NEUTRINO MASS MODELS*

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The observation of flavor oscillations in the leptonic sector constitutes a solid evidence for physics beyond the Standard Model. We review here some of the ideas proposed to explain the origin of neutrino masses, and we briefly address the possible connection of the new physics with the origin of the cosmic matter–antimatter asymmetry and the nature of the dark matter.

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1. Leptonic flavor in the Standard Model and beyond

The leptonic sector of the Standard Model [1] is described by three copies (commonly denominated flavors or generations) of spin 1/2 fermions, doublets under $\mathrm{SU}(2)_\mathrm{L}$ and with hypercharge $Y=-1/2$, denoted as L_i; by three spin 1/2 fermions, singlets under $\mathrm{SU}(2)_\mathrm{L}$ and with hypercharge $Y=-1$, denoted as e_{R_i}; and by one spin-0 boson, doublet under $\mathrm{SU}(2)_\mathrm{L}$ and with hypercharge $Y=1/2$, denoted as Φ. The kinetic part of the fermionic Lagrangian reads

$$\mathcal{L}_\mathrm{kin} = \bar{L}_i i \not{D} L_i + \bar{e}_{\mathrm{R}_i} i \not{D} e_{\mathrm{R}_i} \,, \tag{1}$$

and is clearly invariant under the field transformation in flavor space

$$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} \to U_\mathrm{L} \begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} , \qquad \begin{pmatrix} e_\mathrm{R} \\ \mu_\mathrm{R} \\ \tau_\mathrm{R} \end{pmatrix} \to U_{e_\mathrm{R}} \begin{pmatrix} e_\mathrm{R} \\ \mu_\mathrm{R} \\ \tau_\mathrm{R} \end{pmatrix} , \tag{2}$$

where U_L, U_{e_R} are 3×3 unitary matrices. In other words, the kinetic Lagrangian of the leptonic sector in the Standard Model displays a global $\mathrm{U}(3)_\mathrm{L}\times\mathrm{U}(3)_{e_\mathrm{R}}$ symmetry, which physically means that the free propagation of the fermions (described by the kinetic terms) does not distinguish among generations.

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The Standard Model Lagrangian also contains Yukawa couplings which, upon the breaking of the electroweak symmetry, lead to a 3×3 mass matrix for the charged leptons

$$\mathcal{L}_{\rm Yuk} = -h^e_{ij}\bar{L}_i \Phi e_{{\rm R}_j} + {\rm h.c.} \rightsquigarrow -m^e_{ij}\bar{e}_{{\rm L}_i} e_{{\rm R}_j} + {\rm h.c.} \qquad (3)$$

A priori, the mass matrix m^e_{ij} is not proportional to the identity matrix, therefore, the Yukawa interactions distinguish in general among generations. Indeed, experimentally, the charged lepton mass matrix m^e_{ij} is rank-3 and has non-degenerate eigenvalues, thus the Yukawa matrix breaks the ${\rm U}(3)_{\rm L}\times {\rm U}(3)_{e_{\rm R}}$ symmetry to ${\rm U}(1)^3$.

It is common to redefine the fermionic fields such that the charged lepton mass matrix is diagonal. On this basis, the Lagrangian simply reads

$$\mathcal{L}_{\rm lep} = \bar{L}_i i \not{D} L_i + \bar{e}_{{\rm R}_i} i \not{D} e_{{\rm R}_i} - m^e_i \bar{e}_{{\rm L}_i} e_{{\rm R}_i} + {\rm h.c.}\,, \qquad (4)$$

where the three charged lepton masses are experimentally $m_e \simeq 511$ keV, $m_\mu \simeq 106$ MeV, $m_\tau \simeq 1.78$ GeV [2]. On this basis, it is also explicit the conservation of three leptonic charges, associated to each global U(1) symmetry in the Lagrangian. Namely, the global symmetry of the Lagrangian Eq. (4) is ${\rm U}(1)_e \times {\rm U}(1)_\mu \times {\rm U}(1)_\tau$, which leads to the separate conservation of the electron number, the muon number and the tau number.

The symmetry ${\rm U}(1)_e\times {\rm U}(1)_\mu\times {\rm U}(1)_\tau$ has a series of testable implications. Some of these are:

(i) The processes $e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$ can occur in Nature, but the processes $e^+e^- \to \mu^\pm e^\mp, \mu^\pm\tau^\mp$ are forbidden.

(ii) The decays $\mu\to e\gamma$, $\tau\to\mu\gamma$ and $\tau\to e\gamma$ are forbidden.

(iii) The decay of a positively charged pion produces a prompt positively charged muon. Then, the Standard Model predicts the appearance in a distant target material of a proton accompanied only by a negatively charged muon, and forbids the appearance of an electron or a tau. (This process can be understood from the two-body decay $\pi^+\to\mu^+\nu_\mu$, followed by the charged current interaction at a distant target $\nu_\mu n \to \mu^- p$.)

(iv) The decay of a ^{8}B nucleus produces a prompt positron. Then, the Standard Model predicts the appearance in a distant target of a proton accompanied only by an electron, and forbids the appearance of a muon or a tau. (This process follows from the decay $^8{\rm B}\to{}^8{\rm Be}^* e^+\nu_e$, with the subsequent charged current interaction at the distant target $\nu_e n\to e^- p$.)

Current experiments are in excellent agreement with the predictions *(i)* and *(ii)* from the above list. However, there is strong evidence against the predictions *(iii)* and *(iv)*. Charged pions are naturally produced in collisions of cosmic rays with the nuclei in the Earth atmosphere, thus producing a flux of muon neutrinos. However, the upward muon neutrino flux observed by SuperKamiokande was significantly smaller than expected [3], thus providing evidence for the "disappearance" of muon neutrinos when they propagate long distances. Besides, the Sun contains ^{8}B, which decays producing electron neutrinos. However, the electron neutrino flux measured by SNO, using the charged current reaction $\nu_e\, d \to p\, p\, e^-$, was significantly smaller than the one predicted by solar models. On the other hand, the total neutrino flux, measured from the elastic scattering reaction $\nu_x\, e^- \to \nu_x\, e^-$ with $x = e, \mu, \tau$, is consistent with the expectations, thus revealing the existence of a non-electron flavor active neutrino component in the solar flux [4].

The evidence for the breaking of the global symmetry $\mathrm{U}(1)_e \times \mathrm{U}(1)_\mu \times \mathrm{U}(1)_\tau$ implies new physics beyond the Standard Model. Some possible extensions which break this symmetry group are:

(a) A Dirac neutrino mass term

$$\mathcal{L} \supset -m^\nu_{ij} \bar{\nu}_{\mathrm{L}_i} \nu_{\mathrm{R}_j} + \mathrm{h.c.} \tag{5}$$

which requires the addition of three generations of fermionic gauge singlets, usually denominated right-handed neutrinos ν_{R_j}. This extension leaves only one exactly conserved charge, the total lepton number, associated to the Abelian symmetry $\mathrm{U}(1)_\mathrm{L}$. Namely, this term in the Lagrangian breaks explicitly $\mathrm{U}(1)_e \times \mathrm{U}(1)_\mu \times \mathrm{U}(1)_\tau$ to $\mathrm{U}(1)_\mathrm{L}$.

(b) A Majorana neutrino mass term

$$\mathcal{L} \supset -m^\nu_{ij} \bar{\nu}_{\mathrm{L}_i} \nu^c_{\mathrm{L}_j} + \mathrm{h.c.} \tag{6}$$

In this case, no new particle is introduced. Furthermore, this Lagrangian breaks completely the group $\mathrm{U}(1)_e \times \mathrm{U}(1)_\mu \times \mathrm{U}(1)_\tau$ and does not lead to any conserved global symmetry.

(c) Dimension six operators involving four lepton fields, such as

$$\mathcal{L} \supset -\frac{1}{\Lambda^2} \left(\bar{L}_e \gamma^\rho L_e \right) \left(\bar{L}_e \gamma_\rho L_\mu \right) + \mathrm{h.c.} \tag{7}$$

(d) Flavor-violating neutrino magnetic moment operators of the form of

$$\mathcal{L} \supset \tfrac{1}{2} \mu_{ij} \bar{\nu}_{\mathrm{L}_i} \sigma_{\mu\nu} F^{\mu\nu} \nu_{\mathrm{L}_j} + \mathrm{h.c.} \tag{8}$$

The fact that the neutrino flavor conversion only occurs when the detector is very far away from the source excludes option (c), which induces the flavor transition at distance scales of the order of the Λ^{-1}, which are typically much smaller than the nuclear size. Besides, the observed dependence of the deficit of muon neutrinos with the distance and the energy disfavors neutrino decay [5] as the explanation of the data, thus ruling out option (d). Other explanations to single neutrino experiments, such as quantum decoherence [6] to explain the disappearance of muon neutrinos in the atmospheric fluxes, or the resonant spin-flip flavor conversion [7] to explain the disappearance of electron neutrinos in the solar fluxes, are currently ruled out by data.

