish National Science Center (NCN), grant No. 2013/11/B/ST2/04087.

1. Introduction

Using the recently developed Fourier parametrisation of deformed nuclear shapes [1], shown to be very rapidly converging, we have made a first attempt to obtain the fission-fragment mass distribution of even–even plutonium isotopes with mass numbers $236 \leq A \leq 246$. Three collective coordinates corresponding to elongation (q_2), left–right asymmetry (q_3) and neck-size (q_4) were considered in our analysis. A non-axial degree of freedom (see [1]) was not included in the present investigation since we are dealing here with very elongated systems. The potential energy surfaces of different fissioning nuclei were calculated within the macroscopic–microscopic method [2]. The fragment mass distribution obtained in low-energy fission of light actinides was evaluated in a quantum mechanics framework within the Born–Oppenheimer approximation [3]. The fission yield was obtained from the probability distribution of the collective wave function on the (q_3, q_4) plane in the vicinity of the scission configuration ($q_2 \approx 2$). A neck-size-dependent fission probability [4] was used to evaluate the mass yields from the distribution probability at different elongations of the fissioning nucleus. Due to the limited length of the present contribution, we present below only the results for the lightest isotope, ^{236}Pu.

2. Model

The axial symmetric shape-profile function of a fissioning nucleus written in cylindrical coordinates (ρ, z) is expanded in a Fourier series [1]

$$\frac{\rho^2(u)}{R_0^2} = a_2 \cos(u) + a_3 \sin(2u) + a_4 \cos(3u) + a_5 \sin(4u) + a_6(5u) + \ldots\,, \quad (2.1)$$

where R_0 is the radius of spherical nucleus and $u = \pi/2(z - z_{\rm sh})/z_0$ with $-z_0 + z_{\rm sh} \leq z \leq z_0 + z_{\rm sh}$. The volume conservation condition gives $z_0 = R_0\pi/(a_2 - a_4/3 + a_6/5 - \ldots)/3$. The shift coordinate $z_{\rm sh}$ ensures that the centre of mass is located at the origin of the coordinate system.

It was shown in Ref. [1] that the liquid drop (LD) path to fission proceeds towards decreasing values of a_2 and growing negative values of a_4. It is, therefore, convenient to introduce new, physically more intuitive, collective coordinates which ensure an optimal presentation of the potential energy landscape

$$q_2 = a_2^{(0)}\Big/a_2 - a_2\Big/a_2^{(0)}\,, \qquad q_3 = a_3\,, \qquad q_4 = a_4 + \sqrt{(q_2/9)^2 + \left(a_4^{(0)}\right)^2}\,,$$

$$q_5 = a_5 - a_3(q_2 - 2)/10\,, \qquad q_6 = a_6 - \sqrt{(q_2/100)^2 + \left(a_6^{(0)}\right)^2}\,, \quad (2.2)$$

where $a_2^{(0)} = 1.03205$, $a_4^{(0)} = -0.03822$, and $a_6^{(0)} = 0.00826$ are the expansion coefficients of a sphere.

The bottom of the LD fission valley corresponds roughly to $q_4 = q_6 = 0$, and the definition of q_5 and q_6 ensures the smallest stiffness of the LD energy towards q_3 and q_4, respectively, when $q_5 = q_6 = 0$.

In these coordinates, the collective Hamiltonian has the following form:

$$\widehat{H}_{\rm coll} = -\frac{\hbar^2}{2}\sum_{i,j}|M|^{-1/2}\frac{\partial}{\partial q_i}|M|^{-1/2}M_{ij}^{-1}(\{q_i\})\frac{\partial}{\partial q_j} + V(\{q_i\})\,, \tag{2.3}$$

where $M_{ij}(\{q_i\})$ and $V(\{q_i\})$ denote the inertia tensor and the potential energy, respectively, and $|M| = \det(M_{ij})$.

The eigenproblem of this Hamiltonian can be solved in the BOA in which one assumes that the motion towards fission is much slower than the one in the two other collective coordinates. This implies that the eigenfunction of $\widehat{H}_{\rm coll}$ can be approximated by the following product:

$$\Psi_{nE}(q_2, q_3, q_4) = u_{nE}(q_2)\,\varphi_n(q_3, q_4; q_2)\,. \tag{2.4}$$

Here, $u_{nE}(q_2)$ is the wave function for the fission mode and φ_n are the eigenfunctions of the Hamiltonian which describe the collective motion perpendicular to the fission mode. In the following, we shall take the WKB approximation for the $u_{nE}(q_2)$ wave function and consider only the lowest energy eigenstate $\varphi_{n=0}$ since we are interested in fission at a very low excitation energy, and we are going to compare the calculation with experimental measurements involving spontaneous fission. The effect of taking into account higher states was discussed in Ref. [3].

