LETTERS TO THE EDITOR

SPINLESS INTERACTION OF SLOW NEUTRONS WITH A PARTICULAR TYPE OF RECTANGULAR COMPLEX CAVITY

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(Received July 5, 1961; revised paper received October 31, 1961)

Let us assume a radial potential V of the form

$$V = V_0 = \text{const.}$$

for radius varying from 0 to a value R_0 not exceeding the range of nuclear forces, and

$$V = V_1(1 + \xi i) = \text{const.}$$

within the spherical layer of inner radius R_0 and outer radius R, the latter being given by the range of the nuclear forces. Quite generally, we can put $V_0 \neq V_1$ („steps").

Writing the wave function in the form

$$\psi(r) = \frac{u(r)}{r},$$

Schroedinger's equation for the problem under consideration reduces to

$$\frac{d^2u_i}{dr^2} + k_i^2 \cdot u_i = 0 \qquad i = 0, 1, 2.$$

wherein

$$k_0^2 = \frac{2m}{\hbar^2}[E - V_0] \qquad \text{for} \qquad R_0 > r \geqslant 0$$

$$k_1^2 = \frac{2m}{\hbar^2}[E - V_1(1 + \xi i)] \quad \text{for} \qquad R \geqslant r \geqslant R_0$$

$$k_2^2 = \frac{2m}{\hbar^2} E \qquad \text{for} \qquad r > R$$

since, for the case of low energies considered, we can put $l=0$. We assume V_0, V_1 and ξ to be independent of the energy E; moreover, in order to simplify the notation, we write $k_2 = k$.

On equating the functions u_i and their derivatives in the points of potential jumps, assuming a finite value of $\psi(0)$ and applying the usual method of computing effective cross-sections, we have

$$\sigma_{\text{el}} = \frac{\pi}{k^2} |1 - \eta|^2 \tag{1}$$

$$\sigma_r = \frac{\pi}{k^2} (1 - |\eta|^2) \tag{2}$$

with

$$\eta = \frac{(a + bk) + (c + dk)\, i}{(a - bk) + (c - dk)\, i} \cdot e^{-2ikR}$$

the real quantities a, b, c, d being of the form

$$a = (k_0 \cdot \cos k_0 R_0 \cdot \cos \alpha\varrho - \alpha \cdot \sin k_0 R_0 \cdot \sin \alpha\varrho) \cosh \beta\varrho + \beta \cdot \sin k_0 R_0 \cdot \cos \alpha\varrho \cdot \sinh \beta\varrho$$

$$b = \left(\sin k_0 R_0 \cdot \sin \alpha\varrho - \frac{\alpha k_0 \cdot \cos k_0 R_0 \cdot \cos \alpha\varrho}{\alpha^2 + \beta^2} \right) \sinh \beta\varrho + \frac{\beta k_0 \cdot \cos k_0 R_0 \cdot \sin \alpha\varrho}{\alpha^2 + \beta^2} \cosh \beta\varrho$$

$$c = -(\alpha \cdot \sin k_0 R_0 \cdot \cos \alpha\varrho + k_0 \cdot \cos k_0 R_0 \cdot \sin \alpha\varrho) \sinh \beta\varrho - \beta \cdot \sin k_0 R_0 \cdot \sin \alpha\varrho \cdot \cosh \beta\varrho$$

$$d = \left(\sin k_0 R_0 \cdot \cos \alpha\varrho + \frac{\alpha k_0 \cdot \cos k_0 R_0 \cdot \sin \alpha\varrho}{\alpha^2 + \beta^2} \right) \cosh \beta\varrho + \frac{\beta k_0 \cdot \cos k_0 R_0 \cdot \cos \alpha\varrho}{\alpha^2 + \beta^2} \sinh \beta\varrho$$

with

$$\varrho = R - R_0, \qquad \alpha = \text{Re}\, k_1, \qquad \beta = \text{Im}\, k_1.$$

σ_{el} is the total cross-section for elastic scattering, and σ_r — the sum of the cross-sections for all reactions including non-elastic scattering. Hence, the total cross-section is

$$\sigma_t = \sigma_{\text{el}} + \sigma_r = \frac{2\pi}{k^2} (1 - |\eta| \cos \varphi) \tag{3}$$

wherein

$$\varphi = -2kR + \text{arc tg}\, \frac{c + dk}{a + bk} - \text{arc tg}\, \frac{c - dk}{a - bk}$$

The cross-sections thus defined coincide with the usual definitions of the mean cross-sections [1] in the range of lowest energies under consideration here, as no resonance extremums occur in this energy range.

If we put

$$a^2 + c^2 = p, \qquad 2(ab + cd) = q, \qquad b^2 + d^2 = s$$

we can write

$$\eta = \left[\frac{p + qk + sk^2}{p - qk + sk^2} \right]^{1/2} \cdot e^{i\varphi}$$

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We now consider two cases: $\xi > 0$, and $\xi = 0$. In the former ($\xi > 0$) we have $q < 0$ which implies $|\eta| \neq 1$ although $\lim_{k \to 0} |\eta| = 1$. On applying de l'Hospital's rule once to Eq. (2) and twice to Eq. (1), we obtain the following limiting relations for the cross-sections:

$$\lim_{k \to 0} \sigma_r = \lim_{k \to 0} \sigma_t = -\lim_{k \to 0} \frac{\pi q}{kp} = \infty$$

$$\lim_{k \to 0} \sigma_{\mathrm{el}} = \frac{\pi}{p^2} [q^2 + 4(ad - bc - pR)^2]$$

as in the energy range under consideration E is so small with respect to V_0 and V_1 that we are justified in assuming a, b, c, $d = \mathrm{const}$. This yields the well-known v^{-1} law for σ_t and σ_r.

In the second case ($\xi = 0$) we have $q = 0$, *i.e.* $|\eta| = 1 = \mathrm{const}$. This leads to the obvious result

$$\sigma_r \equiv 0$$

and, on applying twice de l'Hospital's rule to Eq. (3), the relation

$$\lim_{k \to 0} \sigma_t = \lim_{k \to 0} \sigma_{\mathrm{el}} = 4\pi \left(\frac{d}{a} - R \right)^2$$

Moreover, for the low energies considered, we have $k \ll a$, and

$$\sigma_t \equiv \sigma_{\mathrm{el}} = \lim_{k \to 0} \sigma_t = \mathrm{const.}$$

Thus, we have obtained the well-known law of flat $\sigma_t(E)$ curve in the pre-resonance range, holding for a very numerous group of isotopes.

REFERENCES

[1] Feshbach, H., Porter, C. E., Weisskopf, V. F., *Phys. Rev.*, **96**, 448 (1954).