PROPAGATION OF CORRELATION TENSORS OF THE ELECTROMAGNETIC FIELD INTENSITIES

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The analysis of the correlations between the electromagnetic field intensities is performed. Special attention is given to the propagation of the quantities describing the degree of the coherence. In the first part the problem is formulated and the necessery definitions are introduced. In the second part there is derived the dependence of the correlation tensor in two arbitrary space points on the correlation tensors values and the values of their first and second time derivatives on the surface surrounding both the points.

1. *Introduction*

The present paper deals with the problem of the propagation of physical quantities describing the correlations between the electromagnetic field states in different points in space and at different time instants.

In the theory concerning the correlation of these quantities we use the complex formalism for describing the field. The concept of analytical signals is also introduced (Born and Wolf 1959). The analytical signals connected with the intensities of the electric and the magnetic field will be furtheron denoted by $\vec{E}(\vec{x}, t)$ and $\vec{H}(\vec{x}, t)$ respectively, the physical fields will be $\mathrm{Re}\,\vec{E} = \vec{E}^{(r)}$ and $\mathrm{Re}\,\vec{H} = \vec{H}^{(r)}$. One can verify that in vacuum, where no field sources exist, the above quantities satisfy the Maxwell eqations (Roman and Wolf 1960)

$$\mathrm{rot}\,\vec{E} + \frac{1}{c}\dot{\vec{H}} = 0 \qquad \mathrm{rot}\,H - \frac{1}{c}\dot{\vec{E}} = 0$$

$$\mathrm{div}\,\vec{E} = 0 \qquad \mathrm{div}\,\vec{H} = 0 \tag{1}$$

Roman and Wolf have introduced in the description of the correlations of a stationary field

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four tensors constructed from the components of the vectors E and H

$$\begin{aligned}
\mathcal{E}_{jk}(\vec{x}_1, x_2, \tau) &= \langle E_j(\vec{x}_1, t+\tau) E_k^*(\vec{x}_2, t)\rangle \\
\mathcal{H}_{jk}^1(\vec{x}_1, \vec{x}_2, \tau) &= \langle H_j(\vec{x}_1, t+\tau) H_k^*(\vec{x}_2, t)\rangle \\
\mathcal{A}_{jk}(\vec{x}_1, \vec{x}_2, \tau) &= \langle E_j(\vec{x}_1, t+\tau) H_k^*(\vec{x}_2, t)\rangle \\
\tilde{\mathcal{A}}_{jk}(\vec{x}_1, \vec{x}_2, \tau) &= \langle H_j(\vec{x}_1, t+\tau) E_k^*(\vec{x}_2, t)\rangle
\end{aligned} \tag{2}$$

where the expressions in brackets $\langle \quad \rangle$ are averaged over time. It is shown in the following, how the values of these tensors for any arbitrary pair of points $\vec{x}_1'$ and $\vec{x}_2'$ depend on their values and the respective time derivatives given on any surface.

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2. *Derivation of the formulas describing the propagation of the correlation tensors*

The problem of the propagation of the correlation tensors can be solved directly by using the differential equations concerning these tensors which have been derived by Roman and Wolf (1960). One can also derive formulae for the propagation of analytical signals $\vec{E}(\vec{x}, t)$ and $\vec{H}(\vec{x}, t)$, and calculate on their basis the propagation of the correlation tensors. The latter method has been accepted in the present paper. The calculations are based on the principle formula of the vector calculus which is the vector analog of Greens theorem. The derivation of this theorem can be found in the monography by Stratton 1941). In vector notation the formula is as follows

$$\int_V (\vec{Q} \cdot \text{rot rot } \vec{P} - \vec{P} \cdot \text{rot rot } \vec{Q}) dv = \int_S (\vec{P} \times \text{rot } \vec{Q} - \vec{Q} \times \text{rot } \vec{P}) \cdot \vec{n} ds, \tag{3}$$

where S is the surface enclosing the volume V, $\vec{n}$ is a unit vector perpendicular to the surface element ds, and $\vec{Q}$, $\vec{P}$ are arbitrary vector functions which are regular inside the volume V.

