DIFFRACTION SCATTERING OF DEUTERONS ON NON-SPHERICAL NUCLEI

III. SEMITRANSPARENT NUCLEUS WITH DIFFUSE EDGES

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The paper concerns diffraction scattering of deuterons on nuclei. Elastic and inelastic scattering leading to excitation of the collective states in nuclei are investigated. Deuteron scattering is treated as a superposition of the difaction scatterings of both the neutron and proton of the deuteron by applying the Achiezez-Sitenko method. The optical model is used to describe the nucleon-nucleus scattering, instead of the "black nucleus" model used in previous investigations. There is no evidence for the existence of the "universal curve" in the present approach. The results of numerical calculations presented here agree better with experiment than in previous calculations.

I. Introduction

In previous papers (Borysowicz and Dąbrowski 1962; Borysowicz and Dąbrowski II, 1962; hereafter to be quoted as I and II) the Drozdov-Blair model[1] for the diffraction scattering of a point particle from deformed nuclei with arbitrary multipolarity was extended to the case of deuteron scattering. The internal structure of the deuteron was taken into account, *i.e.* both nucleons of the deuteron were supposed to be scattered independently from a target nucleus. In I and II, as well as in the Drozdov-Blair theory, complete absorption of the incident projectile by the target nucleus was assumed ("black nucleus" model). The Drozdov-Blair model has been mainly applied to the case of alpha-particle scattering, where the assumption of a strongly absorbing nucleus is well justified. In the case of nucleon-nucleus scattering, the "black nucleus" model is far from realistic. This note presents an attempt to replace the "black nucleus" model used in I and II with the optical model, well established for the description of nucleon-nucleus scattering.

In order to obtain not only the elastic but also the inelastic scattering amplitude a deformed optical potential dependent on the intrisic state of the nucleus is necessary. Such a potential may be written (see Blair 1962) in the form $\tilde{V}(\boldsymbol{r}) = V(\tilde{r})$, where $V(r)$ is the usual,

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[1] All references concerning the Drozdov-Blair model are given in I.

ACTA PHYSICA POLONICA Vol. XXVIII (1965) Fasc. 2

mula, and the elastic and inelastic scattering amplitudes are then calculated in the adiabatic approximation[2]. All formulae necessary for numerical calculation are given in theAppendix. In Section II the present approach is compared with the previous one, and numerical results are compared with experimental data.

II. Discussion of the model and comparison with experiment

When the optical model is used instead of the "black nucleus" model, the angular distributions are expected to be changed in two ways. First, the absolute value of the diffraction scattering cross-section, which is proportional to the total absorption, should diminish when the semitransparent optical potential is used. The second change concerns the form of the angular distributions in which the strong oscillations and non-physical valleys typical for the Drozdov-Blair theory are expected to be less pronounced. These oscillations and valleys were due to the sharp edge of the "black nucleus" potential and should become smoother or disappear when the optical potential with a diffuse edge is applied.

The numerical examples confirm the above considerations. The cross-section for both elastic and inelastic scattering of deuterons on Mg and Al nuclei were calculated and the results are presented in Figs 2, 3 and 4. In the case of Mg the energy of the incident deuterons

