LATTICE INVESTIGATIONS OF THE QCD PHASE DIAGRAM FROM ANALYTICAL CONTINUATION*

JANA N. GUENTHER\textsuperscript{a,b}, SZABOLCS BORSÁNYI\textsuperscript{b}, ATTILA PÁSZTOR\textsuperscript{b} 
ZOLTAN FODOR\textsuperscript{b,c,d}, MATTEO GIORDANO\textsuperscript{d}, KORNÉL KAPÁS\textsuperscript{d} 
SANDOR D. KATZ\textsuperscript{d}, ISRAEL PORTILLO\textsuperscript{e}, CLAUDIA RATTI\textsuperscript{e}, K.K. SZABÓ\textsuperscript{c}

\textsuperscript{a}University of Regensburg, Universitätsstraße 31, 93053 Regensburg, Germany 
\textsuperscript{b}University of Wuppertal, Gaussstraße 20, 42119 Wuppertal, Germany 
\textsuperscript{c}Jülich Supercomputing Centre, Forschungszentrum Jülich 
52425 Jülich, Germany 
\textsuperscript{d}Institute for Theoretical Physics, Eötvös Loránd University 
Pázmány Péter sétány 1/A, 1117 Budapest, Hungary 
\textsuperscript{e}Department of Physics, University of Houston, Houston, TX 77204, USA

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At zero baryon density, lattice QCD is an established tool that provides precise theoretical results. Calculations at non-zero densities, however, require new techniques to deal with the sign problem. In this work, we will review our recent effort to investigate QCD at non-vanishing baryon chemical potential.

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1. Introduction

Correlations of conserved charges are important observables for the investigations of finite-density QCD. In this work we will summarize our results published in [1]. We will then explore the possibility to constrain the critical endpoint with these fluctuations as presented in [2]. In the following, we will use the notation $\chi_{i,j,k}^{B,Q,S} = \frac{\partial^{i+j+k}(p/T)}{(\partial \mu_B)^i (\partial \mu_Q)^j (\partial \mu_S)^k}$ with $\hat{\mu} = \mu/T$.

2. Fluctuations

We present results of an high-precision analysis on an $48^3 \times 12$ lattice. A more detailed description as well as precise information on the lattice set-up can be found in Refs. [1, 3]. We use analytical continuation from imaginary

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chemical potential to determine the $\chi^B$ fluctuations at $\mu_B = 0$. We analyze data for eight different values of $\mu_B = i\frac{j\pi}{8}$ with $j \in \{0, 1, 2, 3, 4, 5, 6, 7\}$. In our analysis, we use the following Ansatz for the pressure:

$$\chi^B_0(\hat{\mu}_B) = \frac{p}{T^4} = c_0 + c_2\hat{\mu}_B^2 + c_4\hat{\mu}_B^4 + c_6\hat{\mu}_B^6 + \frac{4!}{8!}c_4\epsilon_1\hat{\mu}_B^8 + \frac{4!}{10!}c_4\epsilon_2\hat{\mu}_B^{10},$$

where $\epsilon_1$ and $\epsilon_2$ are drawn randomly from a normal distribution with $\mu = -1.25$ and $\sigma = 2.75$. The values were chosen in a way to allow for $\chi^B_8$ to take the value predicted by the hadron resonance gas, as well as the result from the toy model introduced in Section 3. From the Ansatz, we can calculate the derivatives that can be measured on the lattice:

$$\chi^B_1(\hat{\mu}_B) = 2c_2\hat{\mu}_B + 4c_4\hat{\mu}_B^3 + 6c_6\hat{\mu}_B^5 + \frac{4!}{7!}c_4\epsilon_1\hat{\mu}_B^7 + \frac{4!}{9!}c_4\epsilon_2\hat{\mu}_B^9,$$

$$\chi^B_2(\hat{\mu}_B) = 2c_2 + 12c_4\hat{\mu}_B^2 + 30c_6\hat{\mu}_B^4 + \frac{4!}{6!}c_4\epsilon_1\hat{\mu}_B^6 + \frac{4!}{8!}c_4\epsilon_2\hat{\mu}_B^8,$$

$$\chi^B_3(\hat{\mu}_B) = 24c_4\hat{\mu}_B + 120c_6\hat{\mu}_B^3 + \frac{4!}{5!}c_4\epsilon_1\hat{\mu}_B^5 + \frac{4!}{7!}c_4\epsilon_2\hat{\mu}_B^7,$$

$$\chi^B_4(\hat{\mu}_B) = 24c_4 + 360c_6\hat{\mu}_B^2 + c_4\epsilon_1\hat{\mu}_B^4 + \frac{4!}{6!}c_4\epsilon_2\hat{\mu}_B^6.$$  

