SPIN-WAVE THEORY OF MAGNETOPLUMBITE FERRITES

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The theory of spin waves in ferrites having the magnetoplumbite structure is presented. The approximation of the magnetic quasisaturation is used. The spin-wave spectrum consists of five branches. The dispersion formulas have great uniaxial anisotropy.

I. Introduction

A wide class of ferromagnetic oxides obtained as products of the reaction of Fe_2O_3, BaO and MeO in various proportions are technically useful as permanent-magnetic materials. The crystal structures of these compounds present the hexagonal symmetry and are related to the magnetoplumbite ($PbFe_{12}O_{19}$) structure described by Adelsköld (1938, see also Smit and Wijn 1959). Many compounds, *e.g.* $Ba_{1-\lambda-\mu-\nu}Pb_\lambda Sr_\mu Ca_\nu Fe_{12}O_{19}$ ($0 \leqslant \lambda+\mu+\nu \leqslant 1$), $BaMe_\delta^{III}Fe_{12-\delta}O_{19}$(Me = Al, Ga, Cr, ...), $BaMe_\delta^{II}Me'^{IV}_\delta Fe_{12-2\delta}O_{19}$ (Me = Ni, Co, Zn, ...; Me′ = Ti, Ge, Sn, Zr, ...), crystallize in the magnetoplumbite structure. In this paper the spin-wave theory of ferrites with magnetoplumbite structure is presented.

II. Dispersion formula for spin waves

The magnetic lattice of the magnetoplumbite structure consists of 24 translational sublattices. The magnetic ions interact between themselves in an indirect way through the oxygen ions. In the elementary cell, two magnetic ions are in the pentahedral sites, four in the tetrahedral sites and eighteen in the octahedral sites. The foregoing nomenclature of the sites denotes that the respective magnetic ions are at the centres of the pentahedron, tetrahedron and octahedron in whose vertices there are oxygen ions.

Let us assume that the amplitudes of the spin-waves are equal at nodes having identical environments.

Thus, we assume as equal the spin-wave amplitudes at all nodes within the following groups of nodes: a) tetrahedral with indices ε, b) pentahedral with indices α, c) octahedral with indices β, d) octahedral with indices γ, and e) octahedral with indices δ.

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The position vectors of magnetic nodes are the following:
pentahedral nodes (α)

$$\vec{R}_\alpha = \vec{R}_n + \vec{\varrho}_\alpha, \quad \vec{\varrho}_{\alpha_i} = \left(0, 0, \frac{1}{4}\right), \ \left(0, 0, \frac{3}{4}\right), \ i = 1, 2;$$

octahedral nodes (β)

$$\vec{R}_\beta = \vec{R}_n + \vec{\varrho}_\beta, \quad \vec{\varrho}_{\beta_i} = (0, 0, 0), \ \left(0, 0, \frac{1}{2}\right), \ i = 1, 2;$$

octahedral nodes (γ)

$$\vec{R}_\gamma = \vec{R}_n + \vec{\varrho}_\gamma, \quad \vec{\varrho}_{\gamma_i} = (U, 2U, Z), \ \left(U, 2U, \frac{1}{2} - Z\right), \qquad i = 1, 2, 3, 4$$

$$(2U, U, \bar{Z}), \ \left(2U, U, \frac{1}{2} + Z\right), \ U = \frac{1}{3}, \ Z = \frac{17}{90};$$

octahedral nodes (δ)

$$\vec{R}_\delta = \vec{R}_n + \vec{\varrho}_\delta, \quad \vec{\varrho}_{\delta_i} = (U, 2U, \bar{Z}), \ (U, 5U, \bar{Z}), \ (4U, 5U, \bar{Z}),$$

$$(2U, U, Z), (5U, U, Z), (5U, 4U, Z),$$

$$\left(U, 2U, \frac{1}{2} + Z\right), \ \left(U, 5U, \frac{1}{2} + Z\right), \ \left(4U, 5U, \frac{1}{2} + Z\right),$$

$$\left(2U, U, \frac{1}{2} - Z\right), \ \left(5U, U, \frac{1}{2} - Z\right), \ \left(5U, 4U, \frac{1}{2} - Z\right),$$

$$i = 1, 2, \ldots 12, \ U = \frac{1}{6}, Z = \frac{13}{120};$$

tetrahedral nodes (ε)