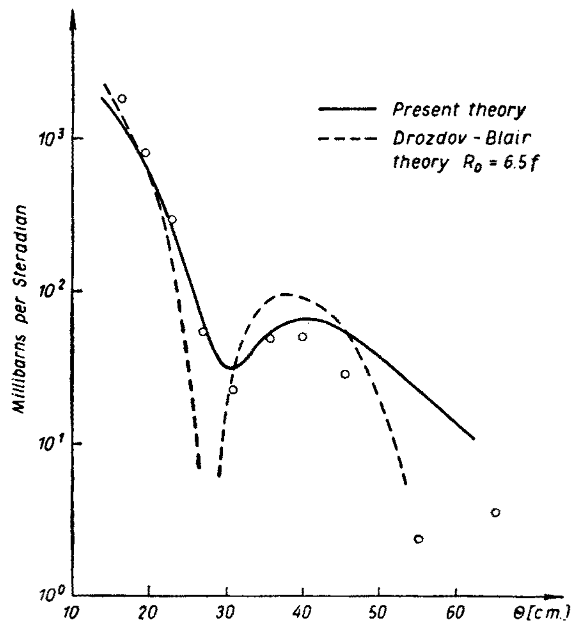

Fig. 2. Experimental (Greenlees and Love 1960) and theoretical distributions for 19.6 MeV deuterons scattered elastically on Mg

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[2] A different formalism for the problem of elastic deuteron scattering has been developed by Zamick (1963) and Rustigi (1964). Unfortunately, because of numerical difficulties, the results obtained in the present paper have not been compared with those of the Zamick and Rustigi theories.

ACTA PHYSICA POLONICA Vol. XXVIII (1965) Fasc. 2

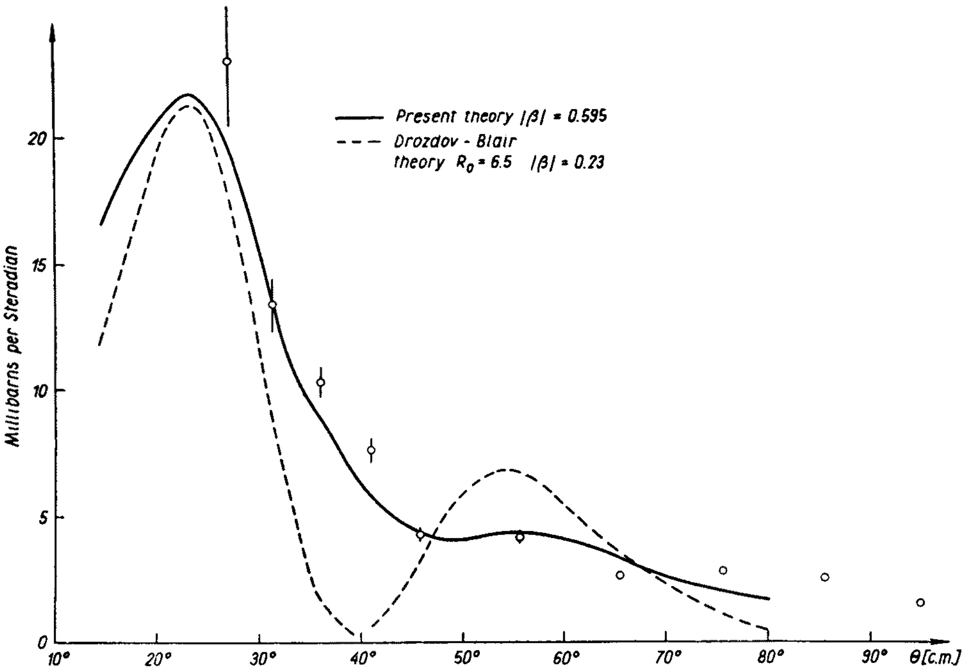

Fig. 3. Experimental (Greenlese and Love 1960) and theoretical angular distributions for inelastic deuteron scattering leading to the 2^+ state with 1.37 MeV excitation energy. Incident deuteron energy 19.6 MeV

