THE EFFECT OF FLEXURAL STRESSES ON THE PIEZOELECTRIC PROPERTIES OF POLYCRYSTALLINE BARIUM TITANATE

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The piezoelectric effect given rise to in polycrystalline barium titanate by flexural stress was investigated by the static and quasi-static methods. Samples in the form of two-layer rods of $1 \times 3 \times 30$ mm were used. Flexural stress in non-polarized $BaTiO_3$ rods was found to induce piezoelectric charge on the surfaces perpendicular to the plane of bending. The effect thereon of strain and temperature was measured. Moreover, the present investigation covered the piezoelectric effect in polarized polycrystalline $BaTiO_3$ rods and its dependence on the strain, the time of polarization and the polarizing field strength. A mechanism accounting for the induction of piezoelectric charge is proposed.

Introduction

In recent years the piezoelectric properties of polycrystalline $BaTiO_3$ have been the subject of investigation by various authors. Papers were published by Rzhanov (1946), Roberts (1947), Mason (1947), (1948), Roy (1950), (1956), Mesnard and Eyraud (1955), Marutake and Ikeda (1957), and others. The investigations dealt chiefly with longitudinal, transversal and radial resonance vibrations and on the effect thereupon of the polarizing field strength and the temperature. The measurements were in most cases carried out by dynamic methods. The elastic and piezoelectric constants of polycrystalline $BaTiO_3$ were computed from the experimental results.

Hitherto but scarce information is available on the piezoelectric effect exhibited by barium titanate rods when subjected to bending. The effect was first investigated by Bauer (1948), who used a quasi-static method. It was one of the aims of the present paper to investigate this piezoelectric effect by a static and a quasi-static method, and to explain the underlying mechanism.

§ *1. Method of Investigation of Piezoelectric Properties*

Polycrystalline ferroelectric samples were prepared in the shape of two-layer rods of $1 \times 3 \times 30$ mm. The piezoelectric charge induced on the surfaces of the sample perpendicular to the plane of bending by lasting flexural stresses applied but once.

was measured with a heterostatically connected Wulf electrometer. In the case of induced flexural vibrations of a frequency of up to 2000 Hz, the piezoelectric charge was determined from the piezoelectric voltage, as measured with a valve voltmeter of an input resistance of about $3 \times 10^6\ \Omega$, and from the dielectric permittivity, as measured by the bridge method at a frequency of 1 kHz and a measuring field strength of 10 V/cm. The piezoelectric voltage was measured separately for either layer of the polycrystalline rod.

The polycrystalline ferroelectric samples in the form of two-layer rods were set in a state of flexural vibration of acoustic frequency ranging up to 2000 Hz with the device shown in Fig. 1. A vibrating coil stiffly connected with a concentrically cor-

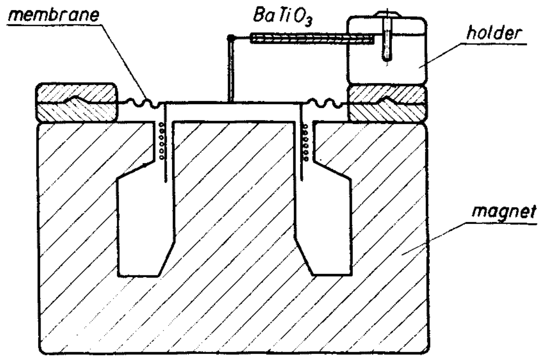

Fig. 1. Device for producing flexural vibrations in polycrystalline rods.

rugated fibre membrane was placed within the field of the slit of a loud-speaker magnet. The membrane together with the coil was fixed to the bulk of the magnet with two brass rings. The surface of the upper ring supported a holder of organic glass provided with electrodes and serving to fix one end of the polycrystalline rod. Its remaining, free end was stuck to a thin aluminium rod that was stiffly attached to the loud--speaker membrane. The deflection of the polycrystalline rod was measured to $\pm 5\mu$ with a microscope having a micrometer scale.

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§ 2. *The Piezoelectric Properties of Non-polarized Barium Titanate*

The piezoelectric effect exhibited by crystals belonging to 20 crystallographic classes consists in electric charge being induced on the crystal faces as a result of externally applied mechanical stress (Cady 1946; Mason 1950). A general criterium for the appertenance of a crystal to one of the piezoelectric classes is given by the absence of a centre of symmetry and the presence of polar axes of symmetry. The piezoelectric effect also appears in ferroelectric polycrystals after the latter have been polarized in a strong external electric field, thus producing non-zero electric polarization.

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The piezoelectric effect has hitherto not been observed to be induced by mechanical stress in non-polarized polycrystalline $BaTiO_3$. In the absence of an external electric field, the resultant polarization P in polycrystalline $BaTiO_3$ is zero because of the absence of orientational ordering in the distribution of the crystallographic axes of the various ferroelectric domains. This is the reason why non-polarized polycrystalline $BaTiO_3$ fails to exhibit anisotropy, notwithstanding the fact that the monocrystals of which it consists are anisotropic.

