**SU(2N_F) SYMMETRY OF QCD AT HIGH TEMPERATURE AND ITS IMPLICATIONS**

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If above a critical temperature not only the SU($N_F$)$_L \times$ SU($N_F$)$_R$ chiral symmetry of QCD but also the U(1)$_A$ symmetry is restored, then the actual symmetry of the QCD correlation functions and observables is SU(2$N_F$). Such a symmetry prohibits existence of deconfined quarks and gluons. Hence, QCD at high temperature is also in the confining regime and elementary objects are SU(2$N_F$) symmetric “hadrons” with not yet known properties.

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## 1. Introduction

Nonperturbatively QCD is defined in terms of its fundamental degrees of freedom, quarks and gluons in the Euclidean space-time. These fundamental degrees of freedom are never observed in the Minkowski space, a property of QCD which is called confinement. Only hadrons are observed. It is believed, however, that at high temperature, QCD is in a deconfinement regime and its fundamental degrees of freedom, quarks and gluons, are liberated. Is it true? Here, we present results of our recent findings [1] that suggest that this is actually not true.

In the Minkowski space-time, the QCD Lagrangian in the chiral limit is invariant under the chiral transformations

\[ SU(N_F)_L \times SU(N_F)_R \times U(1)_A \times U(1)_V. \]  

The axial U(1)$_A$ symmetry is broken by anomaly [2]. The SU($N_F$)$_L \times$ SU($N_F$)$_R$ symmetry is broken spontaneously by the quark condensate in

the vacuum. According to the Banks–Casher relation [3], the quark condensate in the Minkowski space can be expressed through a density of the near-zero modes of the Euclidean Dirac operator

$$\lim_{m \to 0} \langle 0 | \bar{\Psi}(x) \Psi(x) | 0 \rangle = -\pi \rho(0). \quad (2)$$

Consequently, if we remove by hands the near-zero modes of the Dirac operator, we can expect a restoration of the chiral SU$_{L}^{(N_F)} \times$ SU$_{R}^{(N_F)}$ symmetry in correlation functions. If hadrons survive this “surgery”, then the chiral partners should become degenerate. The chiral partners of the $J = 1$ mesons are shown in Fig. 1.

$$\begin{align*}
(0, 0) & \quad f_1(0, 1^{++}) & \quad \omega(0, 1^{--}) \\
(1/2, 1/2)_a & \quad \tilde{b}_1(1, 1^{+-}) & \quad \omega(0, 1^{--}) \\
(1/2, 1/2)_b & \quad \rho(1, 1^{--}) & \quad \tilde{h}_1(0, 1^{+-}) \\
(1, 0) \oplus (0, 1) & \quad \rho(1, 1^{--}) & \quad \tilde{a}_1(1, 1^{++})
\end{align*}$$

Fig. 1. SU$_{L}^{(2)} \times$ SU$_{R}^{(2)}$ and U$(1)_A$ classification of the $J = 1$ meson operators.

It was observed in $N_F = 2$ dynamical simulations with the overlap Dirac operator that, indeed, hadrons survive this truncation (except for the ground states of $J = 0$ mesons) and the chiral partners get degenerate [4–7]. Not only the SU$_{L}^{(2)} \times$ SU$_{R}^{(2)}$ restoration was observed. Mesons that are connected by the U$(1)_A$ transformation get also degenerate. We conclude that the same low-lying modes of the Dirac operator are responsible for both SU$_{L}^{(2)} \times$ SU$_{R}^{(2)}$ and U$(1)_A$ breakings, which is consistent with the instanton-induced mechanism for both breakings [8].

Restoration of the full chiral symmetry SU$_{L}^{(2)} \times$ SU$_{R}^{(2)} \times$ U$(1)_A$ of the QCD Lagrangian assumes degeneracies marked by arrows in Fig. 1. However, a larger degeneracy that includes all possible chiral multiplets in Fig. 1 was detected, see Fig. 2.

