HIGH ENERGY SCATTERING IN SYSTEMS WITH ANOMALOUS THRESHOLDS*

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An S-matrix-treatment of high energy scattering is given in the presence of anomalous thresholds. The high energy behaviour of the elastic amplitudes is governed by the Pomeranchuk pole. In the case of the scattering on a nucleus, the amplitude is proportional to the atomic number. Besides the scattering on individual constituents of the target, a non-vanishing coherent scattering on the target as a whole is found. The possible role of many-nucleon intermediate states is briefly discussed.

1. Introduction

It is now a common belief that scattering of high energy "elementary" particles can be suitably described by means of the exchange of a Regge-pole between them. The question, naturally arises, whether taking systems with anomalous thresholds into consideration would substantially change the picture. Working with a mixed formalism (*scil.* describing the composite target with the help of wave functions, while taking the usual relativistic amplitudes for the elementary process). Udgaonkar and Gell-Mann were able to show that the situation remains more or less unchanged as compared with the scattering of elementary systems. They have found, namely, that nulceon-nucleus scattering is dominated by the same Regge-pole which governs elementary processes. There are two modifications, however, first, the residue of the pole is A times the residue, appearing in NN scattering. Second, a cut appears in the angular momentum plane, due to multiple scattering. The contribution of the latter, however, dies out in the asymptotic region.

The present work aims at formulating and solving the same problem in the framework of S-matrix theory. This seems to be desirable not only from an esthetical point of view, but in order to treat systems where a description in terms of wave functions is inappropriate (*e. g.* scattering on Σ particles). In Sec. 2 we treat $\pi\Sigma$ scattering, while Sec. 3 is devoted to

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the consideration of scattering on a nucleus, taking πd scattering as an example. (Complications arising from the spin and isospin structure of the amplitudes are neglected throughout the calculation). The comparison of our results with the work of others, especially a discussion of the problem of the cuts in the angular momentum plane is found in Sec. 4.

2. $\pi\Sigma$ scattering

Consider the generalized partial wave amplitude in the t-channel. In the two-pion approximation we can write the unitarity condition as follows [2]:

$$g_l \equiv \frac{1}{2i}\,(G_l - G_l^{II}) = 2\varrho_\pi G_l F_l^{II}\,, \tag{2.1}$$

where G_l is the amplitude of the $\pi\pi \to \Sigma\bar{\Sigma}$ process, F that of $\pi\pi$ scatering, F_l^{II}, G_l^{II} the corresponding amplitudes on the second sheet of the Riemann surface of the function $\sqrt{t-4}$; $\varrho_\pi = (t-4)^{\frac{1}{2}} t^{-\frac{1}{2}}$ the pion mass being chosen as unity. Neglecting the normal part of the left hand cut, the dispersion relation for G_l can be written as

$$(z-4)^{-l} G_l(z) = \frac{1}{\pi} \int\limits_4^{\infty} \frac{dt}{(t-4)^l}\,\frac{g_l(t)}{t-z} +$$

$$+ \frac{1}{\pi} \int\limits_4^{a(m^2)} \frac{dt}{(t-4)^l}\;\frac{A_l^{II}(t)(1+2i\varrho_\pi(t)F_l(t))}{t-z}\,. \tag{2.2}$$

In writing down Eq. (2.2), we have explicitly exhibited the threshold behaviour of G_l in order to avoid "kinematical" singularities, when l is non-integer;

$$a(m^2) = -M^{-2}\{m^2-(M+1)^2\}\,\{m^2-(M-1)^2\}$$

is the position of the anomalous threshold, m and M being the masses of Σ and Λ particles, respectively [3]. $A_l(t)$ is given exactly by the contribution of the Λ — pole; if Λ is an elementary particle (*i. e.* described by a standing pole in t and u, rather then by a Regge-trajectory), then

$$A_l(t) = \frac{G^2\pi}{(t-4)^{1/2}\,(t-4m)^{1/2}}\,P_l\left(\frac{2(M^2-m^2-1)+s}{(s-4)^{1/2}\,(s-4m^2)^{1/2}}\right)(1+e^{-i\pi l}) \tag{2.3}$$

and $A_l^{II} = -A_l$, where G is the $\Lambda\Sigma\pi$ coupling constant. In what follows, we work with the symmetrical amplitudes only so we drop the factor $(1+e^{-i\pi l})$.