It is convenient to apply this formula to the Fourier transforms of analytical signals, since the corresponding equations are identical with those describing an electromagnetic field depending harmonically on time. The expansions of the analytical signals into harmonic components are

$$\vec{E}(\vec{x}, t) = \int_{-\infty}^{\infty} \vec{e}(\vec{x}, \omega) e^{-i\omega t} d\omega$$

$$\vec{H}(\vec{x}, t) = \int_{-\infty}^{\infty} \vec{h}(\vec{x}, \omega) e^{-i\omega t} d\omega. \tag{4}$$

Thus from Maxwell's equations it follows that the functions $\vec{e}$ and $\vec{h}$ satisfy the simple equations

$$\text{rot } \vec{e} = \frac{i\omega}{c} \vec{h} \qquad \text{rot } \vec{h} = -\frac{i\omega}{c} \vec{e}$$

$$\text{div } \vec{e} = 0 \qquad \text{div } \vec{h} = 0.$$

Formula (3) can be also applied to the quantities defined as follows:

$$\vec{Q} = \vec{a}\,\frac{1}{r}\,e^{i\omega\frac{r}{c}} \overset{\mathrm{df}}{=} \vec{a}\varphi, \quad \vec{P} = \vec{e} \quad (\text{or } \vec{P} = \vec{h})$$

where $\vec{a}$ is an arbitrary vector, $r = |\vec{x} - \vec{x}'|$, $\vec{x}'$ an arbitrary point contained in V and $\vec{x}$ an arbitrary point on the surface S. In the description of the vectors $\vec{e}$ and $\vec{h}$ we can make use of the formulae derived by Stratton (Stratton p. 466) which have a somewhat simpler form owing to the fact that there are no sources there

$$\vec{e}(\vec{x}', \omega) = -\frac{1}{4\pi}\int\limits_S \left[(\vec{n}\times\vec{e})\times\nabla\varphi + (\vec{n}\cdot\vec{e})\nabla\varphi + \frac{i\omega}{c}(\vec{n}\times\vec{h})\varphi\right] ds \tag{5a}$$

$$\vec{h}(\vec{x}', \omega) = -\frac{1}{4\pi}\int\limits_S \left[(\vec{n}\times\vec{h})\times\nabla\varphi + (\vec{n}\cdot\vec{h})\nabla\varphi - \frac{i\omega}{c}(\vec{n}\times\vec{e})\varphi\right] ds. \tag{5b}$$

The functions in the integral depend on $\vec{x}$ *i.e.* on points of the surface over which we integrate.

In order to obtain formulae for the propagation of the functions $\vec{E}$ and $\vec{H}$ we have to multiply (5) by $\exp|-i\omega t|$ and integrate over $d\omega$

$$\vec{E}_{(\vec{x}', t)} = -\frac{1}{4\pi}\int\limits_S ds \int\limits_{-\infty}^{\infty} d\omega \left[(\vec{n}\times\vec{e})\times\nabla r\cdot\left(\frac{i\omega}{c} - \frac{1}{r}\right)\frac{e^{-i\omega\left(t-\frac{r}{c}\right)}}{r} + \right.$$

$$\left. + (\vec{n}\cdot\vec{e})\cdot\nabla r\cdot\left(\frac{i\omega}{c} - \frac{1}{r}\right)\frac{e^{-i\omega\left(t-\frac{r}{c}\right)}}{r} + \frac{i\omega}{c}(\vec{n}\times\vec{h})\frac{e^{-i\omega\left(t-\frac{r}{c}\right)}}{r}\right]. \tag{6}$$

The integration over $d\omega$ can be here done effectively. *E.g.*

$$\int\limits_{-\infty}^{\infty} d\omega(\vec{n}\times\vec{e})\times\nabla r\cdot\frac{i\omega}{cr}\cdot e^{-i\omega\left(t-\frac{r}{c}\right)} = -\left[\vec{n}\times(-i\omega)\int\limits_{-\infty}^{\infty}\vec{e}\cdot e^{-i\omega\left(t-\frac{r}{c}\right)}d\omega\right]\times\nabla(\ln r)\cdot\frac{1}{c}$$

$$= -\left(\vec{n}\times\left.\frac{\partial\vec{E}}{\partial t}\right|_{t-\frac{r}{c}}\right)\times\nabla(\ln r)\cdot\frac{1}{c} = -\left(\vec{n}\times\left[\frac{\partial\vec{E}}{\partial t}\right]\right)\times\nabla(\ln r)\cdot\frac{1}{c}.$$

The symbol $\left[\frac{\partial\vec{E}}{\partial t}\right]$ denotes the derivative of the field intensity calculated at the retarded time $t - \frac{r}{c}$. In the following we will always use square brackets as denoting time retardation.