The present investigation proved that the mechanical stresses accompanying flexural strain of a non-polarized polycrystalline $BaTiO_3$ plate give rise to electric charges $\pm q$ induced on the surfaces of the plate perpendicular to the plane of bending. In the absence of an external electric field, a positive charge, $+q$, is induced on the surface that undergoes distention by flexural stress, whereas a negative one, $-q$, is induced on the surface undergoing compression. If the direction of bending is reversed, so will be the sign of the charges induced. Fig. 2 shows the distribution of charge as it appears on the surfaces of a non-polarized polycrystalline $BaTiO_3$ plate subjected to bending.

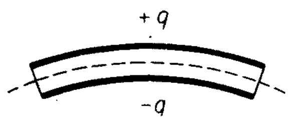

Fig. 2. Distribution of electric charge on surfaces of non-polarized $BaTiO_3$ sample subjected to bending.

Measurements by the present author proved that the piezoelectric voltage U_0 between the electrodes of the polycrystalline sample, resulting from the presence of the charges induced by the flexural stresses, depends on the mean value of the strain $\bar{S}_1$ as given by the following formula (see Appendix, eq. X):

$$\bar{S}_1 = \frac{3}{4}\frac{d}{l^2}\, s \tag{2.1}$$

here, d denotes the thickness of a single layer of the rod, l — its length, and s — the deflection. The voltage U_0 was found to be a linear function of $\bar{S}_1$ throughout the range of strains involved, which were of the order of 4×10^{-4}. The dependence is shown in Fig. 3.

If the piezoelectric voltage U_0 and the dielectric permittivity ε of polycrystalline $BaTiO_3$ are known, the resulting polarization P giving rise to the electric charges on the surfaces of the sample perpendicular to the plane of bending can be computed. Assuming the direction of the bending force to coincide with that of the z-axis of a Cartesian system of reference and the strain $\bar{S}_1$ to occur lengthwise and in the direction of the x-axis, the resultant polarization P_3 in the direction of the z-axis is given by:

$$P_3 = \frac{\varepsilon}{4\pi} E_3 \tag{2.2}$$

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wherein E_3 is the electric field strength arising from the direct piezoelectric effect. For a strain of $\bar{S}_1 = 10^{-4}$ at 20°C the resultant polarization amounts to $P_3 = 0.8 \times 10^{-4}$ μCb/cm^2.

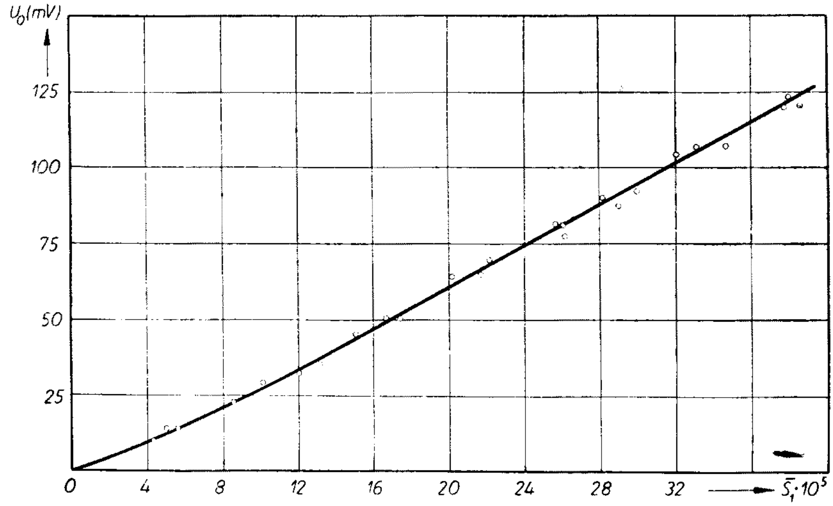

Fig. 3. Piezoelectric voltage U_0 *versus* strain $\bar{S}_1$, for a non-polarized polycrystalline $BaTiO_3$ sample.

In order to give a more general description of the effect observed, use can be made of the equation accounting for the direct piezoelectric effect in crystals. The polarization P_3 is then obtained as follows:

$$P_3 = d_{31}\, T_1 \tag{2.3}$$

wherein T_1 denotes the mechanical stress in the direction of the x-axis, and d_{31} — the piezoelectric modulus of the material under investigation. Substituting in eq. (2.3) the value of eq. (2.2), we have:

$$\frac{\varepsilon}{4\pi} E_3 = d_{31}\, T_1 = d_{31}\, c_{11}\, \bar{S}_1 \tag{2.4}$$