This unexpected degeneracy implies a symmetry that is larger than the chiral symmetry of the QCD Lagrangian. This not yet known symmetry was reconstructed in Refs. [9, 10] and turned out to be

$$\text{SU}(2N_F) \supset \text{SU}(N_F)_L \times \text{SU}(N_F)_R \times \text{U}(1)_A. \quad (3)$$
This group includes as a subgroup the SU(2)_{CS} (chiral spin) invariance. The SU(2)_{CS} chiral spin generators are

\[ \Sigma = \{ \gamma^0, i\gamma^5\gamma^0, -\gamma^5 \}, \quad [\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk} \Sigma^k. \]

The Dirac spinor transforms under a global or local SU(2)_{CS} transformation as

\[ \Psi \rightarrow \Psi' = e^{i\frac{\Sigma}{2}} \Psi. \]

The SU(4) symmetry of \( N_F = 2 \) Euclidean QCD was obtained in lattice simulations. This means that this symmetry must be encoded in the nonperturbative Euclidean formulation of QCD. Obviously, the Euclidean Lagrangian for \( N_F \) degenerate quarks in a given gauge background \( A_\mu(x) \)

\[ \mathcal{L} = \Psi\dagger(x)(\gamma_\mu D_\mu + m)\Psi(x) \]

is not SU(2)_{CS} and SU(2N_F)-symmetric, because the Dirac operator does not commute with the generators of SU(2)_{CS}. A fundamental dynamical
reason for the absence of these symmetries are zero modes of the Dirac operator, $\gamma_\mu D_\mu \Psi_0(x) = 0$. The zero modes are chiral, L or R. With a gauge configuration of a nonzero global topological charge, the number of the left-handed and right-handed zero modes is, according to the Atiyah–Singer theorem, not equal. Consequently, there is no one-to-one correspondence of the left- and right-handed zero modes. The SU(2)$_{\text{CS}}$ chiral spin rotations mix the left- and right-handed Dirac spinors. Such a mixing can be defined only if there is a one-to-one mapping of the left- and right-handed spinors: The zero modes break the SU(2)$_{\text{CS}}$ invariance.

We can expand independent fields $\Psi(x)$ and $\Psi^\dagger(x)$ over a complete and orthonormal set $\Psi_n(x)$ of the eigenvalue problem

$$i \gamma_\mu D_\mu \Psi_n(x) = \lambda_n \Psi_n(x),$$  
$$\Psi(x) = \sum_n c_n \Psi_n(x), \quad \Psi^\dagger(x) = \sum_k \bar{c}_k \Psi^\dagger_k(x),$$

where $\bar{c}_k, c_n$ are Grassmann numbers. The fermionic part of the QCD partition function takes the following form:

$$Z = \int \prod_k \prod_n d\bar{c}_k d c_n e^{\sum_k, n \int d^4 x \bar{c}_k c_n \left( \lambda_n + im \right) \Psi^\dagger_k(x) \Psi_n(x)}.$$ 

In a finite volume, the eigenmodes of the Dirac operator can be separated into two classes. The exact zero modes, $\lambda = 0$, and nonzero eigenmodes, $\lambda_n \neq 0$. It is well-understood that the exact zero modes are irrelevant since their contributions to the Green functions and observables vanish in the thermodynamic limit $V \to \infty$ as $1/V$ [12–14]. Consequently, in the finite-volume calculations, we can ignore the exact zero-modes.

Now, we can read off the symmetry properties of the partition function (8). For any SU(2)$_{\text{CS}}$ and SU($2N_F$) rotation, the $\Psi_n$ and $\Psi^\dagger_k$ Dirac spinors transform as

$$\Psi_n \to U \Psi_n, \quad \Psi^\dagger_k \to (U \Psi_k)^\dagger,$$

where $U$ is any transformation from the groups SU(2)$_{\text{CS}}$ and SU($2N_F$), $U^\dagger = U^{-1}$. It is then clear that the exponential part of the partition function is invariant under global and local SU(2)$_{\text{CS}}$ and SU($2N_F$) transformations, because

$$(U \Psi_k(x))^\dagger U \Psi_n(x) = \Psi^\dagger_k(x) \Psi_n(x).$$

The exact zero modes contributions

$$\Psi^\dagger_0(x) \Psi_n(x), \Psi^\dagger_k(x) \Psi_0(x), \Psi^\dagger_0(x) \Psi_0(x),$$
for which equation (10) is not defined, are irrelevant in the thermodynamic limit and can be ignored. In other words, QCD classically without the irrelevant exact zero modes has in a finite volume $V$ local SU(2)$_{CS}$ and SU(2$N_F$) symmetries. These are hidden classical symmetries of QCD.