The expressions under the integral symbols (6) contain the factor $i\omega$, that is they are of the same type as those calculated above. Besides

$$\int\limits_{-\infty}^{\infty} d\omega(\vec{n}\times\vec{e})\times\nabla\left(\frac{1}{r}\right)\cdot\exp\left\{-i\omega\left(t-\frac{r}{c}\right)\right\} = \left(\vec{n}\times\int\limits_{-\infty}^{\infty}\vec{e}\cdot\exp\left\{-i\omega\left(t-\frac{r}{c}\right)\right\}d\omega\right)\times$$

$$\times\nabla\left(\frac{1}{r}\right) = (\vec{n}\times[\vec{E}])\times\nabla\left(\frac{1}{r}\right)$$

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Formula (6) becomes thus after integration

$$E(\vec{x}', t) = -\frac{1}{4\pi}\int_S ds\left\{(\vec{n}\times[\vec{E}])\times\nabla\left(\frac{1}{r}\right) - \frac{1}{c}\left(\vec{n}\times\left[\frac{\partial\vec{E}}{\partial t}\right]\right)\times\nabla(\ln r) + \right.$$

$$\left. + (\vec{n}\cdot[\vec{E}])\cdot\nabla\left(\frac{1}{r}\right) - \frac{1}{c}\left(\vec{n}\cdot\left[\frac{\partial\vec{E}}{\partial t}\right]\right)\cdot\nabla(\ln r) - \frac{1}{rc}\left(\vec{n}\times\left[\frac{\partial\vec{H}}{\partial t}\right]\right)\right\}. \qquad (7a)$$

Similarly, one can calculate $\vec{H}$ from (5b)

$$\vec{H}(\vec{x}', t) = -\frac{1}{4\pi}\int_S ds\left\{(\vec{n}\times[\vec{H}])\times\nabla\left(\frac{1}{r}\right) - \frac{1}{c}\left(\vec{n}\times\left[\frac{\partial\vec{H}}{\partial t}\right]\right)\times\nabla(\ln r) + \right.$$

$$\left. + (\vec{n}\cdot[\vec{H}])\cdot\nabla\left(\frac{1}{r}\right) - \frac{1}{c}\left(\vec{n}\cdot\left[\frac{\partial\vec{H}}{\partial t}\right]\right)\cdot\nabla(\ln r) + \frac{1}{rc}\left(\vec{n}\times\left[\frac{\partial\vec{E}}{\partial t}\right]\right)\right\}. \qquad (7b)$$

The space argument of the functions $\vec{E}$ and $\vec{H}$ under the integral is of course $\vec{x}$, *i.e.* coordinates of points on the integration surface S.

Now we can start with the construction of formulas describing the propagation of products of the electromagnetic field vector components. From now on we have to abandon the vector notation and shall use the tensor (subscripted variables) notation. We will of course make use of the sum convention. Let us consider for example the product $E_i(\vec{x}_1', t_1)\cdot\cdot E_j^*(\vec{x}_2', t_2)$. After carrying out the multiplication of the integrals on the right hand side of Formula (7a) we obtain products of the functions $E_k\left(\vec{x}_1', t_1-\frac{r_1}{c}\right)$ and $E_m^*\left(\vec{x}_2, t_2-\frac{r_2}{c}\right)$ as well as of their time derivatives of different type (evidently $r_1 = |\vec{x}_1-\vec{x}_1'|$ and $r_2 = |\vec{x}_2-\vec{x}_2'|$. After introducing the notation $\left(\vec{x}_1, t_1-\frac{r_1}{c}\right) \equiv (P_1)$ and $\left(x_2, t_2-\frac{r_2}{c}\right) \equiv (P_2)$, we shall deal with the following quantities:

$$E_k(P_1)E_m^*(P_2),$$

$$E_k(P_1)\cdot\frac{\partial E_m^*(P_2)}{\partial t_2} = \frac{\partial}{\partial t_2}\{E_k(P_1)\cdot E_m^*(P_2)\},$$

$$\frac{\partial E_k(P_1)}{\partial t_1}\cdot\frac{\partial E_m^*(P_2)}{\partial t_2} = \frac{\partial^2}{\partial t_1\partial t_2}(E_k(P_1)\cdot E_m^*(P_2)),$$

$$E_k(P_1)\cdot\frac{\partial H_m^*(P_2)}{\partial t_2} = \frac{\partial}{\partial t_2}(E_k(P_1)\,H_m^*(P_2)),\ldots$$

The behaviour of the products of the components of $\vec{E}$ and $\vec{H}$ will be quite similar.