$$\vec{R}_\varepsilon = \vec{R}_n + \vec{\varrho}_\varepsilon, \ \vec{\varrho}_{\varepsilon_i} = (U, 2U, Z), \ \left(U, \ 2U, \frac{1}{2} - Z\right), \qquad i = 1, 2, 3, 4$$

$$(2U, U, \bar{Z}), \ \left(2U, U, \frac{1}{2} + Z\right), \quad U = \frac{1}{3}, \ Z = \frac{1}{36};$$

where

$$\vec{R}_n = \sum_{i=1}^{3} n_i \vec{e}_i, \quad n_i = 0, 1, 2, \ldots N_i$$

$N_1 \cdot N_2 \cdot N_3 = N$ — the number of elementary cells;
$24N$ — the number of "magnetic" nodes;
$\vec{\varrho}_i (i = \alpha, \beta, \gamma, \delta, \varepsilon)$ — the position vectors "of magnetic" nodes in the elementary cell.

The coordinates of the vectors $\vec{\varrho}_i$ are given in units of a, c lattice constants ($|\vec{e}_1| = |\vec{e}_2| = a$, $|\vec{e}_3| = c$).

The Hamiltonian of superexchange interaction between "magnetic" nodes is of the form:

$$\hat{H} = -\frac{1}{2}\sum_{l_1, l_2} J(l_1, l_2)\,\hat{\vec{S}}_{l_1}\cdot\hat{\vec{S}}_{l_2}, \tag{1}$$

wherein

l_1, l_2 are the indices of „magnetic" lattice nodes;

$J(l_1, l_2)$ are integrals of superexchange interactions between the ions l_1 and l_2;

$\hat{\vec{S}}_l$ is an operator of the spin vector at lattice node l, in $\frac{\hbar}{2}$ units.

We replace the spin operators $\hat{S}_l^x$, $\hat{S}_l^y$, $\hat{S}_l^z$ by Bose creation and annihilation operators $\hat{b}_l^*$, $\hat{b}_l$ (Kowalewski 1961). We assume, furthermore, the Néel orientation of spins in the ground state (Smit, Wijn 1959, p. 182, 193).

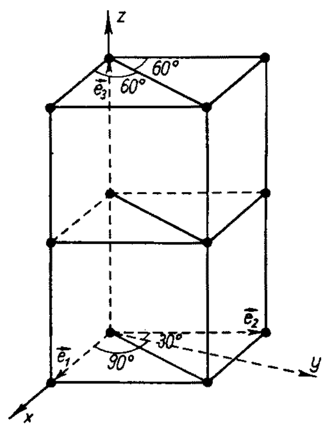

Fig. 1. Elementary cell of the magnetoplumbite structure constructed on the unit vectors $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$. Only the octahedral nodes β are shown in the figure.

$$\hat{S}_l^x = \sqrt{S_l}[g(\hat{n}_l)\hat{b}_l + \hat{b}_l^* g(\hat{n}_l)],$$

$$\hat{S}_l^y = i\eta_l\sqrt{S_l}[g(\hat{n}_l)\hat{b}_l - \hat{b}_l^* g(\hat{n}_l)],$$

$$\hat{S}_l^z = \eta_l[2\hat{b}_l^*\hat{b}_l - S_l],$$

$$g(\hat{n}_l) = \sqrt{1-\frac{\hat{n}_l}{S_l}},\quad \hat{n}_l = \hat{b}_l^*\hat{b}_l,\quad \eta_l = \begin{cases} +1 \text{ for } l = \alpha, \beta, \delta \\ -1 \text{ for } l = \gamma, \varepsilon \end{cases}.$$

Let us introduce also the simplifying assumption of magnetic quasisaturation (Holstein, Primakoff 1940).

