PHENOMENOLOGICAL ESTIMATES OF THE PROFILES OF GIANT DIPOLE RESONANCES BUILT ON GROUND- AND LOW-LYING EXITED STATES*

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Thermal Shape Fluctuation (TSF) Model describes the properties of the Giant Dipole Resonance (GDR) radiation emitted by hot rotating nuclei. This approach turned out to be a very efficient method of extracting the information about the shapes of nuclei and their evolution with increasing angular momentum. The GDR can be built also on the ground-state band but this process needs advanced microscopic methods to be described adequately. We propose to investigate the GDR strength function with the developed for this purpose TSF model. The potential energy surface, which allows to calculate the shape probability using the so-called Boltzmann factor, is obtained within macroscopic–microscopic method, where the Folded-Yukawa model is used for macroscopic part and Folded-Yukawa plus exponential mean-field potential to generate the single-particle energies. The Strutinsky shell- and the particle number projected BCS pairing-correction energies give the microscopic contribution to the energy of a nucleus at each given deformation. The method has been tested for selenium and neodymium isotopes for which the photo-absorption spectra were measured and preliminary results will be presented.

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1. Introduction

The de-excitation of the Giant Dipole Resonance by emission of photons is usually a less probable channel than emission of particles or fission

but it provides a valuable information about nuclear properties. The GDR strength function allows to extract the information about nuclear shapes, both in terms of elongation and non-axiality. The Thermal Shape Fluctuation Method (TSFM) \[4, 6\] combines the potential energy surface (PES) with the Boltzmann factor and the GDR strength function. The TSFM applied to hot rotating nuclei allowed to describe the Jacobi shape transition, where the nucleus changes its deformation from oblate via triaxial to prolate \[7\]. For hot nuclei, the collective properties are described by the macroscopic energy which can be used to define the potential energy surfaces.

The GDR built on the ground state or the low-lying excited states (at temperatures \(T\) varying from 0.1 to 1 MeV) have to be studied with microscopic method since the quantum shell structure is crucial to correctly reproduce various observables. The generalised Separable Random Phase Approximation (SRPA) for self-consistent nuclear models such as time-dependent density functional theory (TDDFT) with Skyrme forces \[1\] reproduces the photo-absorption spectra for neodymium and samarium isotopes. The Quasiparticle RPA (QRPA) with finite amplitude method \[3\] was used to describe the spectra for ytterbium, hafnium and vanadium isotopes. In the present calculations, the phenomenological approach is based on the TSF model taking into account the nuclear structure effects.

2. Total energy

The total energy, \(E_{\text{tot}}\), is obtained within macroscopic–microscopic model, where the Lublin–Strasbourg Drop (LSD) \[5\] and Finite Range Liquid Drop (FRLDM) \[9\] models are used to define the macroscopic energy. The microscopic energy term is based on the mean-field approach with the Folded-Yukawa potential

\[
E_{\text{tot}} = E_{\text{macro}} + E_{\text{micro}} = E_{\text{macro}} + E_{\text{shell}} + E_{\text{proj}}^N. \tag{1}
\]

The Strutinsky shell, \(E_{\text{shell}}\), and pairing, \(E_{\text{proj}}^N\), energies are calculated with single-particle levels of Folded-Yukawa potential \[8, 12\]. The BCS approach with approximate particle number projection method \[10\] is used to calculate the pairing energy

\[
E_{\text{proj}}^N = E_{\text{BCS}} - \epsilon_0^\phi, \tag{2}
\]

where

\[
\epsilon_0^\phi = \left( \sum_\nu \frac{h_\nu^{(1)}}{E_\nu^2} \right) \left( \sum_\nu E_\nu^{-2} \right)^{-1}. \tag{3}
\]
Above, $E_\nu$ is the quasiparticle energy, $h^{(1)}_{\nu}$ is the one-quasiparticle energy contribution in the form of

$$h^{(1)}_{\nu} = (e_\nu - \mu) (u_\nu^2 - v_\nu^2) + 2\Delta u_\nu v_\nu + Gv^4_{\nu} \quad (4)$$

and $\Delta$ is the pairing energy-gap, whereas $v_{\nu}^2$ is the BCS occupation probability of the level $\nu$ and $u_{\nu}^2 = 1 - v_{\nu}^2$. This method is based on the Generator-Coordinate method (GCM) combined with the Gaussian-overlap approximation (GOA).

Figure 1 presents the potential energies $E_{\text{tot}}$ calculated with the LSD and the FRLDM methods in a multidimensional deformation parameter space projected onto ($\beta, \gamma$) plane. The illustration shows the model-dependent differences of the macroscopic energy contributions on the PES for $^{150}$Nd. One may notice that the FRLDM gives slightly steeper energies around the minima.

Fig. 1. Total energy of $^{150}$Nd obtained with microscopic–macroscopic method, when the macroscopic energy is calculated with: (a) Lublin–Strasbourg Drop model and (b) Finite Range Liquid Drop Model.
The free energy calculated in each point of the PES

\[ F(T) = E_{\text{tot}}(\text{def}) + E_{\text{rot}}(\text{def}, I) + a(\text{def})T^2 \]  

(5)
is used in the Boltzmann factor to obtain the probability of each given deformation

\[ P(\text{def}; I; T) = \exp \left\{ - \frac{F(T)}{kT} \right\} . \]  

(6)

3. Giant Dipole Resonances

The shape of the strength function is approximated by Lorentzian or Breit–Wigner equations

\[ f_k(E_\gamma, (\text{def})) = \frac{\sigma_k \Gamma_k E_\gamma^2}{\left( E_\gamma^2 - E_{\text{GDR},k}^2 \right)^2 + E_\gamma^2 \Gamma_k^2} \]  