While restricting the considerations to the stationary case it is common to introduce the parameter $\tau = t_1-t_2$. It is then evident that $\frac{\partial}{\partial t_1} = \frac{\partial}{\partial\tau} = -\frac{\partial}{\partial t_2}$.

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After averaging the particular products with respect to the time t_2, there appear components of different correlation tensors or their derivatives with respect to τ. (Definitions (2) of these tensors introduced by Roman and Wolf will be used here). Thus for example:

$$\left\langle \frac{\partial E_j(\vec{x}_1, t_1)}{\partial t_1} \cdot \frac{\partial E_k^*(\vec{x}_2, t_2)}{\partial t_2} \right\rangle = \left\langle \frac{\partial^2}{\partial t_1 \partial t_2} E_j(\vec{x}_1, t_1) E_k^*(x_2, t_2) \right\rangle$$

$$= -\frac{\partial^2}{\partial \tau^2} \langle E_j(\vec{x}_1, t_2+\tau) \cdot E_k^*(\vec{x}_2, t_2) \rangle = -\frac{\partial^2}{\partial \tau^2} \mathcal{E}_{jk}(\vec{x}_1, \vec{x}_2, \tau).$$

Since in the stationary case the time average does not depend on the selection of zero time, one can introduce a new time parameter $t' = t_2 - \frac{r_2}{c}$ and average over this parameter. If besides this we take into account the time retardation, we will obtain as a result:

$$\frac{\partial^2}{\partial \tau^2} \left\langle E_j \left(t' + \tau - \frac{r_1 - r_2}{c} \right) \cdot E_k^*(t') \right\rangle = \left[\frac{\partial^2}{\partial \tau^2} \mathcal{E}_{jk}(\vec{x}_1, \vec{x}_2, \tau) \right]$$

$$\equiv \frac{\partial^2}{\partial \tau^2} \mathcal{E}_{jk} \left(\vec{x}_1, \vec{x}_2, \tau - \frac{r_1 - r_2}{c} \right).$$

The square bracket symbol introduced here, will from here on denote the retardation of τ by the quantity $\frac{r_1 - r_2}{c}$.

After these remarks one can already write the formulas for the components of the particular correlation tensors. They all are quite similar, if we suitably group the terms of the sum in the integral and extract from the brackets the components of the correlation tensors. Thus the formula describing the tensor $\mathcal{A}_{is}(\vec{x}_1', \vec{x}_2', \tau)$ will look as follows ($\varepsilon_{klm} = 1$, (-1) if the numbers klm are an even (odd) permutation of the numbers 1, 2, and 3, and $\varepsilon_{klm} = 0$ in the remaining cases.):

$$\mathcal{A}_{is}(\vec{x}_1', \vec{x}_2', \tau) = \frac{1}{(4\pi)^2} \int_S \int_S d_2\vec{x}_1 \cdot d_2\vec{x}_2 \Bigg([\partial^2 \tilde{\mathcal{A}}_{mw}] \varepsilon_{ilm} \varepsilon_{stw} n_l^1 n_t^2 \cdot \frac{1}{rc} \cdot \frac{1}{rc} +$$

$$+ [\mathcal{A}_{mw}] \Bigg\{ \varepsilon_{ijk} \varepsilon_{jlm} \varepsilon_{stu} \varepsilon_{tvw} n_l^1 n_v^2 \left(\frac{1}{r_1} \right)_{,k} \left(\frac{1}{r_2} \right)_{,u} + n_m^1 n_w^2 \left(\frac{1}{r_1} \right)_{,i} \cdot \left(\frac{1}{r_2} \right)_{,s} +$$

$$+ \varepsilon_{ijk} \varepsilon_{jlm} n_l^1 n_w^2 \left(\frac{1}{r_1} \right)_{,k} \left(\frac{1}{r_2} \right)_{,s} + \varepsilon_{stu} \varepsilon_{tvw} n_m^1 n_v^2 \left(\frac{1}{r_1} \right)_{,i} \left(\frac{1}{r_2} \right)_{,u} \Bigg\} +$$

$$+ [\partial \mathcal{A}_{mw}] \Bigg\{ \varepsilon_{ijk} \varepsilon_{jlm} \varepsilon_{stu} \varepsilon_{tvw} n_j^1 n_v^2 \left(\frac{1}{r_1} \right)_{,k} \left(\frac{1}{c} \ln r_2 \right)_{,u} + n_m^1 n_w^2 \left(\frac{1}{r_1} \right)_{,i} \left(\frac{1}{c} \ln r_2 \right)_{,s} +$$