We now transform the operators $\hat{b}$, $\hat{b}^*$ to the reciprocal lattice:

$$\hat{b}_l + \hat{b}_l^* = \frac{1}{\sqrt{NN_l}} \sum_{\vec{K}} e^{(-1)^\chi i\vec{K}\cdot\vec{R}_{l(n)}} \hat{Q}^l(\vec{K}),$$

$$\hat{b}_l - \hat{b}_l^* = \frac{i}{\sqrt{NN_l}} \sum_{\vec{K}} e^{-(-1)^\chi i\vec{K}\cdot\vec{R}_{l(n)}} \hat{P}^l(\vec{K}),$$

where

$$\vec{K} = 2\pi \sum_{i=1}^{3} \frac{m_i}{N_i} \vec{b}^i, \qquad \vec{b}^i \cdot \vec{e}_j = \delta_{ij},$$

$$-\frac{1}{2} N_i < m_i \leqslant \frac{1}{2} N_i, \qquad N_\alpha = N_\beta = 2, \qquad N_\gamma = N_\varepsilon = 4, \qquad N_\delta = 12,$$

χ is even for $l = \gamma, \varepsilon$ and odd for $l = \alpha, \beta, \delta$.

The Fourier transforms $\hat{P}$ and $\hat{Q}$ satisfy the following commutation relations:

$$\hat{Q}_l(\vec{K})\hat{P}_{l'}(\vec{K}') - \hat{P}_{l'}(\vec{K}')\hat{Q}_l(\vec{K}) = 2i\delta_{l,l'}\delta_{\vec{K},\vec{K}'}.$$

All other commutators of the operators $\hat{P}$ and $\hat{Q}$ vanish.

On restricting the calculations to the case of strongest interactions between nearest neighbours only, the Hamiltonian becomes

$$\hat{H} = C + \sum_{(\mu,\nu)} A_{\mu\nu} \sum_{\vec{K}} \varphi^e_{\mu\tilde{\nu}} [\hat{Q}_\mu(\vec{K})\hat{Q}_\nu(\vec{K}) - \hat{P}_\mu(\vec{K})\hat{P}_\nu(\vec{K})] +$$

$$+ \sum_{\mu} B_\mu \sum_{\vec{K}} [\hat{Q}_\mu(\vec{K})\hat{Q}_\mu(-\vec{K}) + \hat{P}_\mu(\vec{K})\hat{P}_\mu(-\vec{K})]. \tag{2}$$

The sum $\sum_{(\mu,\nu)}$ consists of four expressions: $(\mu, \nu) = (\alpha, \gamma), (\beta, \varepsilon), (\delta, \gamma), (\delta, \varepsilon)$. The sum $\sum_{\mu}$ consists of five expressions: $\mu = \alpha, \beta, \gamma, \delta, \varepsilon$.

$$C = 6NJ_{\alpha\gamma}(S_\alpha + S_\gamma + S_\alpha S_\gamma) + 6NJ_{\beta\varepsilon}(S_\beta + S_\varepsilon + S_\beta S_\varepsilon) +$$

$$+ 12NJ_{\gamma\delta}(S_\gamma + S_\delta + S_\gamma S_\delta) + 18NJ_{\delta\varepsilon}(S_\delta + S_\varepsilon + S_\delta S_\varepsilon),$$

$$\varphi^e_{\mu\tilde{\nu}} = \sum_{\mu\tilde{\nu}} \cos \vec{K}\cdot\vec{\tau}_{\mu\tilde{\nu}}, \qquad \varphi^{\circ}_{\mu\tilde{\nu}} = \sum_{\mu\tilde{\nu}} \sin \vec{K}\cdot\vec{\tau}_{\mu\tilde{\nu}} = 0.$$

The vectors $\vec{\tau}_{\mu\tilde{\nu}}$ connect an arbitrary node μ with the nearest neighbouring nodes of type ν:

$$\vec{\tau}_{\alpha_i,\tilde{\gamma}_j} = (0, \pm 2y, \pm z), (\pm x, \pm y, \pm z); \; i = 1, 2; \; j = 1, 2, \ldots 6;$$

$$x = \frac{a}{2}, \; y = \frac{\sqrt{3}}{6} a, \quad z = \frac{11}{180} c;$$

$$\vec{\tau}_{\delta_i,\tilde{\gamma}_j} = (\pm x, \pm y, \pm z), (\pm x, \pm y, \pm z), (\pm 2x, 0, \pm z), (\pm 2x, 0, \pm z);$$