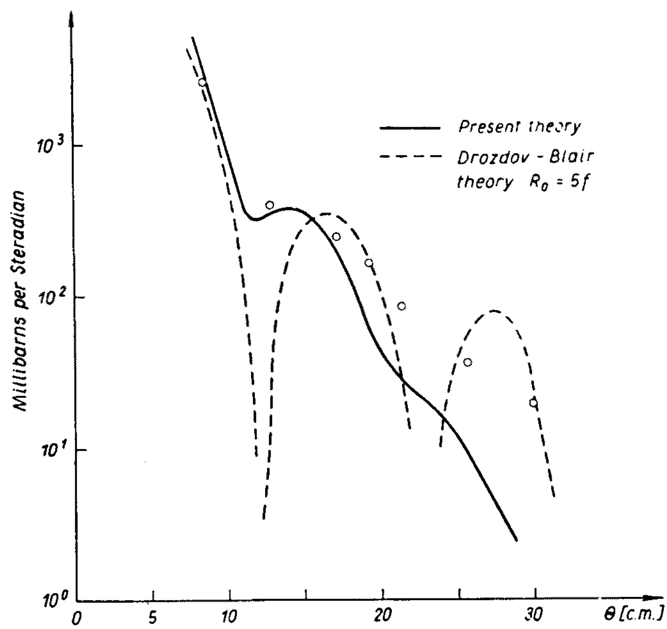

Fig. 4. Experimental (Baldwin *et al.* 1956) and theoretical angular distributions for elastic scattering of 157 MeV deuterons on Al

was 19.6 MeV and the *WKB* approximation is not justified for such low energy. Unfortunately there is a lack of experimental data on enelastic deurteon scattering at higher energies. The validity of all other approximations was discussed in I and II.[3]

The optical potential was assumed to be the same for neutrons and protons, and the spin-orbit term was neglected. In the case of the Mg nucleus, because of the low energy of the incident particle surface absorption was assumed

$$V = U + iW,$$

where
$$V = U_0/[1+\exp(r-R)/a],$$
$$W = W_0 \exp[-(r-R)^2/b^2]$$

and

$$U_0 = -44\text{ MeV},\ a = 0.6f,$$
$$W_0 = -10\text{ MeV},\ b = 1.1f,$$
$$R = 1.3 \times A^{1/3} f.$$

The choice of the parameters U_0, W_0, a, b seems to be well justifield by the optical model analysis of nucleon-nucleus scattering (Saxon 1961—62; see also Woods, Saxon 1954; Melkanoff *et al.* 1959; Bjorklund *et al.* 1956). The deuteron ground state wave function was assumed in the Hulthen form

$$\varphi_0(\mathrm{r}) = N^{-1/2}(e^{-\alpha r} - e^{-\gamma r}),$$

with

$$N^{-1} = \frac{1}{2\pi}\frac{\alpha\gamma(\alpha+\gamma)}{(\gamma-a)^2}.$$

The values $\alpha = 0.232 f^{-1}$ and $\gamma = 1.204 f^{-1}$ were used (Moravcsik 1958; Gartenhaus 1955).

The values of the modulus of the deformation parameter and radius parameter R_0 of the "black nucleus" model were adjusted to get the best fit of the theoretical to the experimental cross-sections. The result is:

$|\beta| = 0.595$ for the present approach, as compared with,

$|\beta| = 0.23$ obtained in I, $R_0 = 6f$ the same as obtained on the basis of alpha-particle scattering by Lair *et al.* (1960). Experimental values of cross-sections are taken from the work of Greenlees and Love (1960). In Fig. 2 the elastic cross-sections are shown. The absolute value of the cross-section according to the present approach is larger at the first valley and smaller at the second peak as compared with the results of I, and thus agrees better with experiment. In the case of inelastic scattering leading to the 2^+ state with 1.37 MeV excitation energy (Fig. 3) the improvement is even more apparent, especially if one notes that other available experimental values of $|\beta|$ are 0.43 (Ofer and Schwarcschild 1959) and

[3] In the present approach it is no longer assumed, as it was necessary to assume in I and II, that the deuteron radius is small in comparizon with the radius of the target-nucleus.

0.56 (Helm 1956), and that the cross-section is proportional to $|\beta|^2$. The present approach and that of I and II fail, however, for angles smaller than 30°. Coulomb interaction, which was neglected in the calculation is not strong enough to explain the discrepancy. The pure Coulomb cross-section calculated in the first Born approximation (Mamasakhlisov and Kopaleshvili 1958) is about 4 mb at zero scattering angle and decreases rapidly for larger angles.