We perform a correlated fit for $\chi^B_1(\hat{\mu}_B)$, $\chi^B_2(\hat{\mu}_B)$, $\chi^B_3(\hat{\mu}_B)$ and $\chi^B_4(\hat{\mu}_B)$ for the different values of $\mu_B$ to determine the fitting parameters $c_2$, $c_4$ and $c_6$. From the parameters, we can determine $\chi^B_2(0) = 2c_2$, $\chi^B_4(0) = 24c_4$, $\chi^B_6(0) = 720c_6$ and $\chi^B_8(0) = 24c_4\epsilon_1$. The results are shown in figure 1. These equations show the relation between $\chi^B_4$ and $\chi^B_8$ that are just related by the factor of $\epsilon_1$ (in the same way $\chi^B_4$ and $\chi^B_{10}$ are related by a factor of $\epsilon_2$). In this way, we take into account the influence of higher order corrections to our fit function. We choose 1000 different values for $\epsilon_1$ and $\epsilon_2$ and, in addition, we include either seven or eight different values of $\mu_B$ in our data. All resulting fits are combined in a histogram and weighted with the Akaike information criteria [4], thus allowing to estimate the systematic error. The statistical error is determined by the Jackknife method and both errors are added quadratically to get the combined error shown in the plots.
3. Looking for the critical point

To look for the critical endpoint in the QCD phase diagram, one can try to calculate the radius of convergence of an expansion in $\mu_B$. Two obvious expansions for this are either the pressure

$$p(\mu) = p_0 + p_2 \hat{\mu}^2 + p_4 \hat{\mu}^4 + p_6 \hat{\mu}^6 + \ldots$$

or the fluctuations that are directly related

$$\chi_2^B(\mu) = 2p_2 + 12p_4 \hat{\mu}^2 + 30p_6 \hat{\mu}^4 + \ldots$$

We define

$$r_{2n}^p = \sqrt{\frac{p_{2n}}{p_{2n+2}}} \quad \text{and} \quad r_{2n}^\chi = \sqrt{\frac{2n(2n-1)}{(2n+1)(2n+2)}} r_{2n}^p .$$

In the limit of $n \rightarrow \infty$, both $r_{2n}^p$ and $r_{2n}^\chi$ converge to the same value which is the radius of convergence and which guarantees that there is no criticality within this radius. However, since we only know the fluctuations up to $\chi_8^B$ as discussed in the previous section, we will first test this procedure for a toy
model in which the critical endpoint is known. We use unimproved staggered fermions on an $N_t = 4$ lattice. For this set up, the critical endpoint has been already determined [2, 6, 7]. The results for $r_{2n}^p$ and $r_{2n}^\chi$ are shown in the left panel of figure 2. For a temperature where the critical endpoint is close by (right site of the left panel of figure 2), the ratios seem to converge to the correct value. However, as discussed in more detail in Ref. [2], due to the structure of $\chi_6^B$, there is always a temperature for which the ratios seem to converge, independent of the real value for the critical point. For the $N_t = 12$ data, the $r_{2n}^\chi$ and the ratios from the hadron resonance gas are shown in the right panel of figure 2. Here, the errors are still large.

Fig. 2. On the left panel: The ratios $r_{2n}^p$ and $r_{2n}^\chi$ (Eq. (8)) on an $N_t = 4$ lattice. On the very left, the temperature is close to the crossover temperature. Next to it, the temperature is close to the temperature for the critical endpoint. The black arrow marks the value for the critical endpoint from [7]. On the right panel: The $r_{2n}^\chi$ (Eq. (8)) ratios for different temperatures [2].

Instead of investigating a toy model with a known critical endpoint, we can also try to describe the data with a toy model without any critical behavior. If one fits the data for $\chi_1^B/\hat{\mu}_B$ at $\mu_B = 0$ with an analytic function of $T$ and assumes that any change with respect to the chemical potential is a linear shift of this function, one can determine all fluctuations analytically (more detail on this toy model can be found in Ref. [5]). The results of this toy model are shown with black curves in figure 1. They agree well with the data.

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