

CENTER REGIONS AS A SOLUTION TO THE GRIBOV PROBLEM OF THE CENTER VORTEX MODEL*

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The center vortex model, capable of explaining confinement and chiral symmetry breaking, has been plagued by the lattice equivalent of Gribov copies: different maxima of the gauge functional lead to different predictions of the string tension. It is possible to resolve this problem using center regions, loops evaluating to center elements, as a guide for the gauge fixing procedure. The success of this approach was already shown, but the algorithms came with an arbitrary free parameter. In recent development this parameter has been fixed, even improving the results.

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1. Introduction

The *center vortex model* [1, 2] is based upon the center symmetry of the action in lattice quantum chromodynamics. It describes the properties of the vacuum by percolating vortices, that is, closed, quantized magnetic flux lines of finite thickness, condensing in the vacuum. It can explain the behaviour of Wilson and Polyakov loops [3], broken scale invariance [4], and chiral symmetry breaking [5], although it still lacks a closed mathematical formulation. To detect vortices in an SU(2) lattice, the gauge is fixed to *Direct Maximal Center Gauge* using simulated annealing (SA): generating random gauge matrices $\Omega(x)$, one numerically looks for maximizing the functional

$$R_{\text{SA}}^2 = \sum_x \sum_\mu \left| \text{Tr} \left[\Omega(x) U_\mu(x) \Omega^\dagger(x + e_\mu) \right] \right|^2, \quad (1)$$

with $U_\mu(x)$ being an element of SU(2) corresponding to the link pointing in direction μ at lattice point x . A projection to the center degrees of freedom

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follows

$$U_\mu(x) \rightarrow Z_\mu(x) = \text{sign Tr} \left[\Omega(x) U_\mu(x) \Omega^\dagger(x + e_\mu) \right]. \quad (2)$$

The string tension σ is estimated using Creutz ratios

$$\chi(R, T) = -\ln \frac{\langle W(R+1, T+1) \rangle \langle W(R, T) \rangle}{\langle W(R, T+1) \rangle \langle W(R+1, T) \rangle}, \quad (3)$$

with $W(R, T)$ being a Wilson loop of the size of $R \times T$ evaluated in the center projected configuration after gauge fixing. From the asymptotic relation $\langle W(R, T) \rangle = e^{-\sigma R T - 2\mu(R+T) + C}$ with sufficiently high R and T follows $\chi = \sigma$.

Simulated annealing can lead to different local maxima of the gauge functional R_{SA} with differing physical properties: an improvement in the value of the gauge functional can lead to an underestimation of the string tension σ , see [6, 7] and Fig. 1, showing calculations for an SU(2) lattice.

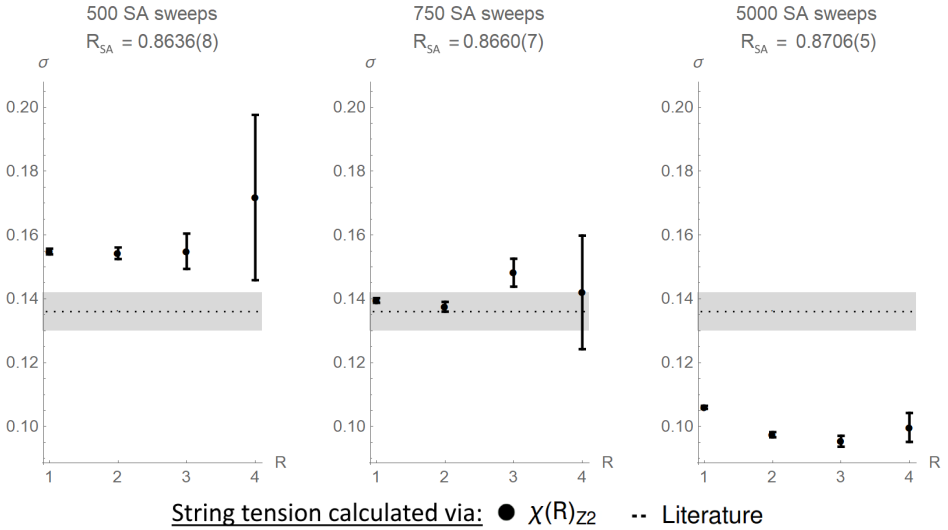
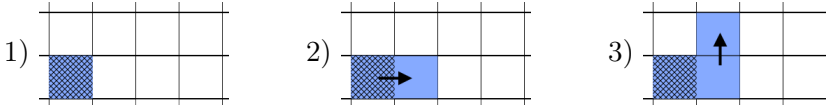


Fig. 1. The string tension was calculated with 300 SU(2) Wilson configurations at $\beta = 2.3$, in lattices of the size of 12^4 (left), 12^4 (middle) and 14^4 (right). The literature value is based on [8]. With the increased number of simulated annealing steps, the value of the gauge functional improves, but the string tension is lost.