Remarkably, almost all current experimental results[1] can be *simultaneously* explained within the Standard Model (with three neutrino flavors) extended by a mass term in the Lagrangian, either of the Dirac type or of the Majorana type (for a review, see [8]). If this is the case, the three neutrino interaction eigenstates $\alpha = e, \mu, \tau$, in general, do not coincide with the three mass eigenstates, $i = 1, 2, 3$, but are instead related by a unitary transformation $U_{\rm lep}$ [9]

$$|\nu_\alpha\rangle = (U_{\rm lep})_{\alpha i}|\nu_i\rangle\,. \tag{9}$$

The leptonic mass matrix depends on three mixing angles θ_{12}, θ_{13}, θ_{23} and one CP-violating phase δ for the case of Dirac neutrinos and three CP-violating phases, δ, ϕ, ϕ' for Majorana neutrinos [10]. The leptonic mixing matrix is conventionally parametrized as

$$\begin{aligned} U_{\rm lep} \;=\; & \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \\ & \times \left(e^{-i\phi/2}, e^{-i\phi'/2}, 1\right)\,, \end{aligned} \tag{10}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.

On the other hand, the neutrino mass eigenstates are labeled such that ν_3 is the eigenvalue which is most split in mass with respect to the other two, while ν_1 and ν_2 are ordered such that ν_1 is the lightest between them. Neutrino oscillation experiments are only sensitive to the mass splittings and not to the masses themselves. Therefore, present experiments allow two possible mass orderings: the "normal" hierarchy, $m_3 > m_2 > m_1$, and the "inverted" hierarchy, $m_2 > m_1 > m_3$. The present status of the determination of neutrino parameters from experiments is summarized in Table I.

Despite the tremendous progress over the last two decades in the determination of the neutrino parameters, many questions still remain open.

[1] Some experiments have reported indications for sterile neutrinos.

TABLE I

Three-flavor oscillation parameters from a global fit to neutrino experiments, assuming normal or inverted hierarchy. Table taken from [11].

	Normal hierarchy		Inverted hierarchy	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.382 \to 0.643$	$0.579^{+0.025}_{-0.037}$	$0.389 \to 0.644$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0186 \to 0.0250$	$0.0219^{+0.0011}_{-0.0010}$	$0.0188 \to 0.0251$
$\delta_{\mathrm{CP}}/^\circ$	306^{+39}_{-70}	$0 \to 360$	254^{+63}_{-62}	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.457^{+0.047}_{-0.047}$	$+2.317 \to +2.607$	$-2.449^{+0.048}_{-0.047}$	$-2.590 \to -2.307$

A particularly pressing question is whether the total lepton number is conserved or violated, which is intimately related to the question whether neutrinos are Dirac or Majorana particles. Current observations leave a two-fold degeneracy in the leptonic Lagrangian, which can be either of the form of

$$\mathcal{L}_{\mathrm{lep}} = \bar{L}_i i \not{D} L_i + \bar{e}_{\mathrm{R}_i} i \not{D} e_{\mathrm{R}_i} + \bar{\nu}_{\mathrm{R}_i} i \not{\partial} \nu_{\mathrm{R}_i} - m^e_i \bar{e}_{\mathrm{L}_i} e_{\mathrm{R}_i} - m^\nu_{ij} \bar{\nu}_{\mathrm{L}_i} \nu_{\mathrm{R}_j} + \mathrm{h.c.} \tag{11}$$

when the neutrino mass is of the Dirac-type, or

$$\mathcal{L}_{\mathrm{lep}} = \bar{L}_i i \not{D} L_i + \bar{e}_{\mathrm{R}_i} i \not{D} e_{\mathrm{R}_i} - m^e_i \bar{e}_{\mathrm{L}_i} e_{\mathrm{R}_i} - m^\nu_{ij} \bar{\nu}_{\mathrm{L}_i} \nu^c_{\mathrm{L}_j} + \mathrm{h.c.} \tag{12}$$

when the neutrino mass is of the Majorana-type. Breaking this two-fold degeneracy would constitute an important step towards understanding the origin of neutrino masses and is the subject of an intense experimental program worldwide.

2. Dirac or Majorana?

If neutrinos are Majorana particles, the nuclear decay $(A, Z) \to (A, Z + 2) + e^- e^-$, namely the double beta decay with the emission of two electrons but no missing energy, is allowed (for reviews, see [12, 13]). If observed, it would constitute a smoking gun for lepton number violation which, in turn, would imply Majorana neutrino masses. No experiment has reported convincing evidence for this process, thus leading to stringent lower limits on the decay width. The strongest current limits are set by the GERDA experiment, which uses germanium and which constrains the half-lifetime to be $T_{1/2}^{\mathrm{Ge}} > 2.1\times 10^{25}$ years [14], and by the KamLAND-Zen experiment, which uses xenon and which constrains $T_{1/2}^{\mathrm{Xe}} > 2.6 \times 10^{25}$ years [15]. On the other hand, no smoking gun for Dirac neutrinos has been identified. Nonetheless, the observation of a neutrino charge would constitute a very strong hint for Dirac neutrinos, since a Majorana mass term would explicitly break the electromagnetic U(1) symmetry, which is known to be very well-preserved in Nature.

From the theoretical and experimental points of view, both Majorana and Dirac neutrino masses are, at present, valid possibilities. Nevertheless, there are some theoretical arguments in favor of Majorana masses:

1. *It is the simplest possibility compatible with the Standard Model gauge symmetry.*

   If one considers just the Standard Model fields, the Lagrangian admits a non-renormalizable term of the form of

$$\mathcal{L} \supset -\frac{\alpha_{ij}}{\Lambda}\left(\bar{L}_i\tilde{\Phi}\right)\left(\tilde{\Phi}^T L_j^c\right) \tag{13}$$

   which leads, after electroweak symmetry breaking, to Majorana neutrino masses. Here, α_{ij} are flavor-dependent couplings and Λ is the scale of the new physics that generates this effective operator.

   If one adds right-handed neutrinos, then the new lowest dimensional operators allowed by the gauge symmetry are

$$\mathcal{L} \supset -h_{ij}^{\nu}\bar{L}_i\Phi\nu_{\mathrm{R}_j} - \frac{1}{2}\bar{\nu}_{\mathrm{R}_i}M_{ij}\nu_{\mathrm{R}_j}^c - \frac{\alpha_{ij}}{\Lambda}\left(\bar{L}_i\tilde{\Phi}\right)\left(\tilde{\Phi}^T L_j^c\right) \tag{14}$$

   which, again, lead to Majorana neutrino masses. In contrast, to have only the Dirac mass term, it is necessary to impose an extra U(1) symmetry for the conservation of the total lepton number.

2. *Majorana neutrino masses automatically lead to an electrically neutral neutrino and to an electrically neutral neutron.*

   In the Standard Model, the Higgs doublet can be chosen to have hypercharge $Y_{\Phi} = 1/2$. Then, the vacuum is invariant under the U(1) symmetry with generator $Q = T_3 + Y$, with T_3 the third component of isospin and Y the hypercharge; this is the standard charge generator. With this assignment, it can be shown that the requirement that the Yukawa couplings are invariant under $\mathrm{U}(1)_Y$ and the requirement of the cancellation of the gauge anomalies fix univocally the hypercharges of all fermion fields: $Y_Q = 1/6$, $Y_{u_{\mathrm{R}}} = 2/3$, $Y_{d_{\mathrm{R}}} = -1/3$, $Y_{L_{\mathrm{L}}} = -1/2$, $Y_{e_{\mathrm{R}}} = -1$.

Let us assume now that neutrinos have a Dirac mass, which requires the addition of right-handed neutrinos. The same calculation shows that the hypercharges are not univocally defined, but instead depend on one free parameter, which can be taken as the right-handed neutrino hypercharge, $Y_{\nu_\mathrm{R}} = -\epsilon$, such that $Y_Q = 1/6 + \epsilon/3$, $Y_{u_\mathrm{R}} = 2/3 + \epsilon/3$, $Y_{d_\mathrm{R}} = -1/3 + \epsilon_3$, $Y_{L_\mathrm{L}} = -1/2 - \epsilon$, $Y_{e_\mathrm{R}} = -1 - \epsilon$[2]. These assignments lead to an electric charge for the proton $Q_p = 1 + \epsilon$, for the electron $Q_e = -1 - \epsilon$, for the neutron $Q_n = +\epsilon$ and for the neutrino $Q_\nu = -\epsilon$, and which imply that the hydrogen atom is still electrically neutral, but both the neutron and the neutrino have an electric charge [16].