The last example is the elastic scattering of deuterons with energy 157 MeV (experiments of Baldwin 1957) on Al (Fig. 4). The optical potential with volume absorption was assumed:

$$V(r) = (U_0+iW_0)/[1+\exp{(r-R)/a}],$$

where

$$U_0 = -22\,\mathrm{MeV},\ W_0 = -20\,\mathrm{MeV},$$

$$a = 0.65\,f,\ R = 1.25\times A^{1/3}f.$$

The references to the parameters of the potential are the same as in the case of Mg. The R_0 parameter of the "black nucleus" model was adjusted to be $4.5\,f$. Experiment shows no oscillations in the angular distribution and the calculated distribution shows a similar behaviour. The previous method of I and II is incapable of producing distributions without strong oscillations as follows from the existence of the "universal curve" (Blair *et al.* 1960). The existence of the "universal curve" in the Drozdov-Blair model folows from the fact that the scattering amplitude divided by k depends on the energy only trough the product $k \sin\theta$. In the present approach this is not the case, because the Ω function calculated with the optical potential depends in a complicated way on the energy of the incident particle.

The numerical integrations were performed partly with the help of the Elliott 803 electronic computer in the Institute of Electrical Engineering in Międzylesie and parthly on the desk computer Supermetall.

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APPENDIX

The amplitude of deuterons scattering on a "frozen" nucleus with position given by parameters α_{lm} is

$$\bar{f}_d(\alpha,\varkappa) = ik_d\cos^2\frac{\theta}{2}\int\!\!\int d\boldsymbol{\rho}'\, e^{-i\varkappa\rho}\omega_d(\boldsymbol{\rho}'),$$

where $\boldsymbol{k}_f$ is the wave vector of the outgoing deuteron, $\boldsymbol{\varkappa}$ its projection on the shadow plane and k_d its modulus. The ω_d function for the deuteron is a superposition of the corresponding Ω functions for both nucleons

$$\omega_d(\boldsymbol{\rho}') = \int\!\!\int\!\!\int d\boldsymbol{r}|\varphi_0(r)|^2\left[1-\Omega\left(\boldsymbol{\rho}'+\frac{\boldsymbol{r}}{2}\right)\Omega\left(\boldsymbol{\rho}'-\frac{\boldsymbol{r}}{2}\right)\right],$$

ACTA PHYSICA POLONICA Vol. XXVIII (1965) Fasc. 2

ACTA PHYSICA POLONICA Vol. XXVIII (1965) Fasc. 2

and the Ω function for a single nucleon is (with $k_n \cong k_d/2$)

$$\Omega(\boldsymbol{\rho}') = \exp\left\{ik_n \int\limits_{-\infty}^{\infty} \left[\sqrt{\frac{E_n - V(\boldsymbol{r})}{E_n}} - 1\right] ds\right\}, \qquad r = \sqrt{s^2 + \varrho'^2}.$$

To the first order in α_{lm} one has

$$\Omega(\boldsymbol{\rho}') = \Omega_{el}(\varrho') + \sum_{lm} \alpha_{lm} \Omega_{lm}(\boldsymbol{\rho}')$$

$$\Omega_{el}(\varrho') = \Omega(\boldsymbol{\rho}')|_{\alpha_{lm}=0} = \exp\left\{ik_n \int\limits_{-\infty}^{\infty} \left[\sqrt{\frac{E_n - V(\boldsymbol{r})}{E_n}} - 1\right] ds\right\}$$

and

$$\Omega_{lm}(\boldsymbol{\rho}') = \frac{\partial}{\partial \alpha_{lm}} \Omega(\boldsymbol{\rho}')|_{\alpha_{lm}=0} = -\frac{ik_n}{2} \Omega_{el}(\varrho') \int\limits_{-\infty}^{\infty} \frac{V'(r)}{\sqrt{E_n[E_n - V(r)]}} r Y_{lm}(\theta'\Phi')\, ds.$$