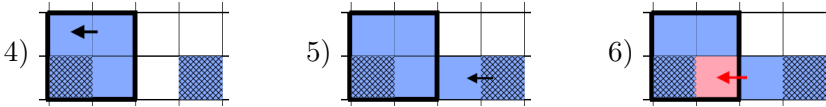
By enforcing restrictions on the gauge matrices $\Omega(x)$, we can prevent this loss of the string tension: only such transformations are allowed that result in non-trivial center regions [9] projecting onto -1 . Previously published versions of the respective algorithms came with a free parameter [10]. Now, we present a parameter-free version, further improving the results.

2. Detecting non-trivial center regions

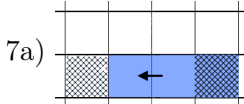
After sorting the plaquettes of a configuration by rising traces, the list is processed taking the plaquettes as origin for an enlargement procedure as shown in Fig. 2. This procedure aims to push the evaluation of the growing



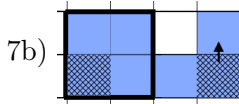
Steps 1-3: Starting with a plaquette that neither belongs to a before identified center region, nor was taken as origin for growing a region, it is tested, which enlargement by a neighboring plaquette brings the new region nearest to a center element.



Steps 4-6: If no enlargement leads to further improvement, a new enlargement procedure is started with another plaquette. During enlargement the new region could grow into an existing one. The following steps describe the collision handling:



Step 7a: The evaluation of the growing region is nearer to a non-trivial center element than the evaluation of the old region: Delete the old region, only keeping the mark on its starting plaquette and allow growing.



Step 7b: The growing region deviates more from a non-trivial center element than the existing one: try other enlargements.

Fig. 2. The algorithm for detecting center regions repeats these procedures until every plaquette either belongs to an identified region or has been taken once as starting plaquette for growing a region. The arrow marks the direction of enlargement, plaquettes belonging to a region are colored, plaquettes already used as origin are shaded.

region as close towards a non-trivial center element as possible. Not all resulting regions evaluate sufficiently near to a non-trivial center element, hence a further selection is necessary. The identified regions are sorted by rising trace. Within this sorted list, an inflection point before a stronger rise in the values of the traces is identified, see Fig. 3. The interval, the inflection point is searched in, is defined by a tangent through the point given by 1.1 times the value of the lowest trace at index 0. This is followed by identifying the lowest possible single inflection point using a method based on second difference quotients. All center regions within the list below this inflection

point are taken into account during the gauge fixing procedure by simulated annealing. This procedure is modified if non-trivial center regions appear or vanish: appearance is always enforced, disappearance always forbidden.