Hence:

$$\frac{E_3}{\bar{S}_1} = \frac{4\pi}{\varepsilon} c_{11}\, d_{31} = h_{31} \tag{2.5}$$

with h_{31} denoting the piezoelectric constant of the material. Using the value of Young's modulus $c_{11} = 0.8 \times 10^{12}$ dyn/cm^2 as computed from dynamic measurements, and the experimental value of the permittivity $\varepsilon = 1600$, the numerical value of the piezoelectric modulus d_{31} and that of the piezoelectric constant h_{31} were computed. At room temperature these values amount to:

$$d_{31} = 3.2 \times 10^{-9} \text{ CGS} \quad \text{and} \quad h_{31} = 20 \text{ CGS}.$$

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If the strain $\overline{S}_1$ is kept constant, the value of the piezoelectric voltage U_0 is found to depend on the temperature. Fig. 4 shows the dependence as measured for a strain of $\overline{S}_1 = 2.5 \times 10^{-4}$. At first the piezoelectric voltage U_0 increases with the temperature, attaining its maximum value at about 50°C. As the temperature continues to rise, the voltage U_0 decreases; at first, this occurs rather steeply, so that the tangent at an arbitrary point of this part of the curve intersects the temperature axis at a point

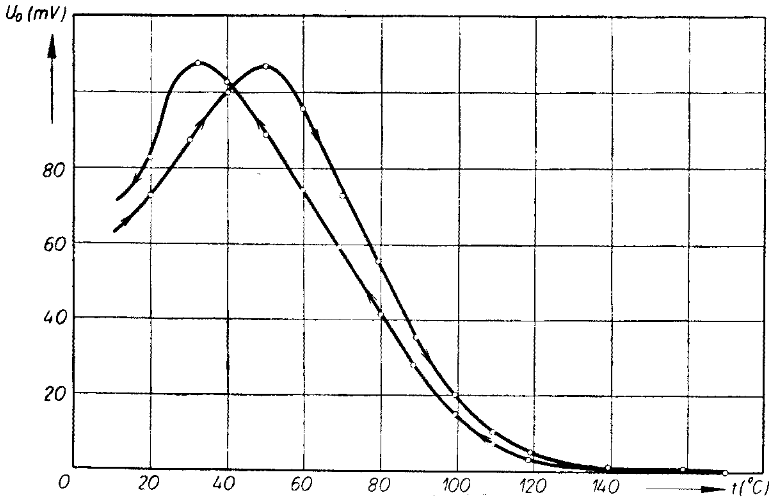

Fig. 4. Piezoelectric voltage U_0 *versus* the temperature, for a non-polarized polycrystalline $BaTiO_3$ sample.

corresponding to the Curie point temperature; the fall in voltage then becomes less and less steep, and it is only at about 170°C that the voltage disappears altogether. When the temperature was made to decrease, a similar dependence of the voltage U_0 on the temperature was observed, the maximum being slightly displaced towards lower temperatures. The displacement is probably due to thermal hysteresis of the dielectric permittivity as observed in polycrystalline $BaTiO_3$ by Piekara and Pająk (1952) and in single crystals by Känzig and Maikoff (1951).

The piezoelectric voltage U_0, especially at higher temperatures, cannot be measured with exactitude because of the rather considerable electric conductivity σ of $BaTiO_3$. The latter was measured in $BaTiO_3$ single crystals by Busch and Merz (1948) and in polycrystalline material by Trzebiatowski and Pigoń (1952) and by Pigoń (1954), who proved σ to rise with the temperature according to the formula:

$$\sigma = \sigma_0 \, e^{\frac{-E_a}{2kT}} \tag{2.6}$$

wherein σ_0 denotes a constant characteristic of the kind of material used in the measurements, E_a — the activation energy, k — Boltzmann's constant, and T — the Kelvin temperature. Within the temperature range of 20°C to 200°C, the conductivity of

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polycrystalline $BaTiO_3$ $\sigma = 10^{-6} \div 10^{-4}\, \Omega^{-1}\, cm^{-1}$. The rise in σ towards higher temperatures leads to a decrease in the experimental values of the piezoelectric voltage corresponding to given values of the strain $\overline{S}_1$, rendering their exact measurement impossible.

§ 3. *Piezoelectric Properties of Polarized Barium Titanate*

The dependence of the piezoelectric voltage U_0 on the strain $\overline{S}_1$ in polycrystalline two layer rods polarized by applying an external electric field, and the effect on the value of U_0 of the polarizing field strength E and of the time during which the polarizing field had been applied, were investigated. The piezoelectric voltage U_0 was measured separately for either layer of the polycrystalline rod.

