DIFFERENTIAL STRUCTURE OF NON-LOCAL THEORIES, I

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It is shown on an example of a one-dimensional linear integro-differential equation how to obtain a differential description of non-local systems. In Section 2 the equivalence of integro-differential and pure differential equations of the same order is proved under certain conditions about the kernel. In Sections 3 and 5 the discussion of the coefficients of the differential equation is carried out. Section 4 is devoted to the canonical quantization of the system. Finally, in Section 6, another direct approach to the considered problem is given.

1. *Introduction*

In the mathematical description of elementary particles non-local theories play an important role. The structure of these theories, however, is not yet fully clarified. This concerns particularly the question to what extent canonical formulation and quantization are possible.

The first to rise this question was Pauli (1953). He considers a particular type of non-local field theory and by use of the method of successive approximations constructs canonical field quantities in the first order approximation in the coupling constant.

We shall approach this problem from a different point of view which allows to develop the canonical formalism in a closed form, at least for the case of linear non-local theories. The basic idea of this approach is the equivalence of the integro-differential equations corresponding to non-local theories with pure differential equations of the same order.

In the present paper the most simple example is treated in some detail, namely the integro-differential equation

$$\ddot{q}(t) + \varkappa^2 q(t) = \lambda \int_a^b K(t, t')dt'q(t') \tag{1.1}$$

Here $q(t)$ is the unknown function, t the independent variable, $\varkappa^2$ and λ are positive constants, dotts denote differentiations with respect to time. $K(t,t')$ is the kernel of the equation; it describes interaction with external systems. Assuming

$$K(t\,t') = \Phi(t)\,G(t - t'), \tag{1.2}$$

one has a complete analogy of the one-dimensional equation (1,1) with the four-dimensional partial integro-differential equation describing non-local interaction of a charged scalar field $q(x, y\ z, t)$ with an external potential $\Phi(x, y, z, t)$. (1,1) may be also considered as an equation for the non-local oscillator.

The ideas developed here for the sake of simplicity for the particular case of equation (1.1) apply also to the general case of an arbitrary ordinary integro-differential equation. For general linear problems the procedure of the present paper may be directly applied. Generalization of the procedure to partial integro-differential equations (non-local field theory) and to non-linear problems (interacting systems) shall be given in a forthcoming paper.

2. Equivalence between integro-differential and differential equations

To relate an integro-differential equation of type (1,1) with a pure differential equation, it is necessary to know the manifold of solutions of the original equation. It may be shown that, with certain assumptions about λ and $K(t, t')$, the manifold of solutions of (1.1) contains two arbitrary parameters. For this purpose let us introduce the function

$$\Delta^r(t) = \eta^r(t)\frac{1}{\varkappa}\sin\varkappa t, \qquad \eta^r(t) = \begin{cases} 1, & t > 1 \\ 0, & t < 1 \end{cases} \tag{2.1}$$

Multiplying (1.1) by $\Delta^r(\tau - t)$ and integrating with respect to t over the interval (a,b) one obtains the integral equation

$$q(\tau) = q^0(\tau) + \lambda\int_a^b N(\tau,t)dt q(t), \tag{2,2}$$

where

$$q^0(t) = \frac{1}{\varkappa}\dot{q}(a)\sin\varkappa(t - a) + q(a)\cos\varkappa(t - a), \tag{2.3}$$

$$N(\tau, t) = \int_a^b \Delta^r(\tau - t')dt'\,K(t', t). \tag{2.4}$$

Obviously, each solution of (1.1) is a solution of (2.2) and vice versa.

Let us assume that $K(\tau, t)$ is bounded in the domain $a\leqslant\tau\leqslant b$, $a\leqslant t\leqslant b$. Then $N(\tau, t)$ is also bounded. If λ is not an eigenvalue of the kernel $N(\tau, t)$, the integral equation (2.2) possesses, according to well known theorems, one unique solution corresponding to each particular $q^0(\tau)$. Since the functions $q^0(\tau)$ form a two-parametric manifold with the arbitrary initial values $\dot{q}(a)$ and $q(a)$ as parameters, the solutions of (2.2) and, therefore, also of (1.1) form a two-parametric manifold. According to another well-known theorem, this manifold satisfies a second order differential equation which may be obtained by elimination of the parameters.

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As stated in the introduction, the above theorems have general validity not restricted to the special type of equation considered in this paper (cf. Krzywicki, Rzewuski, Zamorski, and Zięba 1954).