Now $\omega_d(\boldsymbol{\rho}')\, \bar{f}_d(\boldsymbol{\alpha}, \boldsymbol{\varkappa})$ can too be resolved into elastic (zero order in α_{lm} and inelastic) (first order in α_{lm}) parts

$$\omega_d(\boldsymbol{\rho}') = \omega_{el}(\varrho') + \sum_{lm} \alpha_{lm} \omega_{lm}(\boldsymbol{\rho}'),$$

$$\bar{f}_d(\boldsymbol{\alpha}, \boldsymbol{\varkappa}) = \bar{f}_d(el, \varkappa) + \sum_{lm} \alpha_{lm} \bar{f}(lm, \boldsymbol{\varkappa}),$$

where

$$\omega_{el}(\varrho) = \iiint d\boldsymbol{r} |\varphi_0(r)|^2 \left[1 - \Omega_{el}\left(\left|\boldsymbol{\rho} + \frac{\boldsymbol{r}}{2}\right|\right) \Omega_{el}\left(\left|\boldsymbol{\rho} - \frac{\boldsymbol{r}}{2}\right|\right)\right],$$

$$\omega_{el}(\boldsymbol{\rho}) = -2 \iiint d\boldsymbol{r} |\varphi_0(r)|^2 \,\Omega_{el}\left(\left|\boldsymbol{\rho} + \frac{\boldsymbol{r}}{2}\right|\right) \Omega_{lm}\left(\boldsymbol{\rho} - \frac{\boldsymbol{r}}{2}\right) = e^{im\Phi_{\boldsymbol{\varrho}}} \tilde{\omega}_{lm}(\varrho),$$

$$\tilde{\omega}_{lm}(\varrho) = -2 \iiint d\boldsymbol{r} |\varphi_0(r)|^2 \,\Omega_{el}\left(\left|\boldsymbol{\eta} + \frac{\boldsymbol{r}}{2}\right|\right) \Omega_{lm}\left(\boldsymbol{\eta} - \frac{\boldsymbol{r}}{2}\right) e^{im\Phi_{\boldsymbol{\eta} - \boldsymbol{r}/2}},$$

$$|\boldsymbol{\eta}| = |\boldsymbol{\rho}|, \; \Phi_{\boldsymbol{\eta}} = 0,$$

and

$$\bar{f}(el, \varkappa) = ik_d \cos^2\frac{\theta}{2} \int\limits_0^{\infty} d\varrho\varrho J_0(\varkappa, \varrho)\, \omega_{el}(\varrho),$$

$$\bar{f}(lm, \boldsymbol{\varkappa}) = ik_d \cos^2\frac{\theta}{2}\, e^{im\Phi_{\boldsymbol{\varkappa}}} \int\limits_0^{\infty} d\varrho\varrho i^{|m|} J_{|m|}(\varkappa\varrho)\, \tilde{\omega}_{lm}(\varkappa).$$

Finally, the elastic and inelastic cross-sections can be expressed in terms of the amplitude $\bar{f}_d$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{el}} = |\bar{f}(el, \varkappa)|^2,$$

$$\left(\frac{d\sigma}{d\Omega}\right)(0 \rightarrow l) = |\langle lm|\, \alpha_{lm}|00\rangle|^2 \sum_{m=-l}^{l} |\bar{f}(lm, \varkappa^2)|^2,$$

where

$$|\langle lm|\alpha_{lm}|\, 00\rangle|^2 = \begin{cases} \dfrac{h\omega_l}{2c_l} \\ \text{for vibration excitation of the nucleus} \\ \dfrac{\beta_l^2}{2l+1} \\ \text{for rotation excitation of the nucleus.} \end{cases}$$

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ACTA PHYSICA POLONICA Vol. XXVIII (1965) Fasc. 2