(7)

so that

\[ \sigma(E_\gamma, T, I) = \sum_{(x,y)} P((\text{def}); I; T) \sum_{k=1}^3 f_k(E_\gamma, (\text{def})) , \]  

(8)

where

\[ \Gamma_k = \left( \frac{E_{\text{GDR},k}}{E_{\text{GDR}}} \right)^{1.9} \Gamma_0 , \]  

(9)

and where \( k \) enumerate the axis taking the values \( x, y \) and \( z \). Above, \( E_{\text{GDR},k} \) is the energy of the centroid and \( \Gamma_k \) stands for the GDR function widths. The integration of the GDR profile obtained for each quadrupole nuclear shape over \( (\beta, \gamma) \) plane gives the final GDR strength function. It depends on the \( E_{\text{GDR}} \) and \( \Gamma_0 \) parameters which are fitted to the known GDR functions.

The GDR spectra obtained with and without microscopic energy are presented in Fig. 2 for \(^{142,150}\)Nd. The minimum energy for ground state obtained with macroscopic energy corresponds to the spherical shape.

The macroscopic–microscopic approach allows to obtain the energy minimum for deformed shape in the ground state, cf. Fig. 1. The deformation is manifested by two or three components in the GDR function, which for spherical nucleus is approximated by a single Lorentzian function. The chain of neodymium isotopes is known for the transition from spherical, in \(^{142}\)Nd, to prolate \(^{150}\)Nd. This trend is visible in Fig. 2. Without taking into account the microscopic energy, this evolution could not be reproduced. For the spherical nucleus such as \(^{142}\)Nd, the influence of the microscopic structure is negligible but it is crucial for the deformed one. The minimum energy
of the liquid drop expression for non-rotating system is always at the spher-ical shape, thus for low-lying states, the microscopic energy contributions are essential.

![Graph showing GDR strength functions for neodymium isotopes](image)

**Fig. 2.** The GDR strength functions for neodymium isotopes obtained with TSF model. The influence of microscopic energy on the shape of GDR: dashed lines show the GDR calculated with macroscopic energy and full lines give results for the total energy.

Apart from the PES, there are few elements in the TSF model which can change the GDR shape. They are characterised by the following parameters: (i) level density parameter $a$, (ii) centroid $E_{\text{GDR}}$, (iii) the width $\Gamma_0$ and (iv) the temperature. The level density parameter is taken from deformation-dependent Ignatyuk formula [13], centroid and widths have standard values ($E_{\text{GDR}} = 79A^{-1/3}$ MeV and $\Gamma_0 = 4$ MeV, respectively), but the temperature is unknown. The nucleus in the ground state is cold but still there is present the zero-point vibration energy. This energy can be influenced by the temperature what necessitates employing the Thermal Shape Fluctuation approach for increasing excitations.

Figure 3 shows the GDR function integrated over the GDR profile (full line) calculated for all possible shapes visible in Fig. 1 (b) and for the ground state deformation (dashed line). As the temperature is only 0.1 MeV in this case, the influence of the other shapes can most likely be ignored.
Fig. 3. The GDR strength function obtained with integration over PES and assuming the nuclear shape at energy minimum for $^{150}\text{Nd}$.

Thus in the case of the GDR de-exited to ground state, the information about energy minimum deformation is sufficient to describe the strength function, but as soon as the excited states are considered in which temperature and rotation play an important role, the TSF method is a very useful tool. Therefore, in Fig. 4, we show the GDR calculated at various temperatures for two nuclei: (a) $^{76}\text{Se}$ and (b) $^{150}\text{Nd}$. With the increasing temperature, the probability distributions smooth out as more and more shapes are comparably probable.

Fig. 4. The dependence of the GDR strength function on the temperature of the nuclei: (a) $^{76}\text{Se}$ and (b) $^{150}\text{Nd}$.

The available experimental photo-neutron cross-section data [14] permit to extract important information about nuclear structure and shape using theoretical calculations. Consideration of the microscopic effects allows to reproduce the experimental data for deformed nuclei as it can be seen from figure 5 for (a) selenium and (b) neodymium isotopic chains. The low-lying states for selenium isotopes do not have pure rotational structure. Only
$^{76,78}$Se levels could be assigned to rotational structures up to $8^{+}$; other isotopes have states of non-collective type. The energy of low-lying states is below 5 MeV which suggests that increasing the temperature of the system ameliorates the reproduction of the experimental data.

![Graph](a) $\text{Se, } \Gamma_0=4 \text{ MeV, } T=0.5 \text{ MeV}$

![Graph](b) $\text{Nd, } \Gamma_0=4 \text{ MeV, } T=0.1 \text{ MeV}$

Fig. 5. Comparison between the photo-absorption cross sections [14] and GDR strength functions obtained with TSF model for (a) selenium (at $T = 0.5 \text{ MeV}$) and (b) neodymium (at $T = 0.1 \text{ MeV}$) isotopes.

The neodymium isotopes have low-lying states below 2 MeV with the structure resembling the collective rotation. Thus, one may expect that changing the temperature will have less influence on the discussed profiles of the GDR.

4. Summary

Phenomenological estimates of the Giant Dipole Resonance profiles corresponding to the low-lying states are presented. The Thermal Shape Fluctuation model at the temperature range of 0.1 to 1 MeV is used in conjunction with the Folded-Yukawa+FRLDM macroscopic–microscopic approach. The experimental photo-absorption cross sections for selenium and neodymium isotopes are well-reproduced for the GDR de-excitation to the ground states. In the future, the influence of the rotation will be studied. This approach can help to plan experiments where the shapes of GDR de-excited to different low-lying states vary the most.
REFERENCES