For the linear case (1.1) it is easy to write down the explicit form of the differential equation equivalent to (1.1). For this purpose the solution of the integral equation (2.2) has to be written down by means of the resolving kernel

$$q(\tau) = q^0(\tau) + \lambda \int_a^b R(\tau, t) dt\, q^0(t) \tag{2.5}$$

where

$$R(\tau, t) = N(\tau, t) + \lambda \int_a^b N(\tau, t') dt'\, R(t', t). \tag{2.6}$$

Introducing (2.3) into (2,5), one gets

$$\dot{q}(a)\, r_1(t) + q(a)\, r_2(t) - q(t) = 0 \tag{2.7}$$

where

$$\begin{aligned} r_1(t) &= \frac{1}{\varkappa} \sin \varkappa(t-a) + \lambda \int_a^b R(t, t')\, dt' \frac{1}{\varkappa} \sin \varkappa(t'-a) \\ r_2(t) &= \cos \varkappa(t-a) + \lambda \int_a^b R(t, t') dt' \cos \varkappa(t'-a). \end{aligned} \tag{2.8}$$

Elimination of the arbitrary parameters $\dot{q}(a)$ and $q(a)$ from (2.7) and from its first and second derivative yields

$$\begin{vmatrix} r_1 & r_2 & q \\ \dot{r}_1 & \dot{r}_2 & \dot{q} \\ \ddot{r}_1 & \ddot{r}_2 & \ddot{q} \end{vmatrix} = 0 \tag{2.9}$$

This is a second order linear differential equation of the form

$$a_2 \ddot{q} + a_1 \dot{q} + a_0 q = 0 \tag{2.10}$$

with variable coefficients

$$a_0 = \begin{vmatrix} \dot{r}_1 & \dot{r}_2 \\ \ddot{r}_1 & \ddot{r}_2 \end{vmatrix}, \quad a_1 = - \begin{vmatrix} r_1 & r_2 \\ \ddot{r}_1 & \ddot{r}_2 \end{vmatrix}, \quad a_2 = \begin{vmatrix} r_1 & r_2 \\ \dot{r}_1 & \dot{r}_2 \end{vmatrix} \tag{2.11}$$

We notice the relation

$$a_1 = -\dot{a}_2. \tag{2.12}$$

Equation (2.10) is equivalent to (1,1) in the sense that (with the above assumptions about λ and $K(\tau, t)$) each solution of (1.1) satisfies (2.10) and vice versa.

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The quantization of this equation may be carried out by conventional methods. We shall come back to this question in Section 4.

As stated in the introduction, the above construction of a differential equation which is equivalent to a given integro-differential equation may be carried over without essential difficulties to the general case of linear integro-differential equations of arbitrary order with variable coefficients containing derivatives also under the integral sign (cf. Krzywicki, Rzewuski, Zamorski, and Zięba 1954). In the important case of systems of non-linear integro-differential equations an approximate method must be used. We shall come back to this question in a forthcoming paper.

3. Discussion of the coefficients

For the consideration to follow it is important to carry out a detailed discussion of the coefficients in equation (2,10). Their explicit form is

$$a_1 = -\varkappa^2(1+c_0), \quad a_2 = -(1+c_2) \tag{3.1}$$

with

$$\begin{aligned} C_0 &= -\lambda\int_a^b \dot{R}(t,t')\frac{1}{\varkappa}\sin\varkappa(t-t')dt' - \lambda\int_a^b \ddot{R}(t,t')\frac{1}{\varkappa^2}\cos\varkappa(t-t')dt' \\ &+ \lambda^2\iint_a^b \ddot{R}(t,t')\frac{1}{\varkappa^3}\sin\varkappa(t'-t'')\dot{R}(t,t'')dt'dt'' \\ C_2 &= -\lambda\int_a^b \dot{R}(t,t')\frac{1}{\varkappa}\sin\varkappa(t-t')dt' + \lambda\int_a^b R(t,t')\cos\varkappa(t-t')dt' \\ &- \lambda^2\iint_a^b R(t,t')\frac{1}{\varkappa}\sin\varkappa(t'-t'')\dot{R}(t,t'')dt'dt'' \end{aligned} \tag{3.2}$$

(Dotts denote always differentiation with respect to the first argument).