The integration measure in the partition function is not invariant under a local U(1)$_A$ transformation [2], which is a source of the U(1)$_A$ anomaly. The U(1)$_A$ is a subgroup of SU(2)$_{CS}$. Hence, the axial anomaly breaks SU(2)$_{CS}$ and SU(2$N_F$) → SU($N_F$)$_L \times$ SU($N_F$)$_R$.

In the limit $V \to \infty$ otherwise finite lowest eigenvalues $\lambda$ condense around zero and provide according to the Banks–Casher relation at $m \to 0$ a nonvanishing quark condensate in the Minkowski space. The quark condensate in the Minkowski space-time breaks all U(1)$_A$, SU($N_F$)$_L \times$ SU($N_F$)$_R$, SU(2)$_{CS}$ and SU(2$N_F$) symmetries to SU($N_F$)$_V$. In other words, the hidden classical SU(2)$_{CS}$ and SU(2$N_F$) symmetries are broken both by the anomaly and spontaneously.

3. Restoration of SU(2)$_{CS}$ and SU(2$N_F$) at high temperature [1]

Above the chiral restoration phase transition, the quark condensate vanishes. If, in addition, the U(1)$_A$ symmetry is also restored [15–17] and a gap opens in the Dirac spectrum, then above the critical temperature, the SU(2)$_{CS}$ and SU(2$N_F$) symmetries are manifest. The precise meaning of this statement is that the correlation functions and observables are SU(2)$_{CS}$ and SU(2$N_F$) symmetric.

These SU(2)$_{CS}$ and SU(2$N_F$) symmetries of QCD imply that there cannot be deconfined free quarks and gluons at any finite temperature in the Minkowski space-time. Indeed, the Green functions and observables calculated in terms of unconfined quarks and gluons in the Minkowski space (i.e. within the perturbation theory) cannot be SU(2)$_{CS}$ and SU(2$N_F$) symmetric, because the chromo-magnetic interaction necessarily breaks both symmetries. Then it follows that above $T_c$, QCD is in a confining regime. In contrast, color-singlet SU(2$N_F$)-symmetric “hadrons” (with not yet known properties) are not prohibited by the SU(2$N_F$) symmetry and can freely propagate. “Hadrons” with such a symmetry in the Minkowski space can be constructed [18].

4. Predictions

Restoration of the SU(2)$_{CS}$ and of SU(2$N_F$) symmetries at high temperatures can be tested on the lattice.

Transformation properties of hadron operators under SU(2)$_{CS}$ and SU(2$N_F$) groups are given in Refs. [7,10]. For example, the isovector $J = 1$ operators $\bar{\Psi} \gamma^i \gamma^5 \gamma^i \Psi$, $(1-\cdots)$; $\bar{\Psi} \gamma^0 \gamma^i \gamma^i \Psi$, $(1-\cdots)$; $\bar{\Psi} \gamma^0 \gamma^5 \gamma^i \gamma^i \Psi$, $(1-\cdots)$ form an irre-
ducible representation of $\text{SU}(2)_{\text{CS}}$. One expects that below $T_c$, all three diagonal correlators will be different and the off-diagonal cross-correlator of $(1^{--})$ operators will not be zero. Above $T_c$, an $\text{SU}(2)_{\text{CS}}$ restoration requires that all diagonal correlators should become identical and the off-diagonal correlator of $(1^{--})$ currents should vanish. A restoration of $\text{SU}(2)_{\text{CS}}$ and of $\text{SU}(N_F)_L \times \text{SU}(N_F)_R$ implies a restoration of $\text{SU}(2N_F)$.

A similar prediction can be made with the baryon operators.

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REFERENCES