The piezoelectric voltage U_0 induced when a polycrystalline plate is bent once and kept so, decays exponentially over 10—40 seconds. The curves in Fig. 5 show the

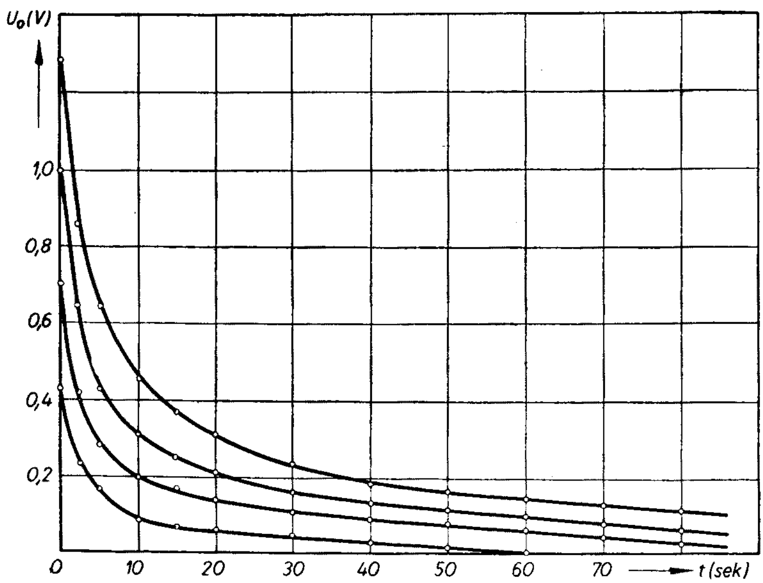

Fig. 5. Piezoelectric voltage U_0 *versus* the time, for a sample bent once and kept so.

time dependence of the voltage U_0, for flexure of various amplitudes. The decay of the piezoelectric voltage is due to the rather high electric conductivity of polycrystalline $BaTiO_3$.

Measurements by the quasi-static methods at a vibration of 400 Hz of the rod showed that the piezoelectric voltage U_0 depends on the value of the strain $\overline{S}_1$. This dependence is illustrated by the curve in Fig. 6. It is linear only for strains of about 2×10^{-4}. The voltage U_0 was not found to depend on the frequency of flexural vibrations throughout the range of 60 to 2000 Hz.

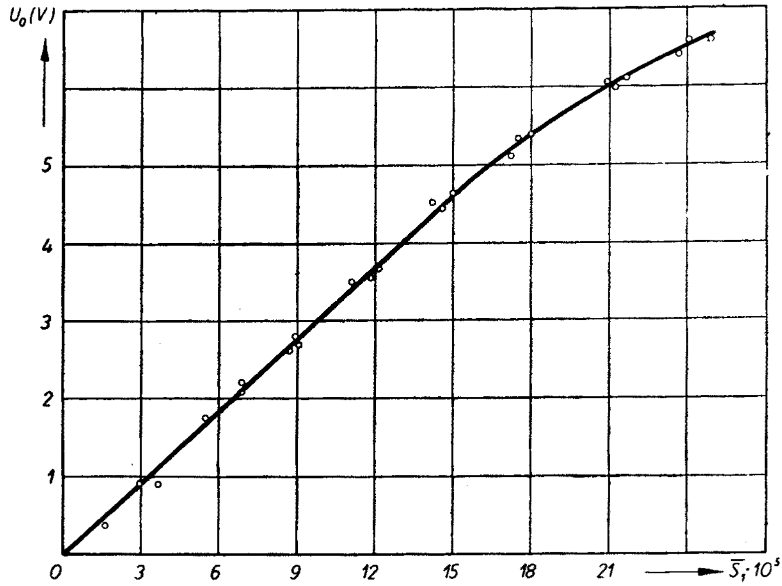

Fig. 6. Piezoelectric voltage U_0 *versus* the value of the strain $\bar{S}_1$.

The value of the piezoelectric voltage U_0 is, moreover, dependent on the field strength E of the electric field applied for polarizing the sample, and on the time during which the latter underwent polarization. The dependence of the piezoelectric voltage U_0 on the time of polarization, for a field of $E = 10$ kV/cm, is shown in Fig. 7. At first, as the polarizing field is applied during 10—20 minutes, the voltage rises

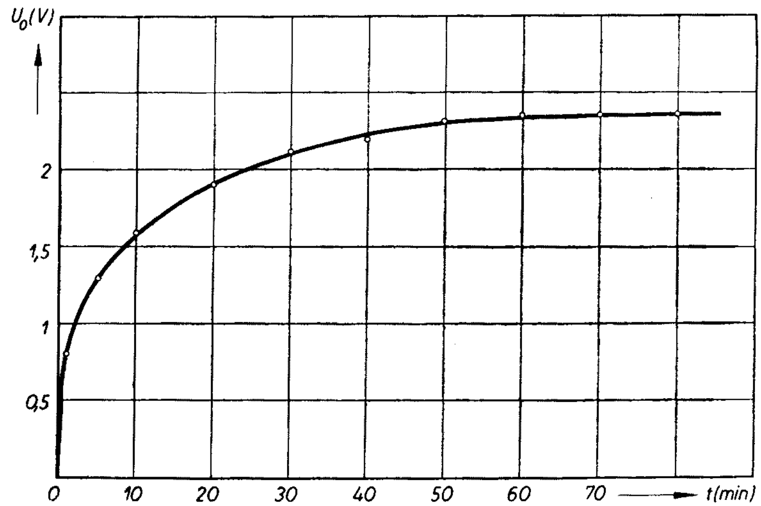

Fig. 7. Piezoelectric voltage U_0 *versus* the time of polarization, at $E = 10$ kV/cm.

rapidly; later on, it tends to saturation. If the polarizing field E is applied for more than 70 minutes, no further time-dependent variations in U_0 are observed.