With help of the notation (3.1), equation (2.10) may be written

$$(1+c_2)\ddot{q} - \dot{c}_2\dot{q} + \varkappa^2(1+c_0)q = 0. \tag{3.3}$$

From (3.2) it is easily seen that for sufficiently small λ, $1+c_2 > 0$ in the interval (a, b) and we may write (3.3) in the form

$$\ddot{q} + \varkappa^2 q = \frac{\dot{c}_2\dot{q} + \varkappa^2(c_2 - c_0)q}{1+c_2} \tag{3.4}$$

Another form, not restricted by the condition of smallness for λ, is

$$\ddot{q} + \varkappa^2 q = -c_2\ddot{q} + \dot{c}_2\dot{q} - \varkappa^2 c_0 q. \tag{3.5}$$

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In both (3.4) and (3.5) the right hand sides are proportional to λ and vanish as $\lambda \to 0$, which corresponds to the case of no external forces.

The transition to a local theory is desribed by

$$G(t) \to \delta(t) \tag{3.6}$$

($\delta(t)$ — Dirac's symbol). In this case, on account of (1.2), the right hand side of the original equation (1.1) goes over into $\lambda\, \Phi\, q$. The same must hold for the right-hand side of (3.4) and (3.5). To verify that this is indeed the case, let us calculate the coefficients $\dot{c}_2$ and $\varkappa^2 (c_2 - c_0)$. From (3.2) one finds

$$\begin{aligned} \varkappa^2 (c_2 - c_0) &= \lambda \int_a^b M(t, t') \cos \varkappa (t - t')\, dt' \\ &- \lambda^2 \iint_a^b M(t, t') \sin \varkappa (t' - t'') \frac{1}{\varkappa} \dot{R}(t, t'')\, dt'dt'' \\ \dot{c}_2 &= - \lambda \int_a^b M(t, t') \frac{1}{\varkappa} \sin \varkappa (t - t')\, dt' \\ &+ \lambda^2 \iint_a^b M(t, t') \sin \varkappa (t' - t'') \frac{1}{\varkappa} R(t, t'')\, dt'dt'' \end{aligned} \tag{3.7}$$

where

$$M(t, t') \equiv \ddot{R}(t, t') + \varkappa^2 R(t, t') = K(t, t') + \lambda \int_a^b K(t, t'')\, dt''\, R(t'', t'), \tag{3.8}$$

the last equation stemming from the properties of the function $\Delta^r(t)$. In the limiting case (3.6)

$$M(t, t') \to \Phi(t) \left[\delta(t, - t') + \lambda R(t, t') \right] \tag{3.9}$$

and (after some calculations)

$$\begin{aligned} \varkappa^2 (c_2 - c_0) &\to \lambda\, \Phi(t)\, (1 + c_2) \\ \dot{c}_2 &\to 0 \end{aligned} \tag{3.10}$$

Finally from the last equation (3.10) the bahaviour of c_2 may be deduced. In fact, we may write

$$c_2(t) = \int_a^t \dot{c}_2(t)\, dt, \tag{3.11}$$

since $c_2(a) = 0$, as follows from the properties of the function $\Delta^r(t)$ (cf.(2.1), (2.4), (2.6), and 3.2)). In the limiting case (3.6) $\dot{c}_2 \to 0$ and, therefore,)

$$c_2(t) \to 0 \tag{3.12}$$

4. Canonical quantization

For the purpose of canonical quantization it is sufficient to reduce equation (3.3) to an equation of the Sturm-Liouville type. If λ is sufficiently small $(1 + c_2 > 0)$ this may be achieved by division of (3,3) by $(1 + c_2)^2$. One obtains in this way

$$\frac{d}{dt}\left(\frac{\dot{q}}{1 + c_2}\right) + \frac{\varkappa^2 (1 + c_0)}{(1 + c_2)^2}\, q = 0 \tag{4.1}$$

The corresponding Lagrangian and Hamiltonian are

$$\mathcal{L} = \frac{1}{2}\left\{\frac{1}{1 + c^2}\, \dot{q}^2 - \frac{\varkappa^2 (1 + c_0)}{(1 + c_2)^2}\, q^2\right\}, \tag{4.2}$$

$$H = \frac{1}{2}\left\{(1 + c_2)\, p^2 + \varkappa^2 \frac{1 + c_0}{(1 + c_2)^2}\, q^2\right\}, \tag{4.2}$$

where

$$p = \frac{\dot{q}}{1 + c_2} \tag{4.3}$$

is the canonical momentum conjugate to q.

