COMPLEX SCALING IN NEUTRINO MASS MATRIX*

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Using the residual symmetry approach, we propose a complex extension of the scaling Ansatz on M_ν which allows a nonzero mass for each of the three light neutrinos as well as a nonvanishing θ_{13}. Leptonic Dirac CP violation must be maximal, while atmospheric neutrino mixing need not be exactly maximal. Each of the two Majorana phases to be probed by the search for $0\nu\beta\beta$ decay has to be zero or π and a normal neutrino mass hierarchy is allowed.

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If $G_i^T M_\nu G_i = M_\nu$ defines a horizontal symmetry for the complex symmetric M_ν and $U^T M_\nu U = M_d$, where M_d has only real positive diagonal nondegenerate elements, then another unitary matrix $V = Ud$ also puts M_ν into a diagonal form, where $d = \mathrm{diag}(d_1, d_2, d_3)$ with $d_{i(i=1,2,3)} = \pm 1$. Moreover, $U^\dagger G_i U = d_i$. Each d_i defines a Z_2 symmetry and the corresponding G_i is also a representation of that Z_2 symmetry. Among eight possible forms of d_i, only two can be shown to be independent, taken as $d_2 = \mathrm{diag}(-1, 1, -1), d_3 = \mathrm{diag}(-1, -1, 1)$. Thus, the two independent representations $G_{2,3}$ describe a residual $Z_2 \times Z_2$ flavor symmetry [1, 2] in M_ν. In this way, we reinterpret the Simple Real Scaling Ansatz [3] in M_ν as a $Z_2 \times Z_2$ symmetry. We further make a complex extension of this invariance and obtain the corresponding M_ν. Interesting phenomenological consequences follow. Here, we sketch our method and present the basic results leaving many details to a future lengthier publication [4]. Throughout, we follow the PDG convention.

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The Simple Real Scaling Ansatz [3] attributes the following structure to the neutrino mass matrix

$$M_\nu^{\rm SRS} = \begin{pmatrix} X & -Yk & Y \\ -Yk & Zk^2 & -Zk \\ Y & -Zk & Z \end{pmatrix} \quad (1)$$

with X, Y, Z as complex mass dimensional quantities and k as a real positive dimensionless scaling factor. It has one vanishing mass eigenvalue with the corresponding eigenvector $(0, \frac{e^{i\frac{\beta}{2}}}{\sqrt{1+k^2}}, \frac{ke^{i\frac{\beta}{2}}}{\sqrt{1+k^2}})^T$. The mixing matrix is

$$U^{\rm SRS} = \begin{pmatrix} c_{12} & s_{12}e^{i\frac{\alpha}{2}} & 0 \\ -\frac{ks_{12}}{\sqrt{1+k^2}} & \frac{kc_{12}e^{i\frac{\alpha}{2}}}{\sqrt{1+k^2}} & \frac{e^{i\frac{\beta}{2}}}{\sqrt{1+k^2}} \\ \frac{s_{12}}{\sqrt{1+k^2}} & -\frac{c_{12}e^{i\frac{\alpha}{2}}}{\sqrt{1+k^2}} & \frac{ke^{i\frac{\beta}{2}}}{\sqrt{1+k^2}} \end{pmatrix} \quad (2)$$

with an arbitrary θ_{12} and Majorana phases α, β. Now, $G_{2,3}$ can be calculated from $Ud_{2,3}U^\dagger$ to be

$$G_2^k = \begin{pmatrix} -\cos 2\theta_{12} & \frac{k\sin\theta_{12}}{\sqrt{1+k^2}} & -\frac{\sin\theta_{12}}{\sqrt{1+k^2}} \\ \frac{k\sin\theta_{12}}{\sqrt{1+k^2}} & \frac{k^2\cos 2\theta_{12}-1}{1+k^2} & \frac{-k(\cos 2\theta_{12}+1)}{1+k^2} \\ -\frac{\sin\theta_{12}}{\sqrt{1+k^2}} & \frac{-k(\cos 2\theta_{12}+1)}{1+k^2} & \frac{\cos 2\theta_{12}-k^2}{1+k^2} \end{pmatrix},$$

$$G_3^{\rm scaling} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1-k^2}{1+k^2} & \frac{2k}{1+k^2} \\ 0 & \frac{2k}{1+k^2} & \frac{k^2-1}{1+k^2} \end{pmatrix}. \quad (3)$$

The form of $U^{\rm SRS}$ in (2) implies a vanishing s_{13}. Since this has been experimentally excluded at $> 10\sigma$, the SRS Ansatz has to be discarded. However, we shall retain G_2^k as well as $G_3^{\rm scaling}$ and propose a complex extension. Our complex extension postulates

$$\left(G_3^{\rm scaling}\right)^T (M_\nu)^{\rm CES} G_3^{\rm scaling} = \left(M_\nu^{\rm CES}\right)^* . \quad (4)$$