The dependence of U_0 on the polarizing field strength E, for long-time polarization as determined from the measurements discussed above, is illustrated by Fig. 8.

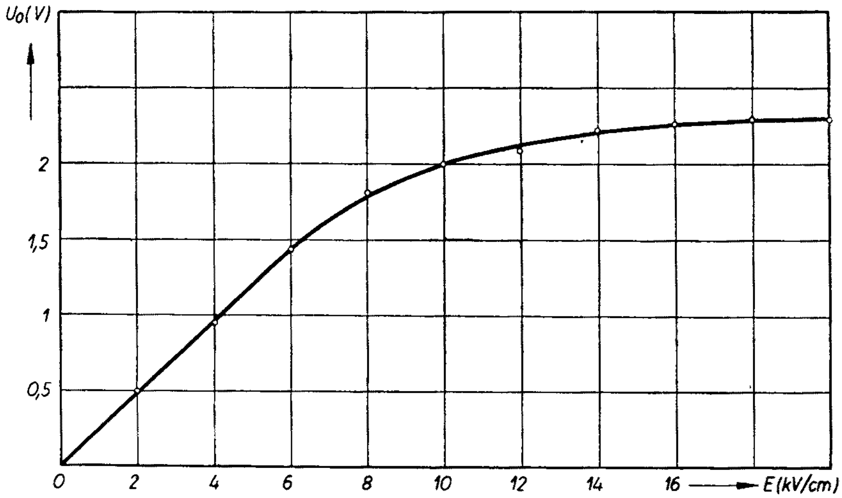

Fig. 8. Piezoelectric voltage U_0 *versus* the polarizing field strength E.

As the polarizing field strength increases, the dependence remains practically linear up to fields of 6 kV/cm; for still stronger fields, there is a tendency to saturation, which occurs at 20 kV/cm and long time polarization. The foregoing results were obtained at 20° C.

The values of the piezoelectric voltage U_0, the strain $\bar{S}_1$, Young's modulus c_{11} and the dielectric permittivity ε as obtained by measurements, were used for computing the piezoelectric constants d_{31}, g_{31}, h_{31} and e_{31} of polycrystalline $BaTiO_3$ polarized to saturation in a field of $E = 20$ kV/cm. The numerical values, in CGS units, are assembled in Table I. For the sake of comparison, numerical values of the piezoelectric

Table I

Numerical values of the dielectric permittivity ε, Young's modulus c_{11} and piezoelectric constants as obtained from measurements by the quasi-static method in bending polarized polycrystalline $BaTiO_3$ rods in the course of the present investigation, and by Bauer (1948). All figures are in CGS units.

ε	c_{11}	d_{31}	g_{31}	h_{31}	e_{31}	measured by
1600	0.8×10^{12}	3.3×10^{-7}	2.4×10^{-9}	2.0×10^{3}	2.2×10^{5}	Present author
2000	0.8×10^{12}	5.2×10^{-7}	3.3×10^{-9}	2.5×10^{3}	4.2×10^{5}	Bauer

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constants as computed from results due to Bauer (1948) and obtained by him quasi-statically when subjecting polycrystalline $BaTiO_3$ rods to bending are also given. For all values of the piezoelectric constants in Table I, there is agreement as to the order of magnitude. The divergences in the numerical values are probably due to different conditions of preparation of the polycrystalline material.

It is seen that the piezoelectric moduli d_{31} of polarized and non-polarized $BaTiO_3$ differ by two orders of magnitude. Since the piezoelectric modulus d_{31} is defined as the variation in resultant polarization P_3 due to unit stress, this variation in samples polarized to saturation is seen to exceed that in non-polarized samples by two orders of magnitude.

§ 4. *Mechanism of Piezoelectric Polarization in Bent Polycrystalline Barium Titanate Plates*

The mechanism underlying the piezoelectric effect resulting from flexural stress in non-polarized polycrystalline $BaTiO_3$ has not been fully elucidated. The mechanical stress that results when the rod is bent causes one of its layers to become distended with respect to the neutral plane of symmetry n (the plane containing the centre of the rod and perpendicular to the plane of flexure), whilst the other layer undergoes compression (Fig. 9 a). The strain is the greater as the distance from the plane of symmetry increases, and hence a gradient of strain arises parallel to the curvature radius R. Considering maximum values S_1' and S_1'' of the strain for two elementary fibres of the bent rod of length l situated at distances $\varDelta R_1$ and $\varDelta R_2$ from the neutral plane of symmetry n, we have (see Appendix, eqs. I and IX):

$$S_1' = \frac{3}{2}\,\frac{\varDelta R_1}{l^2}\,s \quad \text{and} \quad S_1'' = \frac{3}{2}\,\frac{\varDelta R_2}{l^2}\,s \tag{4.1}$$