The strongest experimental upper limit on the neutrino charge is $Q_\nu < 2 \times 10^{-15}\, e$ from the energy spread and the dispersion in arrival times of the neutrinos from SN 1987A [17]. Moreover, from the non-observation of the deflection of neutrons in an electric field, $Q_n = (0.4 \pm 1.1) \times 10^{-21}\, e$ [18]. Then, in the framework of Dirac neutrino masses, it is puzzling why the neutrino and the neutron charges are so small, when *a priori* the parameter ϵ can take any value and even be $\mathcal{O}(1)$. A simple explanation why the neutrino electric charge is so small is to consider that neutrinos are instead Majorana particles. In this case, from the requirement of electric charge conservation, it follows that $\epsilon = 0$, thus explaining why $Q_\nu = 0$ and $Q_n = 0$, as hinted by the experimental data.

3. *Majorana neutrino masses might be the key to understand the striking differences between quark and neutrino parameters.*

   Experiments have revealed striking differences between the neutrino parameters and the quark parameters:

   (i) Quark masses are in the MeV or GeV mass range, while neutrino masses are in the sub-eV mass range. In particular, for the third generation, the top Yukawa coupling is $h_t \approx 1$ and the bottom Yukawa coupling $h_b \approx 0.02$, while the corresponding neutrino Yukawa coupling would be $h_{\nu_3} \approx 0.0000000000003$.

   (ii) The ratios among the up-type quark masses are $m_t/m_c \approx 140$, $m_c/m_u \approx 550$, and among the down-type quark masses, $m_b/m_s \approx 44$, $m_s/m_d \approx 19$, however, the ratio between the two

[2] The same result can be obtained noticing that there are two assignment of charges which are anomaly free with the matter content of the Standard Model extended by right-handed neutrinos, which are the standard hypercharge assignment and $B - L$. Therefore, any linear combination of both generators would lead to an anomaly free U(1) symmetry. In particular, one can take as hypercharges $Y = Y^{\mathrm{SM}} + \epsilon Q_{B-L}$, which are precisely the assignments listed in the main text.

largest neutrino eigenvalues is much milder

$$m_{\text{largest}}/m_{\text{next-to-largest}} \lesssim 6$$

(namely, $\simeq 6$ for normal hierarchy and $\simeq 1$ for inverted hierarchy).

(iii) The CKM mixing matrix displays a hierarchical structure, $|V_{ub}| \ll |V_{cb}| \ll |V_{us}|$, whereas in the leptonic mixing matrix all the entries are comparable in size.

If neutrinos were Dirac particles, it would be very puzzling why neutrino and quark parameters are so different, if they are generated by a similar term in the Lagrangian. On the other hand, if neutrinos were Majorana particles, new possibilities open to explain the differences between quark and neutrino parameters. Namely, the mass term is instead $m^{\nu}_{ij} = \alpha_{ij}\langle\Phi^0\rangle^2/\Lambda$, therefore, the smallness of neutrino masses could be either explained by a small α_{ij} or by a large Λ. Both possibilities are plausible. For the former, no Majorana fermion has been yet identified so there is no guidance about the size of α_{ij}, which could be small. For the latter, it may occur that the mechanism of neutrino mass generation lies at very high energies, thus explaining why the dimension five operator is so suppressed. Furthermore, in the Majorana framework, the mild neutrino hierarchy and the existence of large mixing angles in the leptonic sector are related to the coupling α_{ij}. As above, since there is no guidance from other sectors about the flavor structure of the couplings of Majorana fermions, it is plausible that α_{ij} could have a flavor structure with different characteristics as Dirac Yukawa couplings.

In the following, we will concentrate on the possibility that neutrinos are Majorana fermions.

3. Generation of a Majorana neutrino mass

Our goal is to generate from the Standard Model fields $L_i = \begin{pmatrix}\nu_{\mathrm{L}_i} \\ e_{\mathrm{L}_i}\end{pmatrix}$ and $\Phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$ a mass term of the form of

$$\mathcal{M} = -\tfrac{1}{2}\mathcal{M}_{\nu ij}\bar{\nu}_{\mathrm{L}_i}\nu^c_{L_j} + \text{h.c.} \tag{15}$$

In this mass term, we have the upper components of two SU(2) doublets, therefore, one has to introduce at least two more doublets to construct a gauge invariant term.

We have then to combine four $\mathrm{SU}(2)_{\mathrm{L}}$ doublets and find the combinations which are invariant under $\mathrm{SU}(2)_{\mathrm{L}}$, namely the singlet combinations. From the addition of angular momenta in quantum mechanics, it is well-known that two doublets lead to one singlet and one triplet

$$\mathbf{2}\times\mathbf{2}=\mathbf{3}+\mathbf{1}\,. \tag{16}$$

Therefore,

$$\mathbf{2}\times\mathbf{2}\times\mathbf{2}\times\mathbf{2}=(\mathbf{3}+\mathbf{1})\times(\mathbf{3}+\mathbf{1})\,. \tag{17}$$

Finally, taking into account that $\mathbf{3}\times\mathbf{3}=\mathbf{5}+\mathbf{3}+\mathbf{1}$, one concludes that the singlet combinations of the product of four doublets consist in the product of two triplets or to the products of two singlets. Now, the possible combinations of two doublets are $(L_i\Phi)$, $(\Phi\Phi)$ and (L_iL_j). Therefore, the possible combinations of four doublets which are $\mathrm{SU}(2)_{\mathrm{L}}$ invariant are:

$$(L_i\Phi)_1(L_j\Phi)_1\,,\qquad (L_iL_j)_1(\Phi\Phi)_1\,, \tag{18}$$

$$(L_i\Phi)_3(L_j\Phi)_3\,,\qquad (L_iL_j)_3(\Phi\Phi)_3\,, \tag{19}$$

where the subindex indicates whether the product of the two doublets is in the singlet or in the triplet representation. Clearly, since the singlet is antisymmetric under the exchange of the two doublets, the combination $(\Phi\Phi)_1=0$ and we are left with only three possibilities: $(L_i\Phi)_1(L_j\Phi)_1$, $(L_iL_j)_3(\Phi\Phi)_3$ and $(L_i\Phi)_3(L_j\Phi)_3$. Explicitly,

$$(L_iL_j)\rightsquigarrow\begin{cases}(L_iL_j)_1 & \to(\nu_i\ell_j-\ell_i\nu_j)\,/\sqrt{2}\\ (L_iL_j)_3 & \to(\nu_i\nu_j)\ ,\ \ (\nu_i\ell_j+\ell_i\nu_j)/\sqrt{2}\,,\ \ (\ell_i\ell_j)\end{cases}, \tag{20}$$

$$(L_i\Phi)\rightsquigarrow\begin{cases}(L_i\Phi)_1 & \to\left(\nu_i\phi^0-\ell_i\phi^+\right)/\sqrt{2}\\ (L_i\Phi)_3 & \to(\nu_i\phi^+)\ ,\ \ \left(\nu_i\phi^0+\ell_i\phi^+\right)/\sqrt{2}\,,\ \ \left(\ell_i\phi^0\right)\end{cases}, \tag{21}$$

$$(\Phi\Phi)\rightsquigarrow\begin{cases}(\Phi\Phi)_1 & \to\left(\phi^+\phi^0-\phi^0\phi^+\right)/\sqrt{2}=0\\ (\Phi\Phi)_3 & \to(\phi^+\phi^+)\ ,\ \ \left(\phi^+\phi^0+\phi^0\phi^+\right)/\sqrt{2}\,,\ \ \left(\phi^0\phi^0\right)\end{cases}. \tag{22}$$

Then, the combination which leads to neutrino masses $\nu_i\nu_j\phi^0\phi^0$ arises from

$$(L_i\Phi)_1(L_j\Phi)_1\rightsquigarrow\left(\nu_i\phi^0-\ell_i\phi^+\right)\left(\nu_j\phi^0-\ell_j\phi^+\right)\,, \tag{23}$$

$$(L_iL_j)_3(\Phi\Phi)_3\rightsquigarrow\nu_i\nu_j\phi^0\phi^0-(\nu_i\ell_j+\ell_i\nu_j)\,\phi^0\phi^++\ell_i\ell_j\phi^+\phi^+\,, \tag{24}$$

$$(L_i\Phi)_3(L_i\Phi)_3\rightsquigarrow\left(\nu_i\phi^0+\ell_i\phi^+\right)\left(\nu_j\phi^0+\ell_j\phi^+\right)-2\nu_i\ell_j\phi^+\phi^0-2\ell_i\nu_j\phi^0\phi^+ \tag{25}$$

which all correspond to the $\mathrm{SU}(2)\times\mathrm{U}(1)_Y$ invariant and Lorentz invariant dimension-5 Weinberg operator [19]

$$\mathcal{L}\supset-\frac{\alpha_{ij}}{\Lambda}\left(\bar{L}_i\tilde{\Phi}\right)\left(\tilde{\Phi}^TL_j^c\right)+\mathrm{h.c.} \tag{26}$$

