A NEW AND FINITE FAMILY OF SOLUTIONS OF HYDRODYNAMICS: PART III: ADVANCED ESTIMATE OF THE LIFE-TIME PARAMETER*

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We derive a new formula for the longitudinal HBT radii of the two particle Bose–Einstein correlation function from a new family of finite and exact, accelerating solution of relativistic perfect fluid hydrodynamics for a temperature-independent speed of sound. The new result generalizes the Makhlin–Sinyukov and Herrmann–Bertsch formulae and leads to an advanced life-time estimate of high-energy heavy-ion and proton–proton collisions.

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1. Introduction

This manuscript is the third part of a manuscript series. This series presents various applications of a new, accelerating, finite and exact family of solutions of perfect fluid hydrodynamics, the recently found Csörgő–Kasza–Csanád–Jiang (CKCJ) family of solution of Ref. [1]. The first part of this series [2] fixes the notation, summarizes this class of exact solutions and evaluates the rapidity and pseudorapidity density distributions. The second part [3] evaluates the initial-energy densities in high-energy collisions [1], and provides a fundamental correction to the renowned Bjorken estimate of initial-energy density [4].

In this manuscript, we evaluate the Bose–Einstein correlation functions in a Gaussian approximation from the CKCJ solutions [1]. Given that the considered dynamics is a 1+1 dimensional expansion, we evaluate $R_L$, the Hanbury Brown–Twiss (HBT) radius in the longitudinal (beam) direction. This longitudinal HBT radius parameter is proportional to the mean freeze-out time of the fireball, thus the advanced evaluation of its transverse mass

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dependence and its constant of proportionality for finite, longitudinally non-boost-invariant fireballs may have important physics implications on lifetime determinations.

2. Bose–Einstein correlations and the longitudinal HBT radii

In high-energy heavy-ion collisions, the Bose–Einstein correlation functions (BECF) measure characteristic sizes of the particle emitting source, corresponding to lengths of homogeneity [5]. In high-energy heavy-ion collisions, the particle emitting source can be approximated as a locally thermalized fireball, surrounded by a halo of long-lived resonances, this is the so-called core–halo picture. The momentum-dependent intercept parameter $\lambda_*$ of the two-particle Bose–Einstein correlation function can be interpreted in the core–halo picture of Ref. [6] as follows:

$$\lambda_* = \left( \frac{N_c}{N} \right)^2 = \left( \frac{N_c}{N_c + N_h} \right)^2,$$

where $N = N_c + N_h$ is the total number of the emitted particles with a given momentum, adding the contributions from both the core $N_c$ and the halo $N_h$. The fireball that undergoes a hydrodynamical evolution corresponds to core [7]. For locally thermalized sources, the lengths of homogeneity are expressible in terms of the derivatives of the fugacity, $\exp(\mu(x)/T(x))$ and the locally thermalized momentum distribution, $\exp(-k^\mu u_\mu(x)/T(x))$, corresponding to the so-called geometrical and thermal length scales [7]. Assuming an effective Gaussian source for the core particles, the BECF can be expressed in terms of the Bertsch–Pratt variables as follows:

$$C(\Delta k, K) = 1 + \lambda_* \exp(-R_{\text{side}}^2 Q_{\text{side}}^2 - R_{\text{out}}^2 Q_{\text{out}}^2 - R_L^2 Q_L^2 - 2R_{\text{out},L}^2 Q_{\text{out},L} Q_L).$$