The corresponding mass matrix $M_\nu^{\rm CES}$ can be deduced to be

$$M_\nu^{\rm CES} = \begin{pmatrix} x & -y_1k + i\frac{y_2}{k} & y_1 + iy_2 \\ -y_1k + i\frac{y_2}{k} & z_1 - w_1\frac{k^2-1}{k} - iz_2 & w_1 - i\frac{k^2-1}{2k}z_2 \\ y_1 + iy_2 & w_1 - i\frac{k^2-1}{2k}z_2 & z_1 + iz_2 \end{pmatrix}, \quad (5)$$

where x, y_1, y_2, z_1, z_2 and w are real mass dimensional quantities. Equation (4) implies $U^\dagger G_3 U^* = \tilde{d}$ or

$$G_3 U^* = U\tilde{d}\,. \tag{6}$$

Once again, $\tilde{d}_{lm} = \pm\delta_{lm}$ if the neutrino masses $m_{1,2,3}$ are all nondegenerate. The l.h.s. of (6) can be written out as

$$\begin{pmatrix} -(U_{e1}^{\rm CES})^* & -(U_{e2}^{\rm CES})^* & -(U_{e3}^{\rm CES})^* \\ \frac{1-k^2}{1+k^2}\left(U_{\mu1}^{\rm CES}\right)^* + \frac{2k}{1+k^2}(U_{\tau1}^{\rm CES})^* & \frac{1-k^2}{1+k^2}\left(U_{\mu2}^{\rm CES}\right)^* + \frac{2k}{1+k^2}(U_{\tau2}^{\rm CES})^* & \frac{1-k^2}{1+k^2}\left(U_{\mu3}^{\rm CES}\right)^* + \frac{2k}{1+k^2}(U_{\tau3}^{\rm CES})^* \\ \frac{2k}{1+k^2}\left(U_{\mu1}^{\rm CES}\right)^* - \frac{1-k^2}{1+k^2}(U_{\tau1}^{\rm CES})^* & \frac{2k}{1+k^2}\left(U_{\mu2}^{\rm CES}\right)^* - \frac{1-k^2}{1+k^2}(U_{\tau2}^{\rm CES})^* & \frac{2k}{1+k^2}\left(U_{\mu3}^{\rm CES}\right)^* - \frac{1-k^2}{1+k^2}(U_{\tau3}^{\rm CES})^* \end{pmatrix}. \tag{7}$$

The reality of $(U_{\rm PMNS})_{e1}$ rules out the possibility $(\tilde{d}_i)_{11} = 1$. Now, there are four cases: $\tilde{d}_{\rm a} \equiv {\rm diag}(-1,1,1)$, $\tilde{d}_{\rm b} \equiv {\rm diag}(-1,1,-1)$, $\tilde{d}_{\rm c} \equiv {\rm diag}(-1,-1,1)$, $\tilde{d}_{\rm d} \equiv {\rm diag}(-1,-1,-1)$.

These structures of $\tilde{d}$ and the use of (6) lead to the equations given in the following table.

Elements of $U^{\rm CES}$	Constraint conditions
$\mu1$	$2kU_{\mu1}^{\rm CES} = \left(1-k^2\right)U_{\tau1}^{\rm CES} - \left(1+k^2\right)(U_{\tau1})^*$
$\tau1$	$2kU_{\tau1}^{\rm CES} = -\left(1-k^2\right)U_{\mu1}^{\rm CES} - \left(1+k^2\right)(U_{\mu1})^*$
$\mu2$	$2kU_{\mu2}^{\rm CES} = \left(1-k^2\right)U_{\tau2}^{\rm CES} + \eta\left(1+k^2\right)(U_{\tau2})^*$
$\tau2$	$2kU_{\tau2}^{\rm CES} = -\left(1-k^2\right)U_{\mu2}^{\rm CES} + \eta\left(1+k^2\right)(U_{\mu2})^*$
$\mu3$	$2kU_{\mu3}^{\rm CES} = \left(1-k^2\right)U_{\tau3}^{\rm CES} + \xi\left(1+k^2\right)(U_{\tau3})^*$
$\tau3$	$2kU_{\tau3}^{\rm CES} = -\left(1-k^2\right)U_{\mu3}^{\rm CES} + \eta\left(1+k^2\right)(U_{\mu3})^*$

These equations lead to the result that (i) for the case a, $\alpha = \pi$, $\beta = 0$, (ii) for the case b, $\alpha = \pi$, $\beta = \pi$, (iii) for the case c, $\alpha = 0$, $\beta = 0$ and (iv) for the case d, $\alpha = 0$, $\beta = \pi$. Further, $\cos\delta = 0$, where δ is the Dirac phase in $U_{\rm PMNS}$. In addition, we have the prediction $\tan\theta_{23} = k^{-1}$ which implies that the atmospheric mixing angle need not be strictly maximal. We have taken the 3σ ranges [5] for the quantities $|\Delta m^2_{31}|$, Δm^2_{21}, θ_{12}, θ_{23}, θ_{13} for our phenomenological analysis. We also take the upper bound 0.23 eV on the sum of the light neutrino masses.

Our conclusions are the following:

1. Both types of neutrino mass hierarchy are now allowed.
2. For normal hierarchy, the lightest mass m_1 ranges from 10^{-4} eV to 0.07 eV and for inverted hierarchy the lightest mass m_3 ranges from 10^{-4} eV to 0.068 eV.

3. For both hierarchies, the quantity $|m_{ee}|$ of relevance to $0\nu\beta\beta$ decay can reach up to the value 0.14 eV which will be probed by GERDA phase II data.

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