The difference of these expressions yields the strain gradient in the form:

$$\frac{\varDelta S_1}{\varDelta R} = \frac{3}{2}\,\frac{s}{l^2} \tag{4.2}$$

with $\varDelta S_1 = S_1' - S_1''$, and $\varDelta R = \varDelta R_1 - \varDelta R_2$. The strain gradient existing in the bent rod, and the inhomogeneous mechanical stresses related thereto, may give rise to:

1) variation of the orientation of the ferroelectric domains, so that a direction set by the flexural stresses now becomes the prevailing one, or

2) variation in the shape of the elementary cell and, hence, of the domain, or else

3) ordering, by the flexural stresses, of the distribution of defects present in the crystal.

Assuming that the inhomogeneous mechanical stress resulting from flexure when a polycrystalline rod is subjected to bending involves a change in the shape of the elementary cell, the following model of this change together with a mechanism of the induction of the electric charge, which is positive, $+q$, on the distended and negative, — q, on the compressed surface, can be put forward. The elementary cell of $BaTiO_3$ can be schematically represented as in Fig. 9b (Shirane, Jona and Pepinsky, 1955). The same cell, according to the model proposed here, assumes the shapes shown in Fig. 9 c, d, the former relating to the distended layer and the latter to the one subjected to compression. This change in the shape of the elementary cell should cause

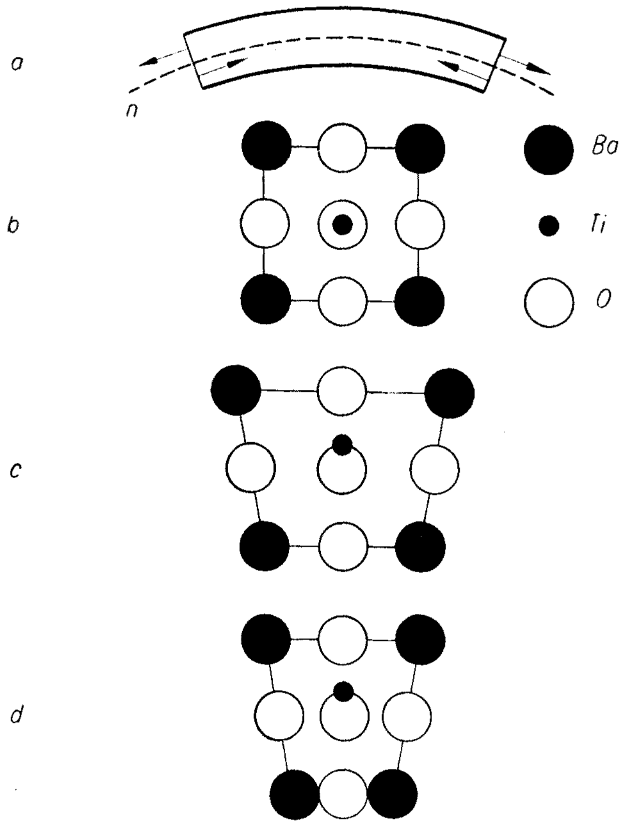

Fig. 9. Model showing change in shape of elementary $BaTiO_3$ cell, as arising from flexural stresses.

displacement (squeezing) of the titanium ion Ti^{4+} from its position of equilibrium in a direction coinciding with that of flexure; such stress-induced polarization will generally give rise to electric charge on the surface layer of the ferroelectric, of positive sign on the convex side and negative on the concave one. The charge in the surface layers will produce electrostatic induction on the electrodes, and the free charges will come to be observed as piezoelectric charge.

From investigations by Frenkel (1926), Wagner and Schottky (1930) and various other authors, a crystal lattice at thermal equilibrium is known always to present a number of ions or atoms situated at interstitial points (Frenkel defects) and a number of non-invested node points (Schottky defects). Other kinds of defects are also possible,

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thus e.g. various types of dislocation. The number of defects present is given by an exponential function of the activation energy and the temperature (Mott and Gurney, 1950). It is also known that, in an external electric field, the defects exhibit a well-defined mobility accounting, in some cases, for the electric conductivity of the crystal. The assumption that the electric charge observed is due to defects or dislocations and to their motion in the inhomogeneous mechanical stress field, and that the direction of their motion and the magnitude of displacement is given by the direction of the stress gradient, would lead to predict an ordered distribution of defects within the crystal.

The present state of the investigation as yet admits of no decision as to which of the interpretations proposed is the correct one in accounting for the mechanism of the effect observed. Investigation on polycrystalline $BaTiO_3$ and on single crystals is proceeding at this Laboratory with the aim of elucidating the mechanism in question.

