INITIAL DEFORMATION OF A QGP DROPLET FROM COLLISIONS WITH POLARIZED DEUTERONS∗

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We discuss the advantages of carrying out ultra-relativistic collisions of heavy projectiles on polarized deuteron targets, which allows to study the build up of elliptic flow through one-body observables. Predictions are extended to the case of other light nuclei with spin \( \geq 1 \). We also mention the forthcoming experimental prospects. Results would shed light on the build-up of collectivity in light systems.

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This paper reports on and extends the results of our recent work [1], where some more details may be found. We propose a new direction of collective flow studies in ultra-relativistic heavy–light collisions, with polarized targets of light nuclei with spin \( \geq 1 \).

The basic idea, illustrated in Fig. 1, is utterly simple. A deuteron polarized along an external magnetic field direction has a wave function which is deformed in space. In particular, as shown in the bottom row of the figure, states of spin projection \( j_3 = 0 \) have a prolate distribution of the location of the nucleons, whereas the distribution in states \( j_3 = \pm 1 \) is oblate. In a

collision with a fast-moving heavy nucleus (indicated in the figure with a flat disk), a fireball is created, with the shape in the transverse plane reflecting the deformation of the deuteron. The deformation is possible due to the admixture of the $D$-wave in the deuteron ground state.

Fig. 1. Top: Ultra-relativistic collision of a heavy nucleus (flattened disk) on an unpolarized deuteron target. The created fireball has no preferential orientation in the transverse plane, and ellipticity relative to a fixed axis, $v_2 \{\Phi_P\}$, vanishes. Bottom: Collision on polarized deuterons. For $j_3 = 0$ (left) the distribution of the nucleons in the deuteron is oblate, which leads to $v_2 \{\Phi_P\} > 0$, where $\Phi_P$ is the polarization axis. On the other hand, for $j_3 = \pm 1$ (right), the distribution is prolate, yielding $v_2 \{\Phi_P\} < 0$.

Let, by convention, the fixed polarization axis $\Phi_P$ be aligned with the $x$-axis. We then define the ellipticity of a generic distribution of $N$ (point-like) sources relative to $\Phi_P$ as the following average over events:

$$
\epsilon_2 \{\Phi_P\} \equiv - \left\langle \frac{\sum_{i=1}^{N} (x_i^2 - y_i^2)}{\sum_{i=1}^{N} (x_i^2 + y_i^2)} \right\rangle,
$$

(1)

where $(x_i, y_i)$ are the transverse coordinates. The overall minus is conventional, as to make the sign the same as in the generated elliptic flow of particles. Definition (1) has been commonly used in simulations of the early phase of the ultra-relativistic nuclear collisions.

The deuteron wave functions with good $j_3$ projection of spin along $\Phi_P$ are

$$
|\Psi_{j_3}(r)\rangle = U(r)|j = 1, j_3, L = 0, S = 1\rangle + V(r)|j = 1, j_3, L = 2, S = 1\rangle.
$$

(2)
The explicit evaluation of ellipticities (1) with the wave functions (2) yields
\[ \epsilon_{j_3=0} |\Psi|_2 \{ \Phi_P \} = -2 \epsilon_{j_3=\pm1} |\Psi|_2 \{ \Phi_P \} = \frac{1}{4} \int r^2 dr \left[ 2\sqrt{2} U(r) V(r) - V(r)^2 \right]. \] (3)

With the Reid93 deuteron wave functions, we find explicitly \( \epsilon_{j_3=0} |\Psi|_2 \{ \Phi_P \} \approx 0.14 \) and \( \epsilon_{j_3=\pm1} |\Psi|_2 \{ \Phi_P \} \approx 0.07. \)

The ellipticity of the fireball is diminished compared to the ellipticity of the deuteron, due to washing out by the interactions with the randomly distributed nucleons from the heavy projectile. The range of the NN interaction, following from the inelastic scattering profile, is about 1 fm. The expected quenching is indeed observed in Monte Carlo simulations with GLISSANDO [2], presented in Fig. 2. We show the results for \( \sqrt{s_{NN}} = 72 \text{ GeV} \), which is the collision energy for the planned future fixed-target experiments at the LHC. We note that the ellipticities decrease with centrality, as expected from geometric arguments. In particular, for large centralities, only one nucleon from the deuteron interacts, and the created fireball carries, on the average, no deformation.

![Fig. 2. The ellipticities evaluated relative to the polarization axis, \( \epsilon_2 \{ \Phi_P \} \), of the fireball created in polarized deuteron–\( ^{208}\text{Pb} \) collisions at \( \sqrt{s_{NN}} = 72 \text{ GeV} \), plotted as functions of centrality (bottom abscissa) and the corresponding number of wounded nucleons (top abscissa).](image)

