Production of pions in proton–nucleus (p + A) reactions outside of a kinematical boundary of proton–nucleon collisions, the so-called cumulative effect, is studied. The kinematical restrictions on pions emitted in the backward direction in the target rest frame are analyzed. It is shown that cumulative pion production requires a presence of massive baryonic resonances that are produced during successive collisions of the projectile with nuclear nucleons. After each successive collision, the mass of created resonance may increase and, simultaneously, its longitudinal velocity decreases. Simulations within Ultra-relativistic Quantum Molecular Dynamics (UrQMD) model reveals that successive collisions of baryonic resonances with nuclear nucleons play the dominant role in the cumulative pion production in p + A reactions.

DOI:10.5506/APhysPolBSupp.10.681

1. Introduction

The cumulative effect in proton–nucleus (p + A) reactions is a production of secondary particles in a kinematic region forbidden in proton–nucleon (p + N) collisions at the same energy of projectile protons. First experiments with detection of cumulative particles were performed at the Synchrophasotron accelerator of the Joint Institute for Nuclear Research in Dubna [1–3]. In
this work, inclusive reactions \( p + A \rightarrow \pi(180^\circ) + X \) are considered with pions emitted in the backward direction, \( i.e. \) at \( 180^\circ \), in the target rest frame.

Let \( E^*_\pi \) denotes the maximal possible energy of the pion emitted at angle \( 180^\circ \) in the laboratory frame in \( p + N \) interaction at the fixed projectile proton momentum \( p_0 \). In \( p + A \) collisions at the same projectile proton momentum \( p_0 \), pions emitted at \( 180^\circ \) in the nucleus rest frame with energies \( E_\pi > E^*_\pi \), even above \( 2E^*_\pi \), were experimentally observed [1–5].

The main physical quantities in our analysis are the masses and longitudinal (\( i.e. \) along the collision axis) velocities of the baryonic resonances created in \( p + A \) reactions. The resonance \( R \) is first produced in \( p + N \rightarrow R + N \) reaction, and it then participates in successive \( R + N \rightarrow R' + N \) collisions. Due to subsequent collisions of the resonances with nuclear nucleons, the resonance mass may increase and its longitudinal velocity decreases. We argue that the cumulative pions in \( p + A \) reactions are created by baryonic resonances with very high masses that are formed due to successive collisions with nuclear nucleons. We also use the UrQMD model to analyze some microscopic aspects of cumulative pion production in \( p + A \) reactions.

2. Successive collisions with nuclear nucleons

Different theoretical models were proposed to describe the cumulative pion production. However, origin of this effect is still not settled. In the present study, we will advocate the approach suggested in Ref. [6]. More details are presented in our recent paper [7], where the references to other theoretical models can be also found. We assume that cumulative particle production takes place due to the large mass of the projectile baryonic resonance created in the first \( p + N \) collision and its further propagation through the nucleus. This baryonic resonance has a chance to interact with other nuclear nucleons earlier than it decays to free final hadrons. It can be shown [7] that during successive collisions of the baryonic resonance with nuclear nucleons, it is possible both to enlarge the resonance mass \( M_R \) and, simultaneously, to reduce its longitudinal velocity \( v_R \).

Production of any additional hadron(s) and/or a presence of non-zero transverse momenta in the final state would require an additional energy and lead to a reduction of \( E^\text{max}_\pi \) value at the fixed projectile momentum \( p_0 \). Thus, to find the maximum of pion energy, one should restrict the kinematical analysis to the one-dimensional (longitudinal) direction, \( i.e. \), all particle momenta should be directed along the collision axis.

If one considers baryonic resonance decay, \( R \rightarrow N + \pi(180^\circ) \), the value of \( E_\pi \) depends on the resonance mass \( M_R \) and its longitudinal velocity \( v_R \). In the resonance rest frame, the pion energy and momentum can be easily found
Cumulative Pion Production via Successive Collisions in Nuclear Medium

\[ E_\pi^0 = \frac{M_R^2 - m_N^2 + m_\pi^2}{2M_R}, \quad p_\pi^0 = \sqrt{(E_\pi^0)^2 - m_\pi^2}. \quad (1) \]

The energy \( E_\pi \) (with neglected pion mass for simplicity), in the laboratory frame, is obtained as

\[ E_\pi = \frac{E_\pi^0 - v_Rp_\pi^0}{\sqrt{1 - v_R^2}}. \quad (2) \]

Therefore, both the increase of \( M_R \) and decrease of \( v_R \) provide an extension of the available kinematic region of \( E_\pi \) for pions emitted at 180°. Thus, the suppression of \( E_\pi \) compared to \( E_\pi^0 \) can be interpreted as the Doppler (“red shift”) effect.

As seen from Eq. (2), both effects of resonance mass increase and its velocity decrease lead to larger values of \( E_\pi \) and, thus, extend the kinematic region for cumulative pion production. The object responsible for the cumulative production of \( \pi(180^\circ) \), i.e., the heavy and slow moving resonance, does not exist inside a nucleus but is formed during the whole evolution process of \( p + A \) reaction.