The quantization may be performed in the usual way. The commutation rules or the can onical coordinates and momenta taken at equal times are

$$[q(t), q(t)] = [p(t), p(t)] = 0, \qquad [q(t), p(t)] = i\hbar. \tag{4.4}$$

For different times one may obtain simple commutation rules by going over from the Heisenberg picture, in which we are working, to a picture in which the equations of motion may be solved explicitly. In this new picture a perturbation procedure may be developed. It must, of course, give the same results as the perturbation method applied directly to the original equation (1.1). We shall verify this statement in the next section.

Heisenberg's and Schroedinger's equations may be written down

$$\dot{F} = \frac{1}{i\hbar}[F, H] + \frac{\partial F}{\partial t} \tag{4.5}$$

$$H\psi(q, t) = i\hbar \frac{\partial \psi(q, t)}{\partial t} \tag{4.6}$$

with H given by (4.2), F being an arbitrary operator which may explicitly depend on the time variable, and $\psi(q, t)$ being the state vector of the system. These equations determine completely the behaviour of the system at each instant of time.

Thus a differential desription of the motion (both quantum-theoretical and classical) is possible for the non-local system described by the integro-differential equation (1.1).

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5. Perturbation method

One may apply the quantum-theoretical perturbation method directly to the original equation (1.1) (integral quantization) as well as to the equivalent equations (3.4) and (3.5). The results will be identical. This follows immediately from the fact that each solution of (1.1) satisfies also (3.4) and (3.5) and, therefore,

$$\frac{\dot{c}_2 \dot{q} + \varkappa^2 (c_2 - c_0) q}{1 + c_2} = - c_2 \ddot{q} + \dot{c}_2 \dot{q} - \varkappa^2 c_0 q = \lambda \int_a^b K(t, t')\, q(t') dt'. \qquad (5.1)$$

Equations (5.1) are operator identities in the space of the solutions of (1.1). They relate the values of $q(t')$ distributed by means of the kernel $K(t, t')$ over the whole interval (a, b) to the values of $q(t)$ and $\dot{q}(t)$ (and possibly also $\ddot{q}(t)$) at an arbitrary point in (a, b). The knowledge of a differential operator which is equivalent to a given integral operator in the space of the solutions of a given integro-differential equation enables us to convert all non-local quantities corresponding to the problem in question into local quantities depending only on one time moment, lying between a and b.

It is instructive to verify the identity (5.1) in the first two approximations in the constant λ. The mechanism of the transition from non-local to local quantities becomes then more apparent.

Expansion of the coefficients of the left-hand side of (5.1) yields up to the second order in λ

$$\frac{\varkappa^2 (c_2 - c_0)}{1 + c_2} = \lambda \int_a^b K(t, t')\, dt' \cos \varkappa (t' - t) + \lambda^2 \iiint_a^b K(t, t')\, dt' \left[\eta^r (t' - t''') - \eta^r (t - t''')\right] \frac{1}{\varkappa} \sin \varkappa (t' - t''')\, dt'''\, K(t''', t'')\, dt'' \cos \varkappa (t'' - t) + \dots \qquad (5.2)$$

$$\frac{\dot{c}_2}{1 + c_2} = \lambda \int_a^b K(t, t')\, dt' \frac{1}{\varkappa} \sin \varkappa (t' - t) + \lambda^2 \iiint_a^b K(t, t')\, dt' \left[\eta^r (t' - t''') - \eta^r (t - t''')\right] \frac{1}{\varkappa} \sin \varkappa (t' - t''')\, dt'''\, K(t''', t'')\, dt'' \frac{1}{\varkappa} \sin \varkappa (t'' - t) + \dots$$

The expansion of $q(t)$ in terms of $q^0(t)$ is given by (2.5) and (2.6)

$$q(t) = q^0(t) + \lambda \int_a^b N(t, t')\, dt'\, q^0(t') + \dots \qquad (5.3)$$

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To obtain the left-hand side of (5.1) in the first approximation in λ, one needs only to consider the first terms in the expansions (5.2) and (5.3)

$$\frac{\dot{c}_2\dot{q} + \varkappa^2 (c_2 - c_0)\, q}{1 + c_2} = \lambda \int_a^b K(t, t')\, dt' \left[\frac{1}{\varkappa} \sin \varkappa (t' - t)\, \dot{q}^0 (t) + \cos \varkappa (t' - t) q^0(t)\right] + \dots$$

$$= \lambda \int_t^b K(t, t')\, dt'\, q^0 (t') + \dots \tag{5.4}$$

It is seen that the basic solutions $\cos \varkappa (t' - t)$ and $\frac{1}{\varkappa} \sin \varkappa(t' - t)$ of the problem in lowest approximation ($\lambda = 0$) play the role of operators which introduce $q(t')$ under the integral sign.