The probability of finding a system, for a given q_2 value, in a defined (q_3, q_4) point is equal to

$$W(q_3, q_4; q_2) = |\Psi(q_3, q_4; q_2)|^2 = |\varphi_0(q_3, q_4; q_2)|^2\,. \tag{2.5}$$

Our model is still simplified further and instead of the square of the collective wave function (2.5), we take the following Wigner function:

$$W(q_3, q_4; q_2) \sim \exp\left\{-\frac{V(q_3, q_4; q_2) - V_{\rm eq}(q_2)}{E_0}\right\}, \tag{2.6}$$

where $V_{\rm eq}(q_2)$ is the potential minimum for a given elongation q_2 and E_0 is the zero-point energy treated here as a free parameter.

The probability distribution integrated over q_4

$$w(q_3; q_2) = \int W(q_3, q_4; q_2)\,{\rm d}q_4 \tag{2.7}$$

is directly related to the fragment mass yield at given elongation q_2.

It is obvious that the fission probability should depend on the neck radius. Following Ref. [4], we assume the neck-rupture probability P in the form of

$$P(q_3,q_4,q_2) = \frac{k_0}{k} P_{\rm neck}(\kappa)\,, \tag{2.8}$$

where k is the momentum in the direction towards fission (or simply the velocity along q_2), while $\kappa = \kappa(q_3,q_4,q_2)$ is the deformation-dependent neck radius (in R_0 units). k_0 plays a role of a scaling parameter. The neck-rupture probability is taken in the form of a Fermi function [4]

$$P_{\rm neck}(\kappa) = \left(1+\exp\left(\frac{\kappa-\kappa_0}{d}\right)\right)^{-1}. \tag{2.9}$$

The parameter $\kappa_0 = 0.16$ is equal to the nucleon size in R_0 units and $d = 0.01$ is fixed by comparing the theoretical fission-fragment mass distribution of ^{236}Pu with the experimental data [8]. The momentum k in Eq. (2.8) has to ensure that the probability depends on the time in which one crosses the subsequent interval in q_2: $\Delta t = \Delta q_2/v(q_2)$, where

$$v(q_2) = \hbar k/\bar{M}(q_2) \tag{2.10}$$

is the velocity towards fission. The inertia $M(q_2)$ is evaluated using the approximation proposed in Ref. [6]

$$\bar{M}(q_2) = \mu\,[1+11.5\,(B_{\rm irr}-1)]\left(\frac{\partial R_{12}}{\partial q_2}\right)^2, \tag{2.11}$$

where $B_{\rm irr}$ is the irrotational inertia corresponding to the distance between the fragments R_{12} and μ is the reduced mass. The value of k in Eq. (2.10) depends on the difference $E-V(q_2)$ and on the part of the collective energy which is converted into heat Q

$$\frac{\hbar^2 k^2}{2\bar{M}(q_2)} = E_{\rm kin} = E - Q - V(q_2)\,. \tag{2.12}$$

Here, we assume that $Q = 0$, *i.e.* we neglect the nuclear dissipation what is a reasonable approximation in low energy fission.

The fission probability (2.7) will be given by the integral

$$w(q_3,q_2) = \int W(q_3,q_4;q_2)P(q_3,q_4,q_2)\,{\rm d}q_4\,. \tag{2.13}$$

Such an approach means that the fission process will be spread over some region of q_2 and that for a given q_2, at fixed mass asymmetry, one has to take into account the probability to fission at a previous q_2 value, *i.e.* one has to replace $w(q_3, q_2)$ by

$$w'(q_3, q_2) = w(q_3, q_2) \left[1 - \int\limits_{q_2' \le q_2} w\left(q_3, q_2'\right) \, \mathrm{d}q_2' \Big/ \int w\left(q_3, q_2'\right) \, \mathrm{d}q_2' \right] . \quad (2.14)$$

The integral mass yield will be the sum of all partial yields at different q_2

$$Y(q_3) = \int w'\left(q_3, q_2\right) \mathrm{d}q_2 \Big/ \int w'(q_3, q_2) \, \mathrm{d}q_2 \, \mathrm{d}q_3 \, . \quad (2.15)$$

As it is seen from (2.15), the scaling factor k_0 in the expression for P, Eq. (2.8), has vanished and does not appear any more in the definition of the mass yield. Our model will thus have only two adjustable parameters, κ_0 and d, that appear in the neck-rupture probability (2.9).

3. Results

The potential energy surfaces were calculated within the macroscopic–microscopic model using the Lublin–Strasbourg Drop [5] for the macroscopic part, while the microscopic part was evaluated as the sum of the Strutinsky shell and the BCS pairing correction obtained using the single-particle energies of the Yukawa-folded Hamiltonian [7].