$$+ \varepsilon_{ijk} \varepsilon_{jlm} n_l^1 n_w^2 \left(\frac{1}{r_1} \right)_{,k} \left(\frac{1}{c} \ln r_2 \right)_{,s} + \varepsilon_{stu} \varepsilon_{tvw} n_m^1 n_v^2 \left(\frac{1}{r_1} \right)_{,i} \left(\frac{1}{c} \ln r_2 \right)_{,u} \Bigg\} +$$

$$- [\partial \mathcal{A}_{mw}] \Bigg\{ \varepsilon_{ijk} \varepsilon_{jlm} \varepsilon_{stu} \varepsilon_{tvw} n_l^1 n_v^2 \left(\frac{1}{c} \ln r_1 \right)_{,k} \left(\frac{1}{r_2} \right)_{,u} + n_m^1 n_w^2 \left(\frac{1}{c} \ln r_1 \right)_{,i} \left(\frac{1}{r_2} \right)_{,s} +$$

$$
\begin{aligned}
&+\varepsilon_{ijk}\varepsilon_{jlm}\, n_l^1 n_w^2 \left(\frac{1}{c}\ln r_1\right)_{,k}\left(\frac{1}{r_2}\right)_{,s} + \varepsilon_{stu}\varepsilon_{tvw} n_m^1 n_v^2 \left(\frac{1}{c}\ln r_1\right)_{,i}\left(\frac{1}{r_2}\right)_{,s}\Bigg\} - \\
&-[\partial^2 \mathscr{A}_{mw}]\Bigg\{\varepsilon_{ijk}\varepsilon_{jlm}\varepsilon_{stu}\varepsilon_{tvw} n_l^1 n_v^2 \left(\frac{1}{c}\ln r_1\right)_{,k}\left(\frac{1}{c}\ln r_2\right)_{,u} + n_m^1 n_w^2 \left(\frac{1}{c}\ln r_1\right)_{,i}\left(\frac{1}{c}\ln r_2\right)_{,s} + \\
&\varepsilon_{ijk}\varepsilon_{jlm} n_l^1 n_w^2 \left(\frac{1}{c}\ln r_1\right)_{,k}\left(\frac{1}{c}\ln r_2\right)_{,s} + \varepsilon_{stu}\varepsilon_{tvw} n_m^1 n_v^2 \left(\frac{1}{c}\ln r_1\right)_{,i}\left(\frac{1}{c}\ln r_2\right)_{,u}\Bigg\} + \\
&-[\partial \mathcal{E}_{mw}]\Bigg\{\varepsilon_{ijk}\varepsilon_{stu}\varepsilon_{stw}\, n_l^1 n_t^2 \left(\frac{1}{r_1}\right)_{,k}\frac{1}{r_2 c} + \varepsilon_{stw}\, n_m^1 n_t^2 \left(\frac{1}{r_1}\right)_{,i}\frac{1}{r_2 c}\Bigg\} + \\
&-[\partial \mathcal{H}_{mw}]\Bigg\{\varepsilon_{ilm}\varepsilon_{stu}\varepsilon_{tvw} n_l^1 n_v^2 \frac{1}{r_1 c}\left(\frac{1}{r_2}\right)_{,u} + \varepsilon_{ilm}\, n_l^1 n_w^2 \frac{1}{r_1 c}\left(\frac{1}{r_2}\right)_{,s}\Bigg\} + \\
&+[\partial^2 \mathcal{E}_{mw}]\Bigg\{\varepsilon_{ijk}\varepsilon_{jlm}\varepsilon_{stw}\, n_l^1 n_t^2 \left(\frac{1}{c}\ln r_1\right)_{,k}\frac{1}{r_2 c} + \varepsilon_{stw}\, n_m^1 n_t^2 \left(\frac{1}{c}\ln r_1\right)_{,i}\frac{1}{r_2 c}\Bigg\} + \\
&-[\partial^2 \mathcal{H}_{mw}]\Bigg\{\varepsilon_{ilm}\varepsilon_{stu}\varepsilon_{tvw} n_l^1 n_v^2 \frac{1}{r_1 c}\left(\frac{1}{c}\ln r_2\right)_{,u} + \varepsilon_{ilm}\, n_l^1 n_w^2 \frac{1}{r_1 c}\left(\frac{1}{c}\ln r_2\right)_{,t}\Bigg\}\Bigg). \qquad (8)
\end{aligned}
$$

In order to make shorter the formulas, the symbols ∂ and ∂^2 have been introduced here, which correspond to the first and the second derivative with respect to τ. Besides n_l^1 and n_t^2 denote the l-th component of the unit vector perpendicular to $d_2\vec{x}_1$ and the t-th component of the unit vector perpendicular to $d_2\vec{x}_2$ respectively.