Similarly, in the next approximation one obtains (after some calculations) due to the properties of the basic solutions

$$\frac{\dot{c}_2\dot{q} + \varkappa^2 (c_2 - c_0)\, q}{1 + c_2} = \lambda \int_a^b K(t, t')\, dt' \left\{q_0 (t') + \lambda \int_a^b N(t, t'')\, dt''\, q^0 (t'') + \dots\right\} \tag{5.5}$$

in agreement with (5.1).

6. A direct method

The considerations of the preceding section open an interesting possibility to solve the problem of converting integro-differential equations into pure differential equations in a direct way. Whenever integral operators occur in an equation, we shall try to find differential operators which are equivalent in the space of solutions of the given equation. If the problem is soluble, we obtain the desired differential equation simply by replacing the integral operators by equivalent differential operators. For the case of the linear integro-differential equation (1.1), the problem reduces to the solution of an infinite set of linear equations with an infinite number of unknowns.

Anticipating the results, we may look for a differential operator equivalent to the right-hand side of (1.1) in the form $A\dot{q} + Bq$, and try to determine A and B in such a way that the identity

$$A\dot{q} + Bq = \int_a^b K(t, t')\, dt'\, q(t') \tag{6.1}$$

is satisfied in the space of solutions of (1.1). In course of derivation it will become obvious that there is no necessity to introduce higher derivatives on the left-hand side of (6.1) if the original equation contains second order derivatives outside the

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integral and no derivatives under the integral sign. Expanding A and B in powers of λ

$$A = \sum_{n=o}^{\infty} \lambda^n A^{(n)} , \qquad B = \sum_{n=o}^{\infty} \lambda^n B^{(n)} , \tag{6.2}$$

using relations (2.5) and (2.6), and equating coefficients in (6.1), one gets an infinite set of equations

$$\sum_{m=o}^{n} \left(A^{(m)} \frac{d}{dt} + B^{(m)} \right) \int_a^b \ldots \int_a^b N(t, t^1)\, dt^1\, N(t^1, t^2)\, dt^2 \ldots N(t^{n-m-1}, t^{n-m})\, q^0(t^{n-m})$$
$$= \int_a^b \ldots \int_a^b K(t, \tau)\, d\tau\, N(\tau, t^1)\, dt^1 \ldots N(t^{n-1}, t^n)\, q^0(t^n), \; (n = 0, 1, \ldots). \tag{6.3}$$

Here $q^0(t')$ may be expressed by its initial values and the initial values of its first derivative at an arbitrary time $t = t^0$ (cf. (2.3)). For convenience we introduce the notation

$$\frac{1}{\varkappa} \sin \varkappa t = \Delta(t), \quad \cos \varkappa t = \dot{\Delta}(t). \tag{6.4}$$

With this notation (2.3) becomes

$$q^0(t') = \Delta(t' - t^0)\, \dot{q}^0(t^0) + \dot{\Delta}(t' - t^0)\, q^0(t^0). \tag{6.5}$$

Putting further

$$\int_a^b \ldots \int_a^b D_1(t, t^1)\, dt^1 D_2(t^1, t^2)\, dt^2 \ldots D_n(t^{n-1}, t^0) = \{D_1 . D_2 \ldots D_n\}(t, t^0), \tag{6.6}$$

we may write equations (6.3) in the simple form

$$\sum_{m=o}^{n} \left(A^{(m)} \frac{d}{dt} + B^{(m)} \right) \left[\{N^{n-m} \Delta\}(t, t^0)\, \dot{q}^0(t^0) + \{N^{n-m} \dot{\Delta}\}(t, t^0)\, q^0(t^0) \right]$$
$$= \{KN^n \Delta\}(t, t^0)\, \dot{q}^0(t^0) + \{KN^n \dot{\Delta}\}(t, t^0)\, q^0(t^0), \; (n = 0, 1, \ldots). \tag{6.7}$$