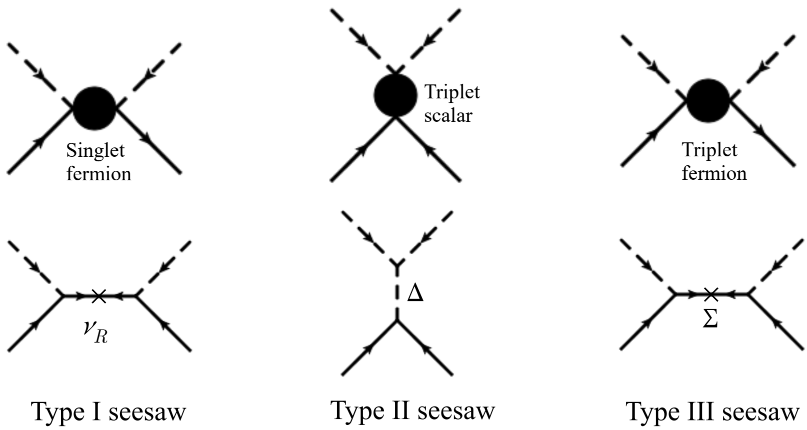

Fig. 1. Possible contractions of the $\mathrm{SU}(2)_\mathrm{L}$ indices leading to the Weinberg operator Eq. (26) and the corresponding realizations at tree level.

The three possibilities of the possible contractions of the $\mathrm{SU}(2)_\mathrm{L}$ indices are shown in the upper panel of Fig. 1. Besides, the Lorentz and gauge invariance of the theory also allows to determine the nature of the heavy particle that induces the corresponding interaction. In the first case, it must be a singlet fermion ν_R, in the second, a triplet scalar Δ and in the third, a triplet fermion Σ, as shown in the lower panel of Fig. 1. These are, respectively, the type I, type II and type III seesaw mechanism that we describe in the following:

Type I seesaw

The type I seesaw mechanism [20–23] introduces right-handed neutrinos (at least two) to the Standard Model particle content. Then, the most general Lagrangian compatible with the Standard Model gauge symmetry reads

$$-\mathcal{L} \subset h^\nu_{ij} \bar{L}_i \tilde{\Phi} \nu_{\mathrm{R}_j} + \tfrac{1}{2} \bar{\nu}^c_{\mathrm{R}_i} M_{ij} \nu_{\mathrm{R}_j} + \mathrm{h.c.} \tag{27}$$

This Lagrangian includes a Majorana mass matrix M_{ij} for the right-handed neutrinos, which explicitly breaks the total lepton number, and a Yukawa coupling h^ν_{ij} to the left-handed lepton doublet, which leads to a Dirac neutrino mass after electroweak symmetry breaking. The type I seesaw mechanism assumes that the mass scale of the right-handed neutrinos is much larger than the Dirac neutrino mass, $M \gg h^\nu \langle \Phi^0 \rangle$, therefore, at the low energies at which neutrino experiments are performed, the theory can be well-described by the effective Lagrangian Eq. (15), where

$$\mathcal{M}_\nu \approx -h^\nu M^{-1} h^{\nu T} \left\langle \Phi^0 \right\rangle^2 , \tag{28}$$

thus giving a suppression of the neutrino mass compared to the Dirac mass $h^\nu \langle \Phi^0 \rangle$ by a factor $\sim h^\nu \langle \Phi^0 \rangle / M \ll 1$.

Type II seesaw

The type II seesaw mechanism [24–26] introduces at least one scalar triplet with hypercharge 1, which can be cast in the form of

$$\Delta = \begin{pmatrix} \Delta^0 - & \Delta^+/\sqrt{2} \\ -\Delta^+/\sqrt{2} & \Delta^{++} \end{pmatrix} . \tag{29}$$

Then, the most general Lagrangian compatible with the gauge symmetry is

$$-\mathcal{L} \subset \left(Y_{ij}^{\Delta} L_i^T C \Delta L_j - \mu \tilde{\Phi}^T \Delta \tilde{\Phi} + \text{h.c.} \right) + M_\Delta^2 \text{Tr} \left(\Delta^\dagger \Delta \right) , \tag{30}$$

where Y_{ij}^{Δ} is a Yukawa coupling, μ is a lepton-number violating coupling with dimensions of mass, and M_Δ is the triplet mass. The seesaw mechanism assumes $M_\Delta^2 \gg \mu \langle \Phi^0 \rangle$, such that at low energies, the triplet effectively decouples, leading to a left-handed neutrino mass term Eq. (15) with

$$\mathcal{M}_\nu \approx \frac{\mu \left\langle \Phi^0 \right\rangle^2}{M_\Delta^2} Y^\Delta \tag{31}$$

which is of the order of $Y^\Delta \langle \Phi^0 \rangle$ times the small factor $\mu \langle \Phi^0 \rangle / M_\Delta^2 \ll 1$.

Type III seesaw

The type III seesaw [27] introduces fermion triplets (at least two) with hypercharge 0, which can be cast as

$$\Sigma_i = \begin{pmatrix} \Sigma_i^0 & \sqrt{2} \Sigma_i^+ \\ \sqrt{2} \Sigma_i^+ & -\Sigma_i^0 \end{pmatrix} \tag{32}$$

with Lagrangian

$$-\mathcal{L} \subset Y_{ij}^{\Sigma} \bar{L}_i \tilde{\Phi} \Sigma_j + \tfrac{1}{2} M_{ij} \text{Tr} \left(\bar{\Sigma}_i \Sigma_j^c \right) + \text{h.c.} , \tag{33}$$

where Y_{ij}^{Σ} is a Yukawa coupling and M_{ij} is a (lepton-number breaking) mass term for the fermion triplets. Under the assumption that

$M \gg Y^{\Sigma}\langle \Phi^0 \rangle$, the fermion triplet decouples at low energies, and the corresponding left-handed neutrino mass term reads

$$\mathcal{M}_\nu \approx -Y^{\Sigma} M_{\Sigma}^{-1} Y^{\Sigma\,T} \left\langle \Phi^0 \right\rangle^2 \tag{34}$$

which is of the order of $Y^{\Sigma}\langle \Phi^0 \rangle$ times the suppression factor $Y^{\Sigma}\langle \Phi^0 \rangle / M \ll 1$.

The type I seesaw mechanism is probably the most studied and most popular mechanism to generate light neutrino masses. In the next section, we will describe in detail its phenomenology and some of its possible observational consequences, apart from neutrino masses.

4. The type I seesaw mechanism

The type I seesaw mechanism is simple, elegant and compelling. Unfortunately, the seesaw tests are impeded by the large parameter space of the model, which allows right-handed masses ranging over many orders of magnitude. More specifically, the seesaw mechanism fixes the combination $\mathcal{M}_\nu = -M_{\mathrm{D}} M_{\mathrm{M}}^{-1} M_{\mathrm{D}}^{T}$ to be $\sim \mathcal{O}(0.1\ \mathrm{eV})$, however both the Dirac mass and the Majorana mass are free parameters. Various possibilities for the Dirac and the Majorana mass have been considered in the literature:

— $M_{\mathrm{D}} \sim 0.1$ eV, $M_{\mathrm{M}} \to 0$, which corresponds to the limit of total lepton number conservation, where neutrinos become Dirac particles.

— $M_{\mathrm{D}} \sim 1$ GeV, $M_{\mathrm{M}} \sim 10^{9}$–$10^{10}$ GeV, which has been discussed in the context of the generation of a cosmic matter–antimatter asymmetry via the out-of-equilibrium decay of the heavy right-handed neutrinos.

— $M_{\mathrm{D}} \sim 10^{-4}$ GeV, $M_{\mathrm{M}} \sim 1$ TeV, which has the interesting feature that the right-handed neutrinos then become *kinematically* accessible to collider experiments.

— $M_{\mathrm{D}} \sim 10$ eV, $M_{\mathrm{M}} \sim 1$ keV, which has been discussed in the context of dark matter, for which keV right-handed neutrinos might be a viable candidate.

In addition to the uncertainty on the mass scales of the Dirac and Majorana terms, there exists an additional uncertainty from the flavor structure of the parameters of the model, and which necessarily must be non-trivial, in order to accommodate the non-vanishing leptonic mixing angles and the neutrino mass ratios inferred from neutrino oscillation experiments.