All the fit parameters ($\lambda_*$, $R_{\text{side}}$, $R_{\text{out}}$, $R_L$ and $R_{\text{out},L}$) depend on the mean momentum of the particle pair, $K^\mu = 0.5(k_1^\mu + k_2^\mu)$. The four-momentum of a given particle is denoted by $k = (E_k, k) = (E_k, k_x, k_y, k_z)$. The three-components of the relative and mean momenta are denoted as

$$\Delta k = k_1 - k_2,$$

$$K = 0.5(k_1 + k_2).$$

In the Bertsch–Pratt decomposition of the relative momentum [8, 9], the principal directions are defined as follows: The out direction is perpendicular to the beam axis and parallel to the mean transverse momentum of the boson pair; the longitudinal direction (indicated by subscript L) is parallel to the beam axis ($r_z$), and the side direction is orthogonal to the previous two.
directions. This Bertsch–Pratt decomposition of the relative momentum is defined as follows:

\[ Q_{\text{side}} = \frac{\left| \Delta k \times K \right|}{|K|}, \]

\[ Q_{\text{out}} = \frac{\Delta k \cdot K}{|K|}, \]

\[ Q_L = k_{1,z} - k_{2,z}. \]

(3) (4) (5)

If the Bose–Einstein correlation function is an approximately Gaussian in terms of the relative momenta, the Gaussian HBT radii \( R^2_{i,j} \) can be introduced, with \( \{i, j\} \in \{\text{side, out, long}\} \). These Gaussian Bertsch–Pratt-radii can be related to the variances of the hydrodynamically evolving core, while the halo of the long-lived resonances is responsible for the effective reduction of the strength of the correlation function:

\[ R^2_{i,j} = \langle \tilde{x}_i \tilde{x}_j \rangle_c - \langle \tilde{x}_i \rangle_c \langle \tilde{x}_j \rangle_c. \]

(6)

Here, the \( \langle A \rangle_c \) stands for the average of quantity \( A \) in the core, \( i, j \) stand for directions (side, out or long) and

\[ \tilde{x}_i = x_i - \beta_i t, \]

\[ \beta_i = \frac{k_{i,1} + k_{i,2}}{E_1 + E_2}. \]

(7) (8)

In this manuscript, we focus on the longitudinal radii, so the radii of the \textit{side} and \textit{out} direction are not discussed, see \textit{e.g.} Ref. \[7\] for more details on this point. As discussed in \[10\] and illustrated in Fig. 1, for a 1+1 dimensional relativistic source, the longitudinal radius in an arbitrary frame reads as

\[ R^2_L = (\beta_L \sinh (\eta^s_\lambda) - \cosh (\eta^s_\lambda))^2 \tau^2_f \Delta \eta^2_\lambda + (\beta_L \cosh (\eta^s_\lambda) - \sinh (\eta^s_\lambda))^2 \Delta \tau^2, \]

(9)

where \( \eta_\lambda \) is the space-time rapidity, and \( \eta^s_\lambda \) is the main emission region of the source, which derived by the saddle-point calculation of the rapidity density, \( \Delta \tau \) and \( \Delta \eta_\lambda \) are characteristic sizes around \( \tau_f \) and \( \eta^s_\lambda \). This formula simplifies a lot in the LCMS (longitudinally co-moving system) frame of the boson pair, where \( \beta_L = 0 \):

\[ R^2_L = \cosh^2 (\eta^s_\lambda) \tau^2_f \Delta \eta^2_\lambda + \sinh^2 (\eta^s_\lambda) \Delta \tau^2. \]

(10)

Our new family of solutions are finite and limited to a narrow rapidity interval around midrapidity \[1\]. At midrapidity, if \( \eta^s_\lambda \approx 0 \), the above equation can be simplified even further:

\[ R_L = \tau_f \Delta \eta_\lambda. \]

(11)
3. Previous results on the longitudinal HBT radius

For a Hwa–Bjorken-type of accelerationless, longitudinal flow [4, 11], Makhlin and Sinyukov determined the longitudinal length of homogeneity in Ref. [5] as

$$R_L = \tau_{\text{Bj}} \sqrt{\frac{T_f}{m_T}}. \quad (12)$$

In this equation, $T_f$ stands for the freeze-out temperature, $m_T$ is the transverse mass of the particle pair and $\tau_{\text{Bj}}$ is the mean freeze-out time of the Hwa–Bjorken solution. This result makes it possible to determine the lifetime, i.e. $\tau_{\text{Bj}}$ of the reaction from the measurement of the longitudinal HBT radius parameter, provided that $T_f \approx m_\pi \approx 140$ MeV can be estimated from the analysis of the single particle spectra.