The deformation energy landscape on the (q_2, q_3) plane is shown for ^{236}Pu in Fig. 1. Each point on the PES was minimized with respect to q_4. At large elongations q_2, a pronounced valley corresponding to asymmetric fission ($q_3 \neq 0$) and a shorter one for the symmetric splitting ($q_3 = 0$) are visible. The cross section of this map at elongation $q_2 = 2.05$ is presented in Fig. 2 on the plane (A_f, q_4), where A_f is the mass-number of the heavier

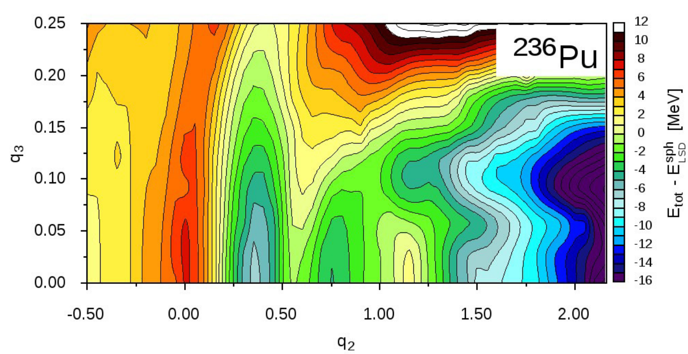

Fig. 1. Potential energy surface of ^{236}Pu on the (q_2, q_3) plane.

fragment. One identifies two minima: one deeper asymmetric around $A_\mathrm{f} = 140$ and the other one corresponding to the symmetric fission. Our estimate (Eq. (2.15)) of the fission fragment mass distribution is compared in Fig. 3 with the experimental yield taken from Ref. [8].

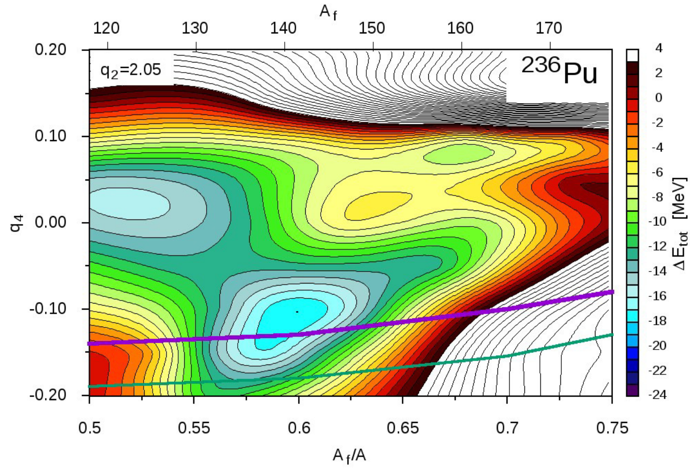

Fig. 2. Potential energy surface of ^{236}Pu on the (A_f, q_4) plane at elongation $q_2 = 2.05$. The black thick/violet line corresponds to the neck radius $r_\mathrm{nk} = 2$ fm, while the gray thin/green one to $r_\mathrm{nk} = 1$ fm.

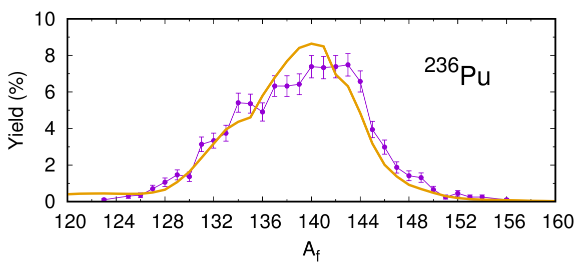

Fig. 3. Experimental fission fragment mass yield for the spontaneous fission of ^{236}Pu [8] compared with our estimate (2.6) for $E_0 = 2$ MeV.

4. Summary

We have shown that the three-dimensional quantum mechanical model which couples the fission, neck and mass asymmetry modes is able to reproduce the main features of the fragment mass distribution when a neck-dependent fission probability is taken into account. Preliminary results for the plutonium isotopes also show that our model gives the fission fragment mass yields close to the data. Further calculations are in progress.

REFERENCES

[1] K. Pomorski, B. Nerlo-Pomorska, J. Bartel, C. Schmitt, *Acta Phys. Pol. B Proc. Suppl.* **8**, 667 (2015).

[2] J. Bartel, K. Pomorski, B. Nerlo-Pomorska, C. Schmitt, *Phys. Scr.* **90**, 114004 (2015).

[3] B. Nerlo-Pomorska, K. Pomorski, F.A. Ivanyuk, *Acta Phys. Pol. B Proc. Suppl.* **8**, 659 (2015).

[4] K. Pomorski, B. Nerlo-Pomorska, F.A. Ivanyuk, `arXiv:1607.00353 [nucl-th]`.

[5] K. Pomorski, J. Dudek, *Phys. Rev. C* **67**, 044316 (2003).

[6] J. Randrup *et al.*, *Phys. Rev. C* **13**, 229 (1976).

[7] A. Dobrowolski, K. Pomorski, J. Bartel, *Comput. Phys. Commun.* **199**, 118 (2016).

[8] L. Dématté *et al.*, *Nucl. Phys. A* **617**, 331 (1997).