Once the matricial structure of the couplings is taken into account, one finds a continuous family of neutrino Dirac masses compatible with the low-energy neutrino data, and which is spanned, in the Standard Model extended by three right-handed neutrinos, by nine parameters. Indeed, the high-energy theory is spanned by the neutrino Yukawa coupling and by the right-handed mass matrix, which contain in total 18 physical parameters: working in the basis where the right-handed neutrino mass matrix is diagonal, the free parameters are the three right-handed masses, the nine moduli of the elements of the neutrino Yukawa coupling, and six phases (since three can be rotated away by a redefinition of the leptonic doublets). On the other hand, the low-energy theory is spanned by the neutrino mass matrix, which only depends on 9 parameters: three masses, three mixing angles and three phases. There are then nine parameters of the type I seesaw Lagrangian which are unconstrained, even if the dimension 5 Weinberg operator could be perfectly determined from experiments.

One possible way to parametrize the high-energy parameters was proposed in [28]. The most general Dirac mass compatible with the low-energy neutrino parameters reads[3]

$$M_{\mathrm{D}} = i U_{\mathrm{lep}}^{*} \sqrt{D_m}\, \Omega \sqrt{D_{\mathrm{M}}}\, V^{\dagger}\,, \tag{35}$$

where U_{lep} is the unitary matrix which diagonalizes $\mathcal{M}_\nu$, $\mathcal{M}_\nu = U^* D_m U^\dagger$, V the matrix which diagonalizes M_{M}, $M_{\mathrm{M}} = V^* D_{\mathrm{M}} V^\dagger$, and Ω is an arbitrary complex orthogonal matrix: $\Omega\Omega^T = 1$. Indeed, it is straightforward to check that this Dirac mass matrix satisfies

$$\mathcal{M}_\nu = -M_{\mathrm{D}} D_{\mathrm{M}}^{-1} M_{\mathrm{D}}^{T} = U_{\mathrm{lep}}^{*} D_m U_{\mathrm{lep}}^{\dagger}\,. \tag{36}$$

Therefore, the nine parameters “lost” in the decoupling process correspond in this parametrization to the three right-handed neutrino masses in D_{M} and to the three complex parameters in Ω. While this parametrization uses as inputs the low-energy neutrino parameters, one should note that from a top–down perspective not all neutrino parameters are equally plausible. In fact, it has been argued that the type I seesaw model tends to generate a mass hierarchy $\Delta m^2_{\mathrm{atm}}/\Delta m^2_{\mathrm{sol}}$ which is much larger than the one required by experiments [30], unless the high-energy parameters take very concrete values. This result can be regarded as a drawback of the seesaw mechanism, or as a hint towards the high-energy parameters leading to the correct neutrino parameters[4].

---

[3] For an alternative parametrization, see [29].

[4] In an extension of the type I seesaw mechanism by a second Higgs doublet, however, the predicted mass hierarchy is naturally in qualitative agreement with the measured value [31, 32].

Just on phenomenological grounds, some regions of the parameter space of the type I seesaw model lead to interesting observational consequences, which we briefly discuss in the following subsections.

4.1. The low-scale seesaw model

From $\mathcal{M}_\nu \simeq -M_\mathrm{D} M_\mathrm{M}^{-1} M_\mathrm{D}^T$, it follows that, naively, when $M_\mathrm{M} \approx 1$ TeV then $m_\mathrm{D} \approx 10^{-4}$ GeV in order to generate neutrino masses not larger than $\mathcal{O}(0.1\ \mathrm{eV})$. This translates into a Yukawa coupling $\mathcal{O}(10^{-6})$, which induces low-energy effects too suppressed to be observed, unless the right-handed neutrino has additional interactions, for instance when it is charged under $\mathrm{U}(1)_{B-L}$. There is, however, a class of seesaw scenarios where the Yukawa couplings can be sizable while being the right-handed neutrinos at the TeV scale. To identify this class of scenarios, we use the parametrization Eq. (35). Assuming for simplicity just two right-handed neutrinos and normal hierarchy for the light neutrinos, the matrix Ω can be written as [33]

$$\Omega = \begin{pmatrix} 0 & 0 \\ \cos\hat{\theta} & \pm\sin\hat{\theta} \\ -\sin\hat{\theta} & \pm\cos\hat{\theta} \end{pmatrix} \tag{37}$$

with $\hat{\theta}$ a complex angle. Interestingly, there are choices of $\hat{\theta}$ which lead to a sizable M_D with entries in U of $\mathcal{O}(0.1)$, in D_m of $\mathcal{O}(10^{-10}\ \mathrm{eV})$ and in D_M of $\mathcal{O}(10^3\ \mathrm{GeV})$. Decomposing $\hat{\theta} = \omega - i\xi$ and taking $\xi \gg 1$, one finds

$$\Omega \simeq \frac{e^{i\omega}e^{\xi}}{2} \begin{pmatrix} 0 & 0 \\ 1 & \mp i \\ i & \pm 1 \end{pmatrix} . \tag{38}$$

Thus, by adjusting ξ, it is possible to construct a viable seesaw scenario with TeV mass right-handed neutrinos and arbitrarily large Dirac masses. It is also important to note that the matrix Ω introduces, in general, new sources of flavor structure in M_D which are unrelated to the flavor mixing observed in neutrino oscillation experiments. However, in the limit of $\xi \gg 1$, the extra flavor structure is fixed, thus rendering a fairly predictive scenario (see also [34–36]).

A large Dirac neutrino mass can significantly modify the charged-current (CC) and neutral-current (NC) leptonic interactions due to the large mixing between left- and right-handed neutrinos. The CC and NC interactions involving the light Majorana neutrinos have the form of

$$
\begin{aligned}
\mathcal{L}_{\mathrm{CC}}^{\nu} &= -\frac{g}{\sqrt{2}}\bar{\ell}\gamma_\rho\,[U_{\mathrm{lep}}]_{\ell i}\,\nu_i W^\rho + \mathrm{h.c.} \\
&\simeq -\frac{g}{\sqrt{2}}\bar{\ell}\gamma_\rho\left[\left(1-\frac{1}{2}(RV)(RV)^\dagger\right)U\right]_{\ell i}\nu_i W^\rho + \mathrm{h.c.}\,, \qquad (39)\\
\mathcal{L}_{\mathrm{NC}}^{\nu} &= -\frac{g}{2c_w}\bar{\nu}_i\gamma_\rho\left[U_{\mathrm{lep}}^\dagger U_{\mathrm{lep}}\right]_{ij}\nu_j Z^\rho + \mathrm{h.c.} \\
&\simeq -\frac{g}{2c_w}\bar{\nu}_i\gamma_\rho\left[U^\dagger\left(1-(RV)(RV)^\dagger\right)U\right]_{ij}\nu_j Z^\rho + \mathrm{h.c.}\,, \qquad (40)
\end{aligned}
$$

with $R^* \simeq M_{\mathrm{D}} M_{\mathrm{M}}^{-1}$. Furthermore, the left–right neutrino mixing gives rise to sizable CC and NC couplings of the heavy Majorana neutrinos ν_{R_j} to the W and Z bosons

$$
\mathcal{L}_{\mathrm{CC}}^{N} \simeq -\frac{g}{2\sqrt{2}}\bar{\ell}\gamma_\rho(RV)_{\ell i}(1-\gamma_5)\nu_{\mathrm{R}_i}W^\rho \;+\mathrm{h.c.}, \qquad (41)
$$

$$
\mathcal{L}_{\mathrm{NC}}^{N} \simeq -\frac{g}{2c_w}\bar{\nu}_{\ell L}\gamma_\rho\,(RV)_{\ell i}\nu_{\mathrm{R}_i}Z^\rho + \mathrm{h.c.} \qquad (42)
$$

Thus, the combination RV parametrizes the effects of the heavy neutrinos in low-energy phenomenology. Using the parametrization discussed above, this matrix can be cast as $RV \simeq -iU_{\mathrm{lep}}\sqrt{D_m}\,\Omega^*\sqrt{D_{\mathrm{M}}^{-1}}$ [37], which allows to express it in terms of the measurable neutrino parameters, the largest eigenvalue y of the matrix of neutrino Yukawa couplings and the heavy neutrino masses. Explicitly [38],

$$
(RV)_{\alpha 1} \;\simeq\; -e^{i\omega} y v\sqrt{\frac{M_2}{(M_1+M_2)}}\sqrt{\frac{m_3}{m_2+m_3}}\left(U_{\alpha 3}+i\sqrt{\frac{m_2}{m_3}}U_{\alpha 2}\right), \qquad (43)
$$

while $(RV)_{\alpha 2} \simeq \pm i\sqrt{M_1/M_2}(RV)_{\alpha 1}$.