(6.7) are identities in the space of solutions of (1.1), hence they must be satisfied for an arbitrary choice of the parameters $\dot{q}^0(t^0)$ and $q^0(t^0)$. The infinite set (6.7) splits therefore into two infinite sets of equations

$$\left.\begin{aligned} \sum_{m=o}^{n} \left(A^{(m)} \frac{d}{dt} + B^{(m)} \right) \{N^{n-m} \Delta\}(t, t^0) &= \{KN^n \Delta\}(t, t^0) \\ \sum_{m=o}^{n} \left(A^{(m)} \frac{d}{dt} + B^{(m)} \right) \{N^{n-m} \dot{\Delta}\}(t, t^0) &= \{KN^n \dot{\Delta}\}(t, t^0) \end{aligned}\right\} \tag{6.8}$$

Their explicit form is

$$
\begin{aligned}
&A^{(o)}\dot{\Delta} + B^{(o)}\Delta = \{K\Delta\} \\
&A^{(o)}\ddot{\Delta} + B^{(o)}\dot{\Delta} = \{K\dot{\Delta}\} \\
&A^{(o)}\{\dot{N}\Delta\} + B^{(o)}\{N\Delta\} + A^{(1)}\dot{\Delta} + B^{(1)}\Delta = \{KN\Delta\} \\
&A^{(o)}\{\dot{N}\dot{\Delta}\} + B^{(o)}\{N\dot{\Delta}\} + A^{(1)}\ddot{\Delta} + B^{(1)}\dot{\Delta} = \{KN\dot{\Delta}\} \\
&A^{(o)}\{\dot{N}N\Delta\} + B^{(o)}\{N^2\Delta\} + A^{(1)}\{\dot{N}\Delta\} + B^{(1)}\{N\Delta\} + A^{(2)}\dot{\Delta} + B^{(2)}\Delta = \{KN^2\Delta\} \\
&A^{(o)}\{\dot{N}N\dot{\Delta}\} + B^{(o)}\{N^2\dot{\Delta}\} + A^{(1)}\{\dot{N}\dot{\Delta}\} + B^{(1)}\{N\dot{\Delta}\} + A^{(2)}\ddot{\Delta} + B^{(2)}\dot{\Delta} = \{KN^2\Delta\} \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&A^{(o)}\{\dot{N}N^{n-1}\Delta\} + B^{(o)}\{N^n\Delta\} + A^{(1)}\{\dot{N}N^{n-2}\Delta\} + B^{(1)}\{N^{n-1}\dot{\Delta}\} + \dots A^{(n)}\dot{\Delta} + B^{(n)}\Delta \\
&\qquad = \{KN^n\Delta\} \\
&A^{(o)}\{\dot{N}N^{n-1}\dot{\Delta}\} + B^{(o)}\{N^n\dot{\Delta}\} + A^{(1)}\{\dot{N}N^{n-2}\dot{\Delta}\} + B^{(1)}\{N^{n-1}\dot{\Delta}\} + \dots A^{(n)}\ddot{\Delta} + B^{(n)}\dot{\Delta} \\
&\qquad = \{KN^n\dot{\Delta}\}
\end{aligned}
$$

From (6.8) or (6.9) $A^{(n)}$ and $B^{(n)}$ may be calculated. The calculation is facilitated by the fact that the characteristic determinant equals unity on account of (6.4). The results are just (5.2) in the first two approximations. Calculation of the n-th approximation and introduction into (6.2) yield for A and B the closed expressions (3.7) divided by $1 + c_2$.

КРАТКОЕ СОДЕРЖАНИЕ

Я. Жевуский, *Дифференциальная струкрура нелокализируемых теорий I.*

Показано на примере линейного интегрально-дифференциального уравнения в одном измерении, как получить дифференциальное описание нелокализируемых систем. В § 2 доказана эквивалентность интегрально-дифференциальных и дифференциальных уравнений того же порядка при определенных условиях относящихся к кернелю. В §§ 3 и 5 проведена дискуссия коэффициентов дифференциального уравнения. § 4 посвящен канонической квантизации системы. В § 6 показан другой непосредственный подход к рассматриваемой проблеме.

REFERENCES

Krzywicki A., Rzewuski J., Zamorski J. and Zięba A. (in preparation, 1954).

Pauli W., *Nuovo Cimento*, **10**, 648-667 (1953).

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