Let us consider successive collisions with nuclear nucleons: \( p + N \rightarrow R_1 + N, R_1 + N \rightarrow R_2 + N, \ldots, R_n + N \rightarrow N + N + \pi(180^\circ) \). The nuclear nucleons are considered as free particles. This approximation can be justified by the fact that the projectile proton energy is typically 3 orders of magnitude larger than the binding energy of nucleons in a nucleus. It is assumed that after \( n^{th} \) collision, the baryonic resonance decays, \( R_n \rightarrow \pi(180^\circ) + N \). The energy and momentum conservation between the initial and final state read as

\[ \sqrt{m_N^2 + p_0^2 + n \cdot m_N} = \sum_{i=1}^{n+1} \sqrt{m_N^2 + p_i^2} + E_\pi, \quad p_0 = \sum_{i=1}^{n+1} p_i - p_\pi. \quad (3) \]

The maximal pion energy \( E_\pi \) after \( n \) successive collisions denoted now \( E_{\pi,n}^* \) can be found from Eq. (3) using the extremum conditions \( \partial E_\pi / \partial p_i = 0 \). This leads to

\[ p_{n+1}^* = p_1 = p_2 = \ldots = p_n^* = \frac{p_0 + p_{\pi,n}^*}{n + 1}, \quad (4) \]

and gives an implicit equation for \( E_{\pi,n}^* \).

The maximal energies \( E_{\pi,n}^* \) of pions emitted at 180° are presented in Fig. 1. A surprising behavior with \( v_{\pi,n}^* < 0 \) for \( n \geq 4 \) is observed at some finite regions of the projectile momentum \( p_0 \), i.e., the heavy resonance may start to move backward after a large number of successive collisions for not too large \( p_0 \). In \( p + N \rightarrow R + N \) reactions, the only \( v_R \) values with \( v_R > 0 \) are permitted.
Fig. 1. Maximal energies $E_{\pi,n}^*$ (a) and velocities $v_n^*$ (b) of the baryonic resonances after $n$ successive collisions with nuclear nucleons. The values of $v_n^*$ are required to provide the maximal energy $E_{\pi,n}^*$ of $\pi(180^\circ)$. They are calculated from Eq. (3) with assumption of $\partial E_{\pi}/\partial p_i = 0$, as functions of the projectile proton momentum $p_0$.

It should be noted that the values of $E_{\pi,n}^*$ and $v_n^*$ found in this section are by no means typical (or average) ones. In fact, the probability to reach these values in $p+A$ reaction is very small. In other words, cumulative pion production is a very rare process.

3. UrQMD simulations

In this section, the analysis of the cumulative production of $\pi(180^\circ)$ within the UrQMD model [8] is presented. The UrQMD gives a unique opportunity to study a history of each individual reaction. In Fig. 2, we compare the spectra of $\pi(180^\circ)$ emitted from resonance decay after $n = 1$ (solid line), $n = 2$ (dashed line), and $n \geq 3$ (dotted line) successive collisions of the projectile with nuclear nucleons.

Fig. 2. UrQMD results for the pion energy spectra at $180^\circ$ in $p+^{208}\text{Pb}$ collisions. Spectra for pions created after different number of collisions with nuclear nucleons: $n = 1$ (solid line) collision, $n = 2$ (dashed line), and $n \geq 3$ (dotted line). Vertical lines correspond to energies $E_{\pi,1}^*$ (solid) and $E_{\pi,2}^*$ (dashed). In (a) $p_0 = 6 \text{ GeV/c}$ and the number of events $N_{ev} = 5.7 \times 10^7$. In (b) $p_0 = 158 \text{ GeV/c}$, $N_{ev} = 4.2 \times 10^7$. 

cumulative pion production via successive collisions in nuclear medium

From Fig. 2, one observes that \( E_\pi \) may exceed \( E^*_{\pi,1} \) even for \( n = 1 \) contribution. This happens because of the nucleon motion inside nuclei (Fermi motion) which exists in the UrQMD model. This effect is, however, not large. The main contribution to the kinematical region forbidden for \( p + N \) collisions (i.e., to \( E_\pi > E^*_{\pi,1} \)) comes from the decays of resonances created within \( n = 2 \) and \( n \geq 3 \) successive collisions with nuclear nucleons. Therefore, the proposed mechanism of the cumulative pion production — the successive interactions of heavy resonances with nuclear nucleons — is supported by the UrQMD analysis.

4. Summary

Pions emitted in \( p + A \) reactions at 180° in the target rest frame are considered. Extension of a kinematical boundary of \( p + N \) reactions due to existence of massive baryonic resonances is studied. These resonances are produced after several successive collisions of the projectile with nuclear nucleons: resonances \( R \) created in \( p + N \) reactions may have further inelastic collisions in the nuclear medium. Due to successive collisions with nuclear nucleons, the masses of these resonances may increase and, simultaneously, their longitudinal velocities decrease. These two effects give an explanation of the cumulative pion production. The simulations of \( p + A \) reactions within the UrQMD model support this physical picture.

This work is supported by the Goal-Oriented Program of the National Academy of Sciences of Ukraine and the European Organization of Nuclear Research (CERN), grant CO-1-3-2016.

REFERENCES