The elements of the matrix RV and the right-handed neutrino masses are constrained by a series of low-energy experiments. The most important constraint in this scenario comes from measurements of neutrinoless double beta decay ($(\beta\beta)_{0\nu}$). In the presence of additional CC interactions, the effective Majorana mass $|(m_\nu)_{ee}|$, which controls the $(\beta\beta)_{0\nu}$ rate, reads [39,40]

$$
|(m_\nu)_{ee}| \simeq \left|\sum_{i=1}^{3} U_{ei}^2\,m_i - \sum_{k=1}^{2}\frac{M_a^2}{M_k}f(A)\,(RV)_{ek}^2\right|, \qquad (44)
$$

where $M_a \approx 0.9$ GeV and $f(A) = 0.079$ for ^{76}Ge. Thus, assuming $M_2 \sim 1000$ GeV and $|(RV)_{e2}| \sim 10^{-2}$, the experimental constraint $|(m_\nu)_{ee}| < 0.35$ eV [41] requires $(M_2 - M_1)/M_1 \lesssim 10^{-2}$. Therefore, in this scenario, the constraints from $(\beta\beta)_{0\nu}$ require the right-handed neutrinos to form a pseudo-Dirac pair in order to suppress the otherwise large lepton number violating effects [37].

Furthermore, the heavy neutrinos have lepton flavor violating CC interactions, which can contribute via quantum effects to the rare lepton decays. In contrast to the standard contribution to the process $\mu \to e\gamma$ from the light neutrinos, which is very suppressed by the tiny factor $\Delta m^2/M_W^2$ due to the GIM mechanism, the contribution from the heavy neutrinos is not GIM suppressed and could lead to large rates, unless the couplings are small [42–44].

Lastly, the sizable left–right neutrino mixing implies a 3×3 leptonic mixing matrix that is non-unitarity, which is severely constrained by present neutrino oscillation data and different measurements of electroweak processes (*e.g.*, on $W^{\pm}$ decays, invisible Z decays or universality tests of eletroweak interactions, see for instance [45]).

Figure 2 shows the summary of constraints on the parameter space of the low-scale seesaw scenario when the light neutrino spectrum has normal hierarchy, for $M_1 = 100$ GeV (left plot) and $M_1 = 1000$ GeV (right plot) and different Yukawa couplings [38]. From the plot, it follows that the most stringent constraint on the parameter space comes from neutrino oscillation experiments and the non-observation of the process $\mu \to e\gamma$, which restricts the Yukawa coupling to be $\lesssim 0.1$ (see also [46–48]). Nevertheless, in some regions of the parameter space, the search for the exotic Higgs decay

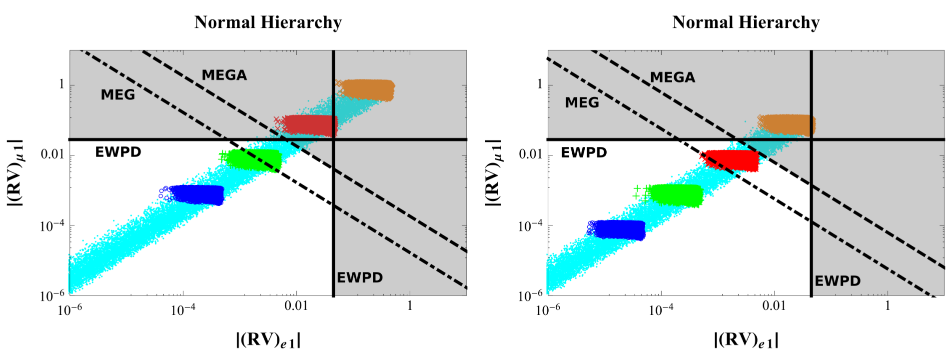

Fig. 2. (Color online) Constraints on the parameter space of the low-scale seesaw scenario, spanned by $|(RV)_{e1}|$ and $|(RV)_{\mu 1}|$ when the light neutrino spectrum has normal hierarchy for $M_1 = 100$ GeV (left plot) and $M_1 = 1$ TeV (right plot), and neutrino Yukawa eigenvalue $y = 0.001$ (blue $\circ$), $y = 0.01$ (green $+$), $y = 0.1$ (red $\times$) and $y = 1$ (orange $\diamond$). The light gray band (cyan points) corresponds to random values of $y \leq 1$. The gray areas are excluded by the present measurements of electroweak precision observables (solid lines) or by the constraint $\mathrm{BR}(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$ from MEGA (dashed line). The dot-dashed line corresponds to $\mathrm{BR}(\mu \to e\gamma) = 10^{-13}$, which is the projected sensitivity of the MEG experiment.

$h \to \nu_{\mathrm{R}_i}\nu_{\mathrm{L}_j}$, where the right-handed neutrino subsequently decays into jets or lepton–antilepton pairs, provides constraints on the seesaw parameters which are competitive with those from rare decays [49, 50].

4.2. Baryogenesis through leptogenesis

Cosmological observations reveal that there exists in our Universe a small excess of baryons over antibaryons. Concretely, the number of baryons minus the number of antibaryons, normalized to the number of photons, is [51]

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.21 \pm 0.16) \times 10^{-10} \,. \tag{45}$$

Whereas the concrete mechanism to generate the baryon asymmetry is yet unknown, it is commonly believed that it was dynamically generated in the very early Universe, through a mechanism denominated baryogenesis. This can occur when the renown Sakharov conditions [52] are simultaneously fulfilled:

1. The baryon number must be violated.
2. C and CP must be violated.
3. There must be a departure from thermal equilibrium.

Notably, these three conditions are satisfied in well-motivated particle physics scenarios, for instance in the decay of the heavy gauge bosons in the SU(5) grand unified theory, as pointed out by Yoshimura [53].

However, it was noticed in Ref. [54] that the three Sakharov conditions do not guarantee the generation of a baryon asymmetry in the decay of heavy particles. As shown by 't Hooft, the lepton and baryon number, which are accidental symmetries of the Standard Model, can be violated by non-perturbative effects [55]. Indeed, there is an infinite number of degenerate vacuum states, which differ from each other by their baryon and lepton numbers in units of three, and separated by a potential barrier with a height $\sim M_W/\alpha_W$. At zero temperature, the transitions among vacua only occur through tunneling, with an inverse rate which is much larger than the age of the Universe. Nevertheless, at high temperatures, higher than the temperature of the electroweak phase transition, the potential barrier can be easily overcome and transitions among vacua with different baryon and lepton numbers occurred very often, violating $B+L$, while preserving $B-L$. These transitions have a very big impact in the evolution of a baryon asymmetry in the early Universe, and can totally washout an initial baryon asymmetry if initially $B = L$, as is the case of Yoshimura's model [54]. Therefore, the first Sakharov condition should be reformulated to "$B - L$ number must be violated".

Interestingly, many models with Majorana neutrino masses satisfy the "revised" Sakharov conditions and lead to a lepton asymmetry in the early Universe through a mechanism denominated leptogenesis. Furthermore, if the lepton asymmetry is generated before the sphaleron transitions become out of thermal equilibrium, the lepton asymmetry can be efficiently converted into a baryon asymmetry. This is the renown leptogenesis mechanism [56], which provides a remarkable link between the mechanism of neutrino mass generation and the generation of the cosmic matter–antimatter asymmetry.

The leptogenesis mechanism can be implemented in the type I seesaw mechanism through the out-of-equilibrium decays of the lightest right-handed neutrino [56]. Indeed, the violation of $B-L$ is guaranteed if the neutrinos are Majorana particles, the violation of C and CP is guaranteed if the neutrino Yukawa couplings contain physical phases, and the departure from thermal equilibrium is guaranteed by the expansion of the Universe. It is now a quantitative question whether the type I seesaw mechanism can generate the observed baryon asymmetry.

Roughly speaking, the generation of the baryon asymmetry through leptogenesis proceeds in three steps. A lepton asymmetry is generated in the decay of the lightest right-handed neutrino into the Higgs and the lepton doublets. Then, the lepton asymmetry is washed-out by inverse decays, which violate lepton number by one unit, and by scatterings, which violate lepton number by two units. Finally, the lepton asymmetry is converted into a baryon asymmetry by the sphaleron transitions (for a review, see [57]). The resulting baryon asymmetry today approximately reads [58]

$$\eta_B \simeq 0.96 \times 10^{-2} \epsilon_1 \kappa_f \,. \tag{46}$$

Here, κ_f is an "efficiency factor" which parametrizes the impact of the inverse decays and scatterings in washing-out the lepton asymmetry, and which is $\kappa_f \lesssim 0.2$ when the abundance of right-handed neutrinos is equal to zero at very high temperatures, and $\kappa_f \lesssim 1$ when the abundance of right-handed neutrinos equals the thermal value. On the other hand, ϵ_1 is the CP asymmetry, which results from the interference between the tree-level and the one-loop decay diagrams shown in Fig. 3. Quantitatively,

$$\begin{aligned}\epsilon_1 &= \frac{\Gamma\left(\nu_{\mathrm{R}_1} \to L\Phi\right) - \Gamma\left(\nu_{\mathrm{R}_1} \to L^c\Phi^c\right)}{\Gamma\left(\nu_{\mathrm{R}_1} \to L\Phi\right) + \Gamma\left(\nu_{\mathrm{R}_1} \to L^c\Phi^c\right)} \\ &\simeq \frac{1}{8\pi}\frac{1}{\left(h_\nu^\dagger h_\nu\right)_{11}} \sum_{i=2,3} \mathrm{Im}\left[\left(h_\nu^\dagger h_\nu\right)^2_{1i}\right]\left[f\left(\frac{M_i^2}{M_1^2}\right) + g\left(\frac{M_i^2}{M_1^2}\right)\right] .\end{aligned} \tag{47}$$

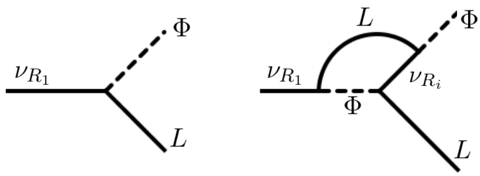

Fig. 3. Diagrams contributing to the generation of a CP asymmetry from the lightest right-handed neutrino decay.