We take the $\pi\pi$ amplitude in the one pole approximation, *i. e.* for $t>4$

$$S_l = 1+2i\varrho_\pi F_l(t) = \frac{l-L^*(t)}{l-L(t)} \tag{2.4}$$

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and continue this expression below threshold with the help of the technique of generalized Jost functions. Thus we put $S_l = f_l(q)(f_l i - q)^{-1}$ with $q = (z-4)^{\frac{1}{2}}$ and then it is easily seen that the Regge-poles are determined by the roots of $f_l(-q)$ (the details of the continuation procedure are explained in the Appendix). Formulae (2.1) to (2.4) constitute an integral equation for G which can be solved by standard methods. In our notation the solution reads:

$$G_l(z) = -(z-4)^l (f_l(-q))^{-1} \frac{1}{\pi} \int_{a(m^2)}^{4} dt' \frac{A^{\mathrm{II}}(t') f_l(q(t'))}{(t'-z)(t'-4)^l}. \tag{2.5}$$

Looking at the expression (2.5) we see that the singularities of G_l in the l-plane (and thus the asymptotic behaviour in the crossed channel) are determined by the denominator, $f_l(-q)$. Hence it follows that the presence of the anomalous threshold does not change the qualitative behaviour of the amplitude for large energies: $\pi\Sigma$ scattering is dominated by the vacuum pole, just like scattering without anomalous thresholds.

3. *πd scattering*

As a second example, we treat pion-deuteron scattering. The essential difference between this process and the one treated in Sec. 2 is that for πd scattering the anomalous threshold comes from $N\overline{N}$ intermediate states. Besides that, in $\pi\Sigma$ scattering we had two contributions to the anomalous cut (from the left-hand cut of the amplitude, passing over to the second sheet and from the left-hand cut of G^{II}, coming out to the first one). In πd scattering, the left-hand cut of the amplitude on the physical sheet does not contribute to the anomalous cut, as the lowest possible state is that with one deuteron. Correspondingly, we have the following dispersion relation for G:

$$|\psi(z)|^{-2}(z-4)^{-l} G_l(z) = \frac{2}{\pi} \int_{4M^2}^{r(M^2)} \frac{dt' \varrho_N(t') R_l^{\mathrm{II}}(t') H_l(t')}{|\psi(t')|^2 (t'-4)^l (t'-z)} +$$

$$+ \frac{1}{\pi} \int_{4}^{\infty} dt' \frac{\varrho_\pi(t') G_l(t') F_l^{\mathrm{II}}(t')}{|\psi(t')|^2 (t'-4)^l (t'-z)}. \tag{3.1}$$

In writing Eq. (3.1) we neglected both the left-hand cut and the normal part of the two-nucleon cut. The notation applied is as follows: $\varrho_N(t) = \left(\frac{t-4M^2}{t}\right)^{1/2}$, $H_l(t)$ is the pion-nucleon-amplitude, $R_l(t)$ is the absorptive part of the $N\overline{N} \to d\overline{d}$ amplitude in the pole approximation (corresponding to the amplitude A_l of the previous section). $\psi(t)$ is the Fourier transform of the deuteron wave function, while the threshold value $r(m^2)$ is given by:

$$r(m^2) = 16MB + O(B^2). \tag{3.2}$$

(Actually, B, the binding energy of the deuteron is very small: $B \approx 1/70$, so formula (3.2) is completely satisfactory for every practical purpose). The factor 2 before the first integral can be understood, if one takes into account that the latter arose from the contribution of two diagrams. (π scattered on p, plus π scattered on n; the πp and πn amplitudes are taken to be equal).

The other details of the derivation of Eq. (3.1) are completely anologous to the considerations made in the previous section. Now, in the one-pole approximation one has

$$H_l(t) = \frac{h_l(t)}{f_l(-q(t))}\,, \tag{3.3}$$

where $h_l(t)$ contains the contributions of left-hand- and inelastic cuts. Taking into account the expression (3.3), we can write the solution of Eq. (3.1) as follows:

$$G_l(z) = -|\psi(z)|^2(z-4)^l \frac{2}{\pi} \int\limits_{r(M^2)}^{4(M^2)} \frac{dt'}{t'-z} \frac{h_l(t')R^{\mathrm{II}}(t')}{|\psi(t')|^2(t'-4)^l} (f_l(-q))^{-1}. \tag{3.4}$$

We see that in complete analogy with $\pi\Sigma$ scattering, the singularities in the l-plane are determined by $f_l(-q)$ and thus the considerations at the end of Sec. 2 apply to this case as well. Let us only notice at this place that — as can be seen from Eq. (3.4) — the residue of G_l at the vacuum pole will be proportional to the number of nucleons contained in the target nucleus.

4. *Discussion*

Summarizing the main qualitative features of our results, we see that the asymptotic behaviour of the $\pi\Sigma$ or πd amplitudes is determined by the Pomeranchuk-pole, just as in the cases without anomalous thresholds. In the case of the scattering on a nucleus (A) the residue of the pole will be proportional to the atomic number. So, in particular, the well--known relations between total cross sections of (πA), ($\pi\pi$) and (AA) scattering:

$$\sigma^2_{\pi A} = \sigma_{\pi\pi}\sigma_{AA}$$

remain valid, as in the "elementary" case [4].