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$$i = 1, 2, \dots 12; \quad j = 1, 2; \quad x = \frac{1}{4}a, \quad y = \frac{\sqrt{3}}{4}a, \quad z = \frac{29}{360}c;$$

$$\vec{r}_{\delta_i, \tilde{\varepsilon}_j} = (\pm x, \pm y, \pm z_1), (\pm x, \pm 3y, \pm z_2), (\pm x, \pm 3y, \pm z_2),$$

$$(0, \pm 2y, \pm z_1), (\pm 2x, 0, \pm z_2), (\pm 2x, 0, \pm z_2); \quad i = 1, 2, \dots 12; \ j = 1, 2, 3;$$

$$x = \frac{1}{4}a, \quad y = \frac{\sqrt{3}}{12}a, \quad z_1 = \frac{49}{360}c, \quad z_2 = \frac{29}{360}c;$$

$$\vec{r}_{\beta_i, \tilde{\varepsilon}_j} = (\pm x, \pm y, \pm z), (0, \pm 2y, \pm z); \quad i = 1, 2; \ j = 1, 2, \dots 6;$$

$$x = \frac{1}{2}a, \quad y = \frac{\sqrt{3}}{6}a, \quad z = \frac{1}{36}c.$$

The figures in parentheses denote vector components in the orthogonal x, y, z system (see, Fig. 1).

$\sum\limits_{\mu, \tilde{\nu}}$ denotes summation over all μ nodes in an elementary cell and over all their nearest neighbouring nodes of type ν.

$$A_{\alpha\gamma} = -\frac{\sqrt{S_\alpha S_\gamma}}{4\sqrt{2}} J_{\alpha\gamma}, \quad A_{\beta\varepsilon} = -\frac{\sqrt{S_\beta S_\varepsilon}}{4\sqrt{2}} J_{\beta\varepsilon},$$

$$A_{\delta\gamma} = -\frac{\sqrt{S_\delta S_\gamma}}{8\sqrt{3}} J_{\delta\gamma}, \quad A_{\delta\varepsilon} = -\frac{\sqrt{S_\delta S_\varepsilon}}{8\sqrt{3}} J_{\delta\varepsilon},$$

$$B_\alpha = -\frac{3}{2} S_\gamma J_{\alpha\gamma}, \quad B_\beta = -\frac{3}{2} S_\varepsilon J_{\beta\varepsilon}, \quad B_\gamma = -\frac{3}{4} S_\alpha J_{\alpha\gamma} - \frac{3}{2} S_\delta J_{\delta\gamma},$$

$$B_\delta = -\frac{1}{2} S_\gamma J_{\delta\gamma} - \frac{3}{4} S_\varepsilon J_{\delta\varepsilon}, \quad B_\varepsilon = -\frac{3}{4} S_\beta J_{\beta\varepsilon} - \frac{9}{4} S_\delta J_{\delta\varepsilon}.$$

The quantum equations of motion for the operators $\hat{P}$ and $\hat{Q}$ are following:

$$\dot{\hat{Q}}_\alpha(\vec{K}) = H_1 \hat{P}_\alpha(-\vec{K}) - R_1 \hat{P}_\gamma(\vec{K}),$$

$$\dot{\hat{Q}}_\beta(\vec{K}) = H_2 \hat{P}_\beta(-\vec{K}) - R_2 \hat{P}_\varepsilon(\vec{K}),$$

$$\dot{\hat{Q}}_\gamma(\vec{K}) = H_3 \hat{P}_\gamma(-\vec{K}) - R_1 \hat{P}_\alpha(\vec{K}) - R_3 \hat{P}_\delta(\vec{K}),$$

$$\dot{\hat{Q}}_\delta(\vec{K}) = H_4 \hat{P}_\delta(-\vec{K}) - R_3 \hat{P}_\gamma(\vec{K}) - R_4 P_\varepsilon(\vec{K}),$$

$$\dot{\hat{Q}}_\varepsilon(\vec{K}) = H_5 \hat{P}_\varepsilon(-\vec{K}) - R_2 \hat{P}_\beta(\vec{K}) - R_4 \hat{P}_\delta(\vec{K}),$$

where

$$H_1 = 4B_\alpha, \ H_2 = 4B_\beta, \ H_3 = 4B_\gamma, \ H_4 = 4B_\delta, \ H_5 = 4B_\varepsilon,$$

$$R_1 = 2A_{\alpha\gamma}\varphi^e_{\alpha\tilde{\gamma}}, \ R_2 = 2A_{\beta\varepsilon}\varphi^e_{\beta\tilde{\varepsilon}}, \ R_3 = 2A_{\delta\gamma}\varphi^e_{\delta\tilde{\gamma}}, \ R_4 = 2A_{\delta\varepsilon}\varphi^e_{\delta\tilde{\varepsilon}}.$$

On interchanging $\hat{Q}$ and $\hat{P}$ and on putting $-H_i$ in place of H_i ($i = 1, 2, \dots 5$), we get another valid group of five equations of motion.