Evaluating the HBT radii from the same Hwa–Bjorken solution [4, 11], Herrmann and Bertsch obtained a more accurate result in Ref. [12], using a Gaussian approximation for the longitudinal HBT radius at midrapidity, in terms of Bessel functions $K_1(z)$ and $K_2(z)$, as follows:

$$R_L = \tau_f \sqrt{\frac{T_f}{m_T}} \sqrt{\frac{2K_2(m_T/T_f)}{K_1(m_T/T_f)}}. \quad (13)$$

This formula improves the Sinyukov–Makhlin formula (12) for lower $m_T/T_f$ values, and approaches it in the large $m_T/T_f$ limit.

If the flow is accelerating, the estimated origin of the trajectories is shifted back in proper-time, thus $\tau_{\text{Bj}}$ is underestimating the life-time of the reaction. The correction was estimated, based on the modification of the flow-profile, from the Csörgő–Nagy–Csanád (CNC) solution [13, 14] as follows:

$$R_L = \frac{\tau_f}{\lambda} \sqrt{\frac{T_f}{m_T}}, \quad (14)$$
where $\tau_f$ stands for the freeze-out time. In the $\lambda \to 1$ boost-invariant limit, this formula also reproduces the Makhlin–Sinyukov formula, but for the realistic $\lambda > 1$ parameter values, it yields larger life-times as compared to the Makhlin–Sinyukov formula.

4. The longitudinal HBT radius parameter of the CKCJ solution

Let us evaluate the emission function for the CKCJ solution of Refs. [1–3]. The integration of the Cooper–Frye formula is performed by the saddle-point approximation. Near to midrapidity, the fluid rapidity is well-approximated by a linear function of the space-time rapidity: $\Omega \approx \lambda \eta_x$. Using a saddle-point integration in $\eta_x$, we obtain the rapidity distribution

$$\frac{dN}{dy} \approx \frac{(2\pi \Delta \eta^2_x)^{1/2}}{2\pi \hbar} \left[ k_\mu u^\mu \frac{\tau(\eta_x)}{\cosh(\Omega - \eta_x)} \exp \left( -\frac{k_\mu u^\mu}{T_f(\eta_x)} \right) \right]_{\eta_x = \eta^*_x}. \quad (15)$$

Here, $\eta^*_x$ stands for the saddle-point, which is found to be proportional to the rapidity $y$: $\eta^*_x \approx \frac{y}{2\lambda - 1}$. At midrapidity, the saddle-point vanishes and the emission function can be well-approximated by a Gaussian centered on zero. The width of this Gaussian is given by $\Delta \eta_x$ as

$$\Delta \eta_x \approx \sqrt{\frac{T_f}{m_T \lambda (2\lambda - 1)}}. \quad (16)$$

At midrapidity, these considerations lead to the following longitudinal HBT radius parameter:

$$R_L = \tau_f \Delta \eta_x \approx \frac{\tau_f}{\sqrt{\lambda (2\lambda - 1)}} \sqrt{\frac{T_f}{m_T}}. \quad (17)$$

Surprisingly, this result is independent of the equation of state, and it is formally different from the CNC estimate.

Our result thus presents an important step forward: once the parameter $\lambda$ of the acceleration is determined from the fits to the (pseudo)rapidity distributions [2], this parameter combined with the longitudinal HBT radius measurement can be used to provide an advanced estimate of the life-time of the reaction, solving Eq. (17) for the life-time $\tau_f$. The significance of our advanced formula is illustrated in figure 2.
Fig. 2. (Color online) The HBT radius $R_L(m_T)$ (left) and $1/R_L^2(m_T)$ (right) of the CKCJ solution are shown with solid red lines and compared to earlier estimations. The parameters correspond to the fit results of the CKCJ solution to $p+p$ collisions at $\sqrt{s} = 7$ TeV [2].

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