One can explore the link between leptogenesis and the low-energy neutrino parameters substituting in this expression the most general Yukawa coupling compatible with the low-energy neutrino parameters, Eq. (35). Assuming a hierarchical spectrum for the right-handed neutrinos, one obtains

$$\epsilon_1 = \frac{3}{16\pi} \frac{M_1}{\langle \Phi^0 \rangle^2} \frac{m_j^2 \mathrm{Im}\left(\Omega_{1j}^2\right)}{m_j^2 \left|\Omega_{1j}\right|^2} \tag{48}$$

which depends on the light neutrino masses m_j, as well as on the lightest right-handed neutrino mass M_1 and on the unknown elements of the matrix Ω. One can show that the CP asymmetry is bounded from above by [59]

$$|\epsilon_1| \leq \frac{3}{16\pi} \frac{M_1}{\langle \Phi^0 \rangle^2} (m_3 - m_1)\,, \tag{49}$$

and, in particular, for a hierarchical neutrino spectrum

$$|\epsilon_1| \leq \frac{3}{16\pi} \frac{M_1 \sqrt{\Delta m^2_{\mathrm{atm}}}}{\langle \Phi^0 \rangle^2}\,. \tag{50}$$

Combining this expression with Eq. (46), it follows a lower limit on the right-handed neutrino mass from the requirement of successful baryogenesis through leptogenesis

$$M_1 \gtrsim \frac{6 \times 10^8 \text{ GeV}}{\kappa_f} \tag{51}$$

which translates into $M_1 \gtrsim 3 \times 10^9$ (6×10^8) GeV for a vanishing (thermal) initial abundance of right-handed neutrinos. Therefore, if leptogenesis is the correct mechanism to generate the observed matter–antimatter asymmetry, the new physics responsible for neutrino masses must lie at a very high energy scale, which makes this mechanism difficult to test[5].

[5] In supersymmetric scenarios, on the other hand, there are new opportunities to test leptogenesis, from the lepton flavor violation which is necessarily induced through quantum corrections on the slepton parameters [60].

4.3. keV-mass right-handed neutrinos as dark matter

The right-handed neutrinos in the type I seesaw are massive, electrically neutral and, in some regions of the parameter space, long lived. Therefore, they constitute a dark matter candidate [61] (for reviews, see [62, 63]). In the simplest version, the model contains just two free parameters, the sterile neutrino mass and the mixing angle between the active and the sterile neutrino, which arises after the electroweak symmetry breaking. In this simple scenario, the parameter space is constrained by the following considerations:

1. Sterile neutrinos can be produced in the early Universe via the active–sterile neutrino mixing [61]. The requirement that sterile neutrinos should not be overproduced sets an upper limit on the mixing angle as a function of the dark matter mass.

2. The existence of a lepton asymmetry can resonantly enhance the dark matter production [64]. An upper limit on the cosmic lepton asymmetry then implies a lower limit on the active–sterile neutrino mixing angle as a function of the sterile neutrino mass.

3. Sterile neutrinos are fermions and obey the Pauli exclusion principle, hence it is not possible to have an arbitrarily large dark matter number density in a region of space. The determination of the mass and the size of galaxy clusters or dwarf galaxies then allows to set a lower limit on the dark matter mass [65].

4. Sterile neutrinos are not absolutely stable particles but decay via quantum effects into an active neutrino and a photon, with a rate proportional to the fifth power of the mass and to the second power of the active–sterile mixing angle [66]. The photon produced in the decay could be detected in X- or γ-ray telescopes, thus leading to an upper limit on the mixing angle as a function of the mass.

The allowed parameter space of the model is shown in Fig. 4. As apparent from the plot, X-ray observations play a pivotal role in constraining the parameter space of the model. Conversely, the observation of a line in the X-ray sky which cannot be identified with nuclear transitions would constitute a strong hint for sterile neutrino decay.

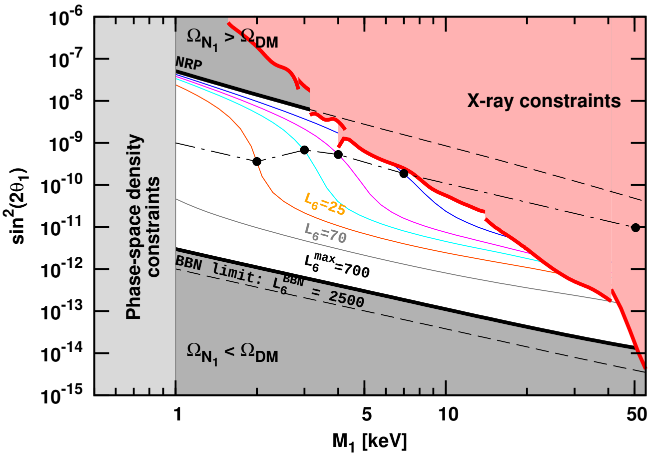

Fig. 4. Allowed parameter space of the sterile neutrino dark matter scenario, spanned by the sterile neutrino mass and the active–sterile mixing angle. Figure taken from [63].

5. Radiative Majorana neutrino masses

A possible explanation to the smallness of neutrino masses consists in postulating new particles with gauge (or discrete) quantum numbers which do not allow the generation of neutrino masses at tree level, but only at the N-loop level. Then, the neutrino mass is

$$\mathcal{M}^{\nu}_{ij} \sim \left(\frac{1}{16\pi^2}\right)^N \frac{\alpha_{ij}}{\Lambda} \left\langle \Phi^0 \right\rangle^2 , \tag{52}$$

where Λ is the scale of the new physics and α_{ij} are effective couplings generating the flavor structure of the neutrino mass matrix. Notably, the suppression of the radiatively generated neutrino masses by the loop factor allows to lower the scale of the new physics, Λ, even when the coupling constants α_{ij} are $\mathcal{O}(1)$, thus opening the exciting possibility of producing signals of the new physics in low-energy experiments.

At the one-loop level and in view of the gauge quantum number of the lepton doublet and the Higgs doublet, only four topologies can be constructed leading to the effective Weinberg operator [67]. In the loop, there must be a fermion and a scalar and the various topologies arise from *(i)* attaching one external Higgs line to the internal scalar line and to the internal fermion line, *(ii)* attaching both external Higgs lines to the internal scalar line or *(iii)* attaching both external Higgs lines to the internal fermion line. The various topologies are shown in Fig. 5.

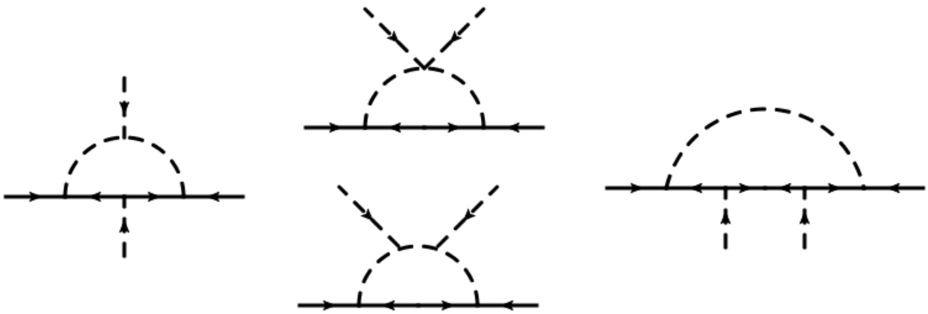

Fig. 5. Topologies leading to a Majorana neutrino mass at the one-loop level.