The second order equations of motion for $\hat{Q}$ and $\hat{P}$ can be written (taking into account their exponential time dependence) in the form:

$$\begin{bmatrix} H_1^2-R_1^2-\omega^2, & 0, & -(H_3-H_1)R_1, & -R_1R_3, & 0 \\ 0, & H_2^2-R_2^2-\omega^2, & 0, & -R_2R_4, & -(H_5-H_2)R_2 \\ (H_3-H_1)R_1, & 0, & H_3^2-R_1^2-R_3^2-\omega^2, & -(H_4-H_3)R_3, & -R_3R_4 \\ -R_1R_3, & -R_2R_4, & (H_4-H_3)R_3, & H_4^2-R_3^2-R_4^2-\omega^2, & -(H_5-H_4)R_4 \\ 0, & (H_5-H_2)R_2, & -R_3R_4, & (H_5-H_4)R_4, & H_5^2-R_2^2-R_4^2-\omega^2 \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} \hat{Q}_\alpha(\pm \vec{K}) \\ \hat{Q}_\beta(\pm \vec{K}) \\ \hat{Q}_\gamma(\mp \vec{K}) \\ \hat{Q}_\delta(\pm \vec{K}) \\ \hat{Q}_\varepsilon(\mp \vec{K}) \end{bmatrix} = 0. \tag{3}$$

The condition for obtaining non-zero solutions of Eqs (3) is:

$$\begin{vmatrix} H_1 \pm \omega, & 0, & R_1, & 0, & 0 \\ 0, & H_2 \pm \omega, & 0, & 0, & R_2 \\ -R_1, & 0, & -H_3 \pm \omega, & -R_3, & 0 \\ 0, & 0, & R_3, & H_4 \pm \omega, & R_4 \\ 0, & -R_2, & 0, & -R_4, & -H_5 \pm \omega \end{vmatrix} = 0. \tag{4}$$

In the special cases of $H_1 = H_2 = H_4 = H$ and $H_3 = H_5 = H'$ ($J_{\varepsilon\beta} = J_{\varepsilon\delta} = S_\gamma S_\varepsilon^{-1} J_{\alpha\gamma} = J < 0$, $J_{\alpha\gamma} = \frac{2}{3} J_{\delta\gamma}$, $S_\gamma S_\varepsilon^{-1} = (S_\alpha + 3S_\delta)(S_\beta + 3S_\delta)^{-1}$) the dispersion formulas (positive solutions of Eqs (4)) take the form:

$$\omega_3 = H,$$

$$\omega_{1,4} = \frac{1}{2}\left\{\pm(H-H') - \sqrt{(H+H')^2 - 2(R_1^2+R_2^2+R_3^2+R_4^2) - 2\sqrt{(R_1^2-R_2^2+R_3^2-R_4^2)^2 + 4R_3^2R_4^2}}\right\}$$

$$\omega_{2,5} = \frac{1}{2}\left\{\pm(H-H') - \sqrt{(H+H')^2 - 2(R_1^2+R_2^2+R_3^2+R_4^2) + 2\sqrt{(R_1^2-R_2^2+R_3^2-R_4^2)^2 + 4R_3^2R_4^2}}\right\}.$$

If $S_\alpha = S_\beta = S_\gamma = S_\varepsilon = S_\delta = S$ we get for small values of K the following dispersion formulas:

$$\omega_3 = -6JS,$$

$$\omega_{1,4} \cong -(3 \mp 3)JS - \frac{9}{4} JSa^2[(K^x)^2 + (K^y)^2] - 0.056 \cdot JSc^2(K^z)^2,$$

$$\omega_{2,5} \cong -3(\sqrt{7} \mp 1)JS - \frac{\sqrt{7}}{8} JSa^2[(K^x)^2 + (K^y)^2] - 0.0137 \cdot JSc^2(K^z)^2. \tag{5}$$

It is readily seen, that the dispersion formulas (5) have the great uniaxial anisotropy (the minimum of the spin-wave energy occurs for the c-axis).

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