In Fig. 3, we show the event-by-event distributions of \( \epsilon_2 \{ \Phi_P \} \) for the two polarization cases. We note a clear shift of the \( j_3 = 0 \) case to the right and \( j_3 = \pm 1 \) case to the left, in accordance to Fig. 2. We remark that the support in Fig. 3 is \([-1, 1]\), as we evaluate ellipticity relative to the fixed polarization axis.
The ellipticity of shape of the fireball results in elliptic flow, as follows from the shape-flow transmutation mechanism due to copious rescattering (hydrodynamic evolution) in the intermediate phase of the collision process. The basic feature we use for the estimate of the elliptic flow is the approximate linearity of the response of the system to the initial deformation

$$\ddot{v}_2 \approx k \dddot{e}_2,$$

where the response coefficient $k \sim 0.2$ for small systems. Experimentally, the deuteron target is not perfectly polarized, but a high degree of polarization can be achieved. For $j = 1$ states, the tensor polarization, relevant in our study, is defined $P_{zz} = n(1) + n(-1) - 2n(0)$, with $n(j_3)$ denoting the fraction of states with a given $j_3$. Combing this with Eq. (4) gives

$$v_{2}\{\Phi_{P}\} \approx k e_{2}^{j_3=\pm1}\{\Phi_{P}\} P_{zz}.$$  

Thus, the elliptic flow is largest and positive for $P_{zz} = -2$, reaching about 1.5%, and smallest negative for $P_{zz} = 1$, reaching about $-0.75\%$ for the case of most central collisions. As for the deuteron the highest experimentally accessible polarization is $-1.5 \lesssim P_{zz} \lesssim 0.7$ [3, 4], Eq. (5) tells us that

$$-0.5\% \lesssim v_{2}\{\Phi_{P}\} \lesssim 1\%.$$  

The effect of this size in a one-body quantity can be easily measured with the available experimental accuracy.
Now, we pass to the case of heavier polarized targets, where presented above for the deuteron can be estimated in simple terms from the known experimental nuclear properties. For this purpose, we first note that for large $N$, Eq. (1) can be approximated as

$$
\epsilon_2 \{\Phi_P\} = -\frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle} + O\left(\frac{1}{N}\right),
$$

where we have replaced the event average of ratios with the ratio of event averages. For the considered case $N$, which is the mass number of the light nucleus, is not so large, so the correction may be substantial, but this is good enough for our rough estimate of the size of the effect.

Elementary calculations allow to rewrite Eq. (7) in terms of the nuclear r.m.s. radius, $\langle r^2\rangle$, and the electric quadrupole moment, $Q_2$, namely

$$
\epsilon_2 |\psi|^2 \{\Phi_P\} = -\frac{\langle x^2 - y^2 \rangle}{\frac{2}{3} \langle x^2 + 2y^2 \rangle + \frac{1}{3} \langle x^2 - y^2 \rangle} \simeq -\frac{3Q_2}{4Z\langle r^2 \rangle},
$$

where we keep only the leading term in $Q_2$, which is small. The strong-interaction nuclear radius may be obtained from the charge radius by unfolding the proton charge radius, $\langle r^2 \rangle = \langle r^2 \rangle_{ch} = \langle r^2 \rangle_p$. Our resulting estimates for several light nuclei are collected in Table I. We note quite large values for the proxy of Eq. (8), even larger than for the case of the deuteron.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$j_3$</th>
<th>$\langle r^2 \rangle_{ch}^{1/2}$ [fm]</th>
<th>$Q_2$ [fm$^2$]</th>
<th>$-\frac{3Q_2}{4Z\langle r^2 \rangle_{ch}}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^7$Li</td>
<td>$\frac{3}{2}$</td>
<td>$\pm\frac{3}{2}$</td>
<td>2.444(42)</td>
<td>-4.03(4)</td>
</tr>
<tr>
<td></td>
<td>$\pm\frac{1}{2}$</td>
<td>$\times(-1)$</td>
<td>$\times(-1)$</td>
<td></td>
</tr>
<tr>
<td>$^9$Be</td>
<td>$\frac{3}{2}$</td>
<td>$\pm\frac{3}{2}$</td>
<td>2.519(12)</td>
<td>5.29(4)</td>
</tr>
<tr>
<td></td>
<td>$\pm\frac{1}{2}$</td>
<td>$\times(-1)$</td>
<td>$\times(-1)$</td>
<td></td>
</tr>
<tr>
<td>$^{10}$B</td>
<td>$\pm3$</td>
<td>$\pm3$</td>
<td>2.428(50)</td>
<td>8.47(6)</td>
</tr>
<tr>
<td></td>
<td>$\pm2$</td>
<td>$\times0$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\pm1$</td>
<td>$\times(-3/5)$</td>
<td>$\times(-3/5)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>$\times(-4/5)$</td>
<td>$\times(-4/5)$</td>
<td></td>
</tr>
</tbody>
</table>
We note that from the Wigner–Eckart theorem, \(Q_2\) as a rank-2 angular tensor is non-zero only in states with \(j \geq 1\), thus the effect does not appear for spin-\(\frac{1}{2}\) nuclei, such as \(^3\)He or tritium.

Hopefully, the ideas outlined in this paper can be verified in the planned LHCb fixed target run (SMOG) \([8, 9]\). If confirmed, they would make another case for collectivity in small systems formed in ultra-relativistic nuclear collisions. Other opportunities that emerge from possible studies with polarized targets involve the hard probes (jets, photons, heavy flavor mesons) analyzed relative to the polarization axis \(\Phi_P\) and the interferometric correlations relative to \(\Phi_P\).

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REFERENCES