It is possible to systematically identify the gauge quantum numbers of the fermions and scalars necessary to construct the diagrams leading to one-loop neutrino masses. Let us for concreteness focus on the diagram where one Higgs line is attached to the internal scalar line and the other to the internal fermion line. To construct the diagram, one must introduce two fermions χ and ψ, and two scalars ω and η (see Fig. 6). The gauge invariance requires χ and ψ to have opposite color charges, namely χ has charge q_1 and ψ, q_1^*. Also, both χ and ψ must be a $\mathrm{SU}(2)_\mathrm{L}$ doublet. Then, we fix χ to be a doublet and hence ψ must be a singlet or a triplet, namely we assign to ψ an $\mathrm{SU}(2)_\mathrm{L}$ charge q_3 which is either 1 or 3. Finally, we assign a hypercharge q_2 to χ and hence ψ must have hypercharge $-q_2+1/2$. With this assignment, we can now identify the gauge quantum numbers of the particle η in the loop, from the requirement that the interaction with L_j and ψ is gauge invariant. Given the quantum number of the lepton doublet, the only possibility is that η has color charge q_1, hypercharge q_2 and $\mathrm{SU}(2)_\mathrm{L}$ charge 2. Finally, ω must have color charge q_1^*, hypercharge $-q_2+1/2$, and

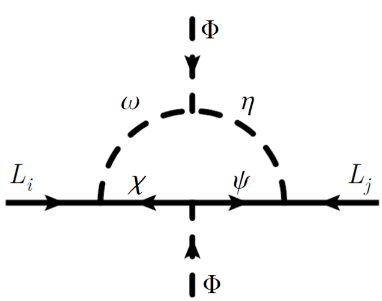

Fig. 6. One-loop diagram leading to Majorana neutrino masses and the various fields involved in the internal lines.

must be in a singlet or a triplet representation of $\mathrm{SU}(2)_\mathrm{L}$, since it couples to the two doublets χ and L_i (or to the doublets η and Φ). The gauge quantum numbers of the particles in the model are summarized in Table II.

TABLE II

Assignment of quantum numbers of the fields involved in the diagram Fig. 6 leading to Majorana neutrino masses at the one-loop level.

	L	Φ	χ	ψ	η	ω
$\mathrm{SU}(3)_\mathrm{C}$	1	1	q_1	q_1^*	q_1	q_1^*
$\mathrm{SU}(2)_\mathrm{L}$	2	2	2	q_3	2	q_4
$\mathrm{U}(1)_Y$	$-1/2$	$1/2$	q_2	$-q_2+1/2$	q_2	$-q_2+1/2$

Some exemplary choices of the quantum numbers are:

(i) $q_1=1$, $q_2=-1/2$, $q_3=1$, $q_4=1$. Then, $\chi=(1,2,-1/2)$ and $\psi=(1,1,1)$, namely, χ has the same quantum numbers as a lepton doublet L, and ψ has the same quantum numbers as the complex conjugate of a right-handed lepton singlet e_R^c. On the other hand, η and ω are new scalars with gauge quantum numbers $(1,2,-1/2)$ and $(1,1,1)$, respectively. This is the renown Zee model [68].

It is interesting to note that the scalar η (ω) has exactly the same quantum numbers as the spin 1/2 fermion χ (ψ), which can be identified with a lepton doublet (lepton singlet conjugate). The states η and ω arise naturally in supersymmetric frameworks as the superpartners of the lepton doublet and the lepton singlet, and can be identified as $\eta=\widetilde{L}$ and $\omega=\widetilde{e}_\mathrm{R}^c$, and the corresponding interactions in Fig. 6 arise when R-parity is violated. This diagram was proposed as a source of neutrino masses in supersymmetric scenarios with R-parity violation by Hall and Suzuki [69].

(ii) $q_1=3$, $q_2=1/6$, $q_3=1$, $q_4=1$. In this case, $\chi=(3,2,1/6)$ and can be identified with the quark doublet Q and $\psi=(3^*,1,1/3)$ can be identified with the conjugate of the down-type quark singlet, d_R^c. Besides, $\eta=(3,2,1/6)$ and $\omega=(3^*,1,1/3)$ have the same gauge quantum numbers as Q and d_R^c, but have instead zero spin. Again, in a supersymmetric framework, these particles could be identified with the supersymmetric partners of the quark doublet and down-type quark singlet, and the diagram in Fig. 6 leads, when R-parity is violated, to neutrino masses generated at the one-loop level, this time with colored articles circulating in the loop [69].

At the two-loop level, many more topologies arise (for a systematic study, see [70]). A renown example is the Zee–Babu model [71, 72], which includes two new fields: a singly charged singlet scalar $\omega^\pm$ and a doubly charged singlet scalar $k^{\pm\pm}$. The two-loop diagram leading to Majorana neutrino masses is shown in Fig. 7, left panel. Furthermore, some models have been constructed where neutrino masses arise at the three-loop level. One example is the Krauss–Nasri–Trodden model [73], which introduces three new fields: two singly charged singlet scalars, $S_1^\pm$, $S_2^\pm$ and one neutral singlet fermion N_R. The corresponding diagram is shown in Fig. 7, right panel.

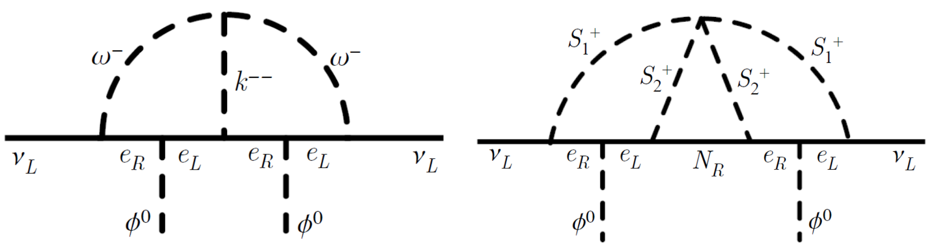

Fig. 7. Some explicit models of Majorana neutrino masses generated at the two-loop (left panel) or the three-loop (right panel) level.

Many neutrino mass models contain a neutral particle with the characteristics of the dark matter. However, in many of these models, the neutral particle quickly decays into Standard Model particles; this is the case in particular of the type I seesaw model. It has been argued, however, that the particles in the loop may carry a conserved (or approximately conserved) global quantum number. Then, the lightest particle charged under this new global charge is long-lived and may constitute the dark matter of the Universe. Furthermore, its couplings with the Standard Model particles would allow their production in the early Universe through the freeze-out mechanism, with a relic abundance which can be compatible with the dark matter abundance measured by the Planck satellite.

A simple example is the "scotogenic" model proposed by Ma in Ref. [74]. It introduces only two new fields: a scalar doublet with hypercharge 1/2, η, and a fermion singlet with zero hypercharge, χ. The model postulates that the vacuum displays an exact Z_2 symmetry under which the new fields are odd, while the SM fields are even. In this model, neutrino masses are generated at the one-loop level, through the diagram shown in Fig. 8 (particles which are odd under the Z_2 symmetry are shown in gray/red). Furthermore, the Z_2 symmetry prevents tree-level charged lepton flavor violation and renders stable the lightest odd particle in the spectrum, which becomes a dark matter candidate. In this model, the role of dark matter can be played by

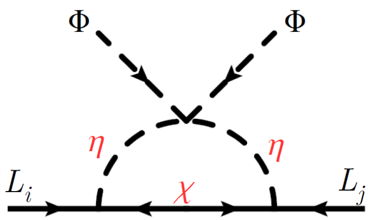

Fig. 8. Diagram leading to Majorana neutrino masses in the "scotogenic" model. The fields in gray/red are odd under a conserved Z_2 symmetry, while the fields in black are even.

the neutral scalar or pseudoscalar or by the lightest singlet fermion. As the tree level type I seesaw model, the scotogenic model tends to generate too large neutrino mass hierarchies; this rawback of the model can be remedied by introducing a second Z_2-odd scalar doublet, η' [75].

6. Conclusions

Understanding the origin of the fermion masses is one of the most pressing open questions in Fundamental Physics. The discovery of neutrino oscillations, which imply that neutrinos are massive, has added a new element to the puzzle of the origin of the fermion masses, especially in view of the striking differences between the quark and the neutrino parameters. Many interesting ideas have been proposed in the literature, however, the actual mechanism that generates fermion masses, and concretely neutrino masses, is yet to be determined. Unfortunately, the most appealing theoretical frameworks to generate neutrino masses introduce new physics at very high energies, or new particles very weakly coupled to the neutrinos, thus making these models very difficult to test. On the other hand, some possibilities are actually testable and some regions of the parameter space of the models have been already ruled out or will be probed by the next generation of experiments at the energy frontier and at the intensity frontier.

The origin of the fermion masses is not the only open question in Fundamental Physics. Namely, the nature of the dark matter, the origin of the cosmic matter–antimatter asymmetry, the strong CP problem, the hierarchy problem, the cosmological constant problem, *etc.* are still not understood. It is an interesting possibility that the solution to some of these problems could also be related to the mechanism of neutrino mass generation. In fact, some neutrino mass models naturally predict, as a bonus, a cosmic matter–antimatter asymmetry (generated through leptogenesis) and/or a dark matter candidate. Therefore, experiments at the cosmic frontier might also shed light on the origin of neutrino masses.

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