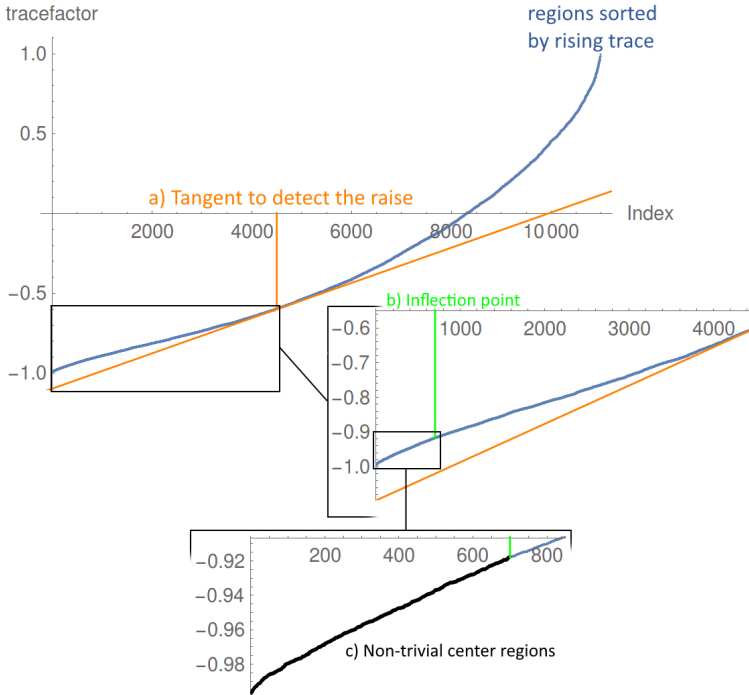


Fig. 3. Non-trivial center regions are selected within a list of detected regions sorted by rising trace. In this list, a second rise in trace is identified by fitting a tangent (a). Within the interval from the lowest trace to this tangent point, the lowest single inflection point is identified (b). All regions below this inflection point are taken for further usage (c).

3. Results

The original simulated annealing procedure has the problem to result in an underestimation of the string tension. By preserving non-trivial center regions, the string tension can be recovered at the cost of a small reduction in the value of the gauge functional and an increase in the size of the errorbars, see Fig. 4. The dependence on the number of simulated annealing steps, shown in Fig. 5, indicates that the improved procedure stays on literature value after a sufficient number of steps.

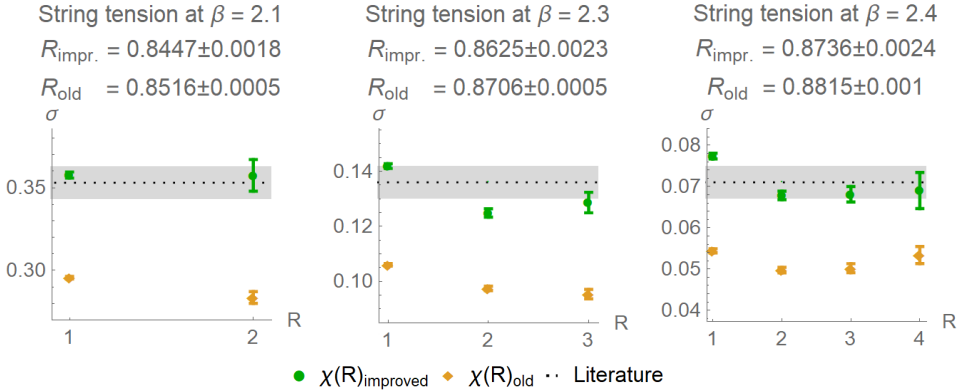


Fig. 4. We compare the original simulated annealing with our improved procedures after 5 000 simulated annealing steps for 300 SU(2) Wilson configurations in lattices of size 12^4 (left), 14^4 (middle) and 12^4 (right). The literature value is based on [8]. Our improvements clearly recover the string tension.

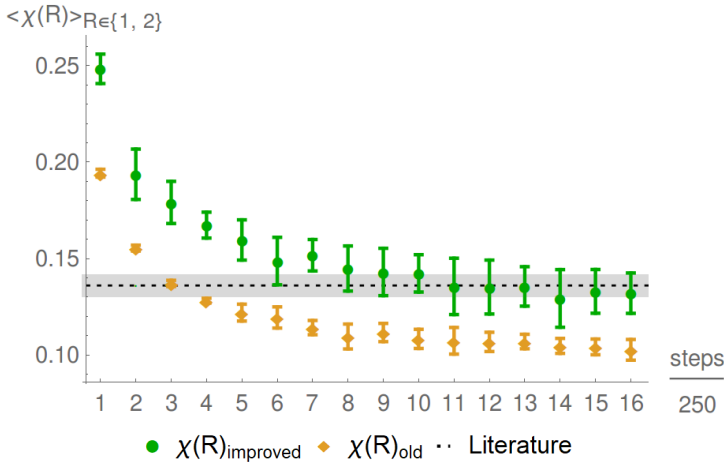


Fig. 5. With a rising number of simulated annealing steps, the original procedure leads to an underestimation of the string tension, while our improvements stay on literature value beginning with 2 500 steps. The data was calculated in a lattice of the size of 14^4 for $\beta = 2.3$. The literature value is based on [8].

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