The distribution of the piezoelectric charge induced by flexural stress on the surfaces of a polarized sample perpendicular to the plane of bending is dependent on the residual polarization P_r from the external polarizing field E and on the direction of the strain $\overline{S}_1$. Such charge results from the variation in residual polarization P_r, caused by the flexural stresses. The static method used for investigating the piezoelectric effect yields the exact relationships between the sign of the piezoelectric charge induced on a surface perpendicular to the plane of bending by variation of the residual polarization P_r, on the one hand, and the original direction of the residual polarization and that of strain, on the other. The results obtained by the static method are assembled in Fig. 10.

According to Mason (1948) and Smolensky (1951), $BaTiO_3$ belongs to the group of ferroelectrics exhibiting positive volume electrostriction. By theoretical considera-

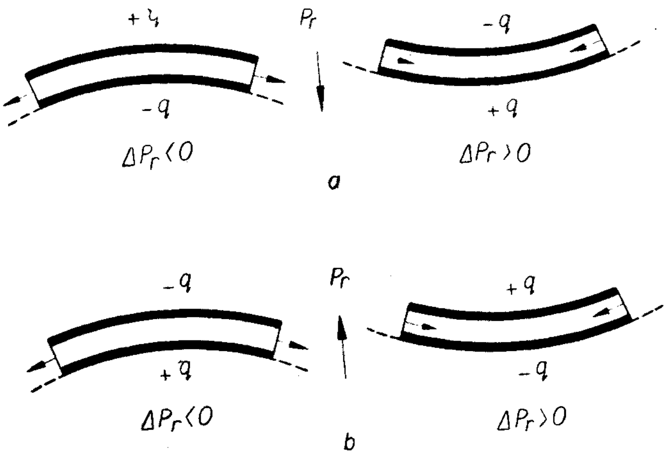

Fig. 10. Distribution of piezoelectric charge as dependent on direction of residual polarization P_r and direction of the strain S_1.

tions due to Kholodenko and Shirobokov (1951) and from measurements carried out by Mason (1955), resultant polarization in a ferroelectric exhibiting the positive effect of volume electrostriction assumes the direction parallel to that of the distention brought about by the external mechanical stresses, and perpendicular to that of the compression. Accounting for the way the direction of resultant polarization in $BaTiO_3$ depends on those of distention and compression, and accounting for the distribution of piezoelectric charge in its dependence on the direction of the residual polarization P_r and that of the strain $\overline{S_1}$ as observed in the paper cited, the following mechanism of the induction of these charges in polarized $BaTiO_3$ may be proposed. Polarization gives rise to surface charges of density $\varrho = \pm P_r$ in the surface layer of the ferroelectric; these induce charge in the electrodes. If external stress $T_1 = 0$, the bound charges induced in the electrodes fully compensate the polarization charges, whereas the free charges are compensated as a result of the electric conductivity of the sample. If the residual polarization P_r is directed towards the plane of junction (denoted by the dashed line in Fig. 10) as shown in Fig. 10a, then bending associated with distention of the layer induces positive charge on the convex surface and negative charge on the concave side. The mechanical stress arising in the process of bending, and causing distention of the upper layer of the rod, reduces the polarization P_r by the amount ΔP_r, arraying the latter in the direction of distention. Due to the decrease in the polarization P_r, the positive charge of the electrode, which hitherto compensated the negative charge of polarization P_r, now becomes a free charge and is available for observation in the electrometer as piezoelectric charge. If the direction of bending is reversed so as to compress the layer of the rod in the direction perpendicular to that of the polarization P_r, the latter will be increased by the amount ΔP_r. This, in turn, gives rise to induction on the electrodes, as a result of which the free charge, negative in the case under consideration, comes to be observed as piezoelectric charge. On the other hand, if the residual polarization is directed from the plane of junction, as shown in Fig. 10 b, distention diminishes it by the amount ΔP_r, as a result of which charges hitherto compensating the polarization are set free on the surfaces of the layer subjected to flexure. Since the direction of the polarization P_r is now reversed with respect to the foregoing case, the signs of the charges observed will be reversed, too.

Thus, from the experimental distribution of piezoelectric charge, distention of a polycrystalline sample perpendicular to the residual polarization P_r is seen always to diminish this polarization, whereas compression of the sample perpendicular to the direction of P_r always results in an increase of the polarization P_r.

Conclusions

Mechanical stress appearing when a non-polarized polycrystalline $BaTiO_3$ plate is subjected to flexure causes electric charge to be induced on its surfaces perpendicular to the plane of flexure. The effect may be due to variation in the orientation of the ferroelectric domains occurring in the direction favoured by flexural stress,

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or to variation in the shape of the elementary cell and domain, or to ordering of the distribution of defects in the crystal lattice.

The distribution of piezoelectric charge induced by flexural stress on the surfaces of the polarized $BaTiO_3$ sample perpendicular to the plane of bending depends on the direction of the residual polarization resulting from the polarizing field, and on the direction of strain. Distention of the polycrystalline sample in the direction perpendicular to that of the residual polarization always produces a decrease whereas compression in the same direction always leads to an increase in the latter.