There are some complications, however, which may obscure the above picture. First of all, besides the twonucleon intermediate state, one has in a nucleus, heavier than deuteron two-deuteron, two-triton *etc.*, intermediate states, giving rise to anomalous thresholds. They obviously spoil the linear dependence on A of the amplitude. In principle, they could be calculated by successively determining the (πd), (πt), (πHe), ... amplitudes.

It is not clear from the beginning, how the result of such a calculation would look like, but if *e. g.* the individual terms simply added up, they would give rise to an essentially--exponential dependence of the amplitude on A. (Note that in this case the relation between total cross sections remains valid). A second complication arises from the presence of a coherent scattering on the nucleus as a whole. One can see, namely, that to the solutions (2.2) and (3.4) the solution of the corresponding homogeneous equation can be added, multiplied by an arbitrary constant. (This homogeneous solution corresponds to the presence of the two-pion intermediate state only). Now, obviously, this homogeneous solution has a pole in the l-plane, corresponding to the Pomeranchuk pole, and thus gives rise to the same

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energy dependence as the other terms. The "arbitrary constant" plays the role of the effective coupling constant of the Pomeranchuk pole to the nucleus (or Σ respectively). In view of a remark of Freund [5] it will be proportional to A, so its presence would not spoil the relation between total cross sections.

Finally, we want to comment on the absence of the "eclipse term" [2] in our results. As can be seen from [2], this term (giving rise to a cut in the l-plane) arises from double scattering inside the nucleus. This implies the exchange of four pions, a contribution we have not taken into account.

In conclusion, we take the opprotunity to express our sincere thanks to Professors M. Mięsowicz and Z. Koba for pleasant discussions on the subject.

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APPENDIX

We want to explain the details of the continuation of the one-pole approximation below physical thresholds. Taking a complex partial wave in the one-pole approximation, we have:

$$S_l \equiv e^{2i\delta_l} = 1+2i\varrho F_l = \frac{l-L^*}{l-L} \tag{A.1}$$

$$F_l = \frac{1}{\varrho}\frac{\operatorname{Im} L}{l-L}. \tag{A.2}$$

Obviously (A.1) or (A.2) as it stands is valid above the physical threshold only (*e. g.* for a two-pion pole, for $t>4$). In order to continue this expression for any value of the energy variable, we make use of the generalized Jost functions (g.J.f.) [6].

Introduce the variable $q = \frac{1}{2}(t-4)^{1/2}$. Then S_l can be written as follows:

$$S_l(q) = \frac{f_l(q)}{f_l(-q)}, \tag{A.3}$$

where the g.J.f. ($f_l(q)$ has the following properties (*cf.* ref. [6]):

a) $f_l(q)$ regular for Im $q<0$

b) $\lim\limits_{|q|\to\infty} f_l(q) = 1$

c) $f_l(-q^*) = f_l(q)^*$

d) For Im $q \to -0$, the phase of $f_l(q)$ is δ_l.

e) For Im $q>0$, $f_l(q) = S_l(q) f_l(-q)$.

(The properties a) to e) can be generalized for complex l in a straightforward way).

One can easily write down a formal expression which is valid on the whole q-plane.

Writing $q = q_1+iq_2$ (q_1, q_2 real) the expression

$$f_l(q) = \exp\left\{\frac{1}{\pi}\int\limits_0^\infty \frac{\delta_l(\nu)\,d\nu}{\nu-q_1^2+q_2^2+2iq_1|q_2|}\right\} \tag{A.4}$$

satisfies the requirements a) to e) ($\nu = 1/4(t'-4)$).

With the help of (A.4) we have:

$$S_l(q) = \exp\left\{2i \int_0^\infty d\nu\delta_l(\nu) \frac{2q_1|q_2|}{(\nu - q_1^2 + q_2^2) + 4q_1^2q_2^2}\right\}. \tag{A.5}$$

Inserting from (A.1)

$$\delta_l(\nu) = \frac{1}{2i} \log \frac{l - L^*(\nu)}{l - L(\nu)},$$

fA.4) and (A.5) gives the result we needed. It is clear from (A.3) that the Regge-poles arise from the roots of $f_l(-q)$.

The relation of (A.3) with an N/D representation is clear: one can write

$$F_l(t) = N_l(t)D_l(t)^{-1},$$

with

$$D_l(t) = f_l(-q(t))$$

$$N_l(t) = (2i\varrho(t))^{-1}[f_l(q(t)) - f_l(-q(t))].$$

Comparing this with the usual N/D method, the appearence of the factors $f_l(-q)$ in expressions like Eq. (2.5) is easily understood.

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