Appendix

Method of Measuring Flexural Stresses

Bending of a rod, of constant cross section along its length l, causes distention of one of the layers and compression of the other one with respect to the neutral surface n through the centre of the rod and perpendicular to the plane of bending. The strain S_1 arising from distention of e.g. the upper layer of the rod in Fig. 11 is

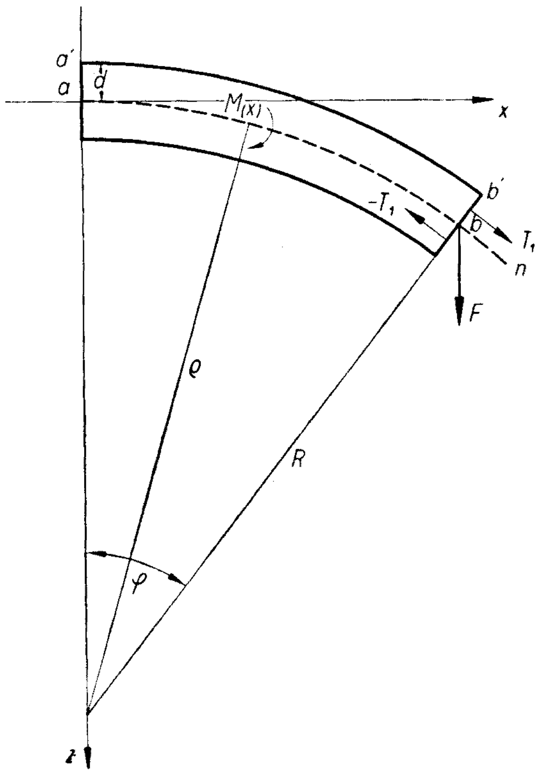

Fig. 11. Section through bent rod, illustrating derivation of eq. $\overline{S_1} = \frac{3}{4}\frac{d}{l^2}s$.

given by the relative variation of the latter's length $\Delta l/l$. With the notation of Fig. 11, we have:

$$S_1 = \frac{\Delta l}{l} = \frac{d}{R} \tag{I}$$

wherein R is the radius of curvature of the neutral surface n at point b, and d — the thickness of the layer subjected to bending. Since the length l of the rod on the neutral surface remains unchanged in the process of bending, the mean strain of a layer of thickness d is given by:

$$\bar{S}_1 = \frac{1}{2}\frac{d}{R} \tag{II}$$

The curvature at any point of the neutral surface, in the coordinates x, z of Fig. 11, is given by:

$$\frac{1}{\varrho} = \frac{\dfrac{d^2z}{dx^2}}{\left[1 + \left(\dfrac{dz}{dx}\right)^2\right]^{3/2}} = \frac{M(x)}{B} \tag{III}$$

wherein $M(x)$ denotes the bending moment acting upon the cross section of the rod at a distance $l-x$ from the point of application of the bending force F, and B — the stiffness modulus of the rod for bending. To all purposes, at small deflexions, $\dfrac{dz}{dx} \ll 1$. Hence:

$$\frac{1}{\varrho} = \frac{d^2z}{dx^2} = \frac{M(x)}{B} = \frac{F(l-x)}{B} \tag{IV}$$

The angle of bending φ of the neutral surface is given by:

$$\varphi \approx \operatorname{tg}\varphi = \frac{dz}{dx} = \frac{F}{B}\int_0^l (l-x)\,dx = \frac{1}{2}\frac{Fl^2}{B} \tag{V}$$

and the deflection s by:

$$s = z = \frac{F}{B}\int_0^l \left[\int (l-x)\,dx\right] dx = \frac{1}{3}\frac{Fl^3}{B} \tag{VI}$$

By eqs. V and VI, a relation between the angle of bending φ and the deflection s is obtained in the form:

$$\varphi = \frac{3}{2}\frac{s}{l} \tag{VII}$$

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For small values of the angle of bending φ, the bending curve n of Fig. 11 can be replaced by a circular arc of radius R. For an arc of length l, the angle φ is now given by:

$$\varphi = \frac{l}{R} \tag{VIII}$$

Comparing eqs. VII and VIII, the curvature of the neutral surface of a rod of length l is obtained in the form:

$$\frac{1}{R} = \frac{3}{2} \frac{s}{l^2} \tag{IX}$$

Substituting the value of $1/R$ of eq. IX in eq. II, the mean strain $\bar{S}_1$ is obtained as a function of the deflection:

$$\bar{S}_1 = \frac{3}{4} \frac{d}{l^2} s \tag{X}$$

According to Hooke's law, the mechanical stresses arising in the rod from bending are given by:

$$T_1 = c_{11} \bar{S}_1 = \frac{3}{4} c_{11} \frac{d}{l^2} s \tag{XI}$$

wherein c_{11} is Young's modulus in the direction of the x-axis. Eq. XI, which was derived according to Southwell's monograph (1941), served here for computing the flexural stresses arising in polycrystalline barium titanate.

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