An analogous formula for $(\tilde{\mathscr{A}}_{is})$ can be obtained from the above one, if we replace the expressions $[\mathscr{A}_{mw}]$, $[\partial \mathscr{A}_{mw}]$ and $[\partial^2 \mathscr{A}_{mw}]$ by $[\tilde{\mathscr{A}}_{mw}]$, $[\partial \tilde{\mathscr{A}}_{mw}]$ and $[\partial^2 \tilde{\mathscr{A}}_{mw}]$ and then $[\partial \mathcal{E}'_{mw}]$, $[\partial^2 \mathcal{E}_{mw}]$, $[\partial \mathcal{H}_{mw}]$ and $[\partial^2 \mathcal{H}_{mw}]$ by $[-\partial \mathcal{H}_{mw}]$, $[-\partial^2 \mathcal{H}_{mw}]$, $[-\partial \mathcal{E}_{mw}]$ and $[-\partial^2 \mathcal{E}_{mw}]$, as well as $[\partial^2 \mathscr{A}_{mw}]$ by $[\partial^2 \mathscr{A}_{mw}]$. The formula for the tensor $\mathcal{E}_{is}$ can be obtained from (8), after substituting on the right hand side $[\mathcal{E}_{mw}]$, $[\partial \mathcal{E}_{mw}]$, and $[\partial^2 \mathcal{E}_{mw}]$ instead of $[\mathscr{A}_{mw})]$, $[\partial \mathscr{A}_{mw}]$ and $[\partial^2 \mathscr{A}_{mw}]$. Besides, we will have $[-\partial \mathscr{A}_{mw}]$ and $[-\partial^2 \mathscr{A}_{mw}]$ instead of $[\partial \mathcal{E}_{mw}]$ and $[\partial^2 \mathcal{E}_{mw}]$, then $[\partial \tilde{\mathscr{A}}_{mw}]$ and $[\partial^2 \mathscr{A}_{mw}]$ instead of $[\partial \mathcal{H}_{mw}]$ and $[\partial^2 \mathcal{H}_{mw}]$, and $[-\partial^2 \mathcal{H}_{mw}]$ instead of $[\partial^2 \mathscr{A}_{mw}]$. The last formula for $[\mathcal{H}_{is}]$ can be obtained from (8) after substituting everywhere $[\mathcal{H}_{mw}]$, $[\partial \mathcal{H}_{mw}]$ and $[\partial^2 \mathcal{H}_{mw}]$ instead of $[\mathscr{A}_{mw}]$, $[\partial \mathscr{A}_{mw}]$ and $[\partial^2 \mathscr{A}_{mw}]$. Next we have to write $[-\partial \mathscr{A}_{mw}]$ and $[-\partial^2 \mathscr{A}_{mw}]$ instead of $[\partial \mathcal{H}_{mw}]$ and $[\partial^2 \mathcal{H}_{mw}]$. Besides, $[\partial \mathcal{E}_{mw}]$ and $[\partial^2 \mathcal{E}_{mw}]$ have to be replaced by $[\partial \tilde{\mathscr{A}}_{mw}]$ and $[\partial^2 \tilde{\mathscr{A}}_{mw}]$, and $[\partial^2 \tilde{\mathscr{A}}_{mw}]$ by $[-\partial^2 \mathcal{E}_{mw}]$.

The formula thus obtained is a certain counterpart to the Huyghens principle. It describes the way of propagation of the quantities characterizing the wave, the main difference being in the fact that we have here a two points phenomenon in the sense of space time. This fact leads to a considerable complexity of the formula, compared with those describing the Huyghens principle.

If we want to find by means of (8) the value of any of the components of the correlation tensors in two points of the region V, we have to know not only the values of all components of this tensor together with their respective first and second τ — derivatives given on the surface S surrounding V, but also the values of the first and second τ — derivatives of all

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remaining tensors. A similar situation exists in the theory of the propagation of the values of the electromagnetic field strengths, where if we want to describe *e.g.* the electric field strength, in a point, we have to put the values of all components of this field on the surface surrounding this point, their time derivatives, but also the time derivatives of the magnetic field strength. Not till all this information is known, can the values of the electric field be determined so that both the wave and the Maxwells equations would be satisfied.

Formula (8) can be derived more directly from the equations given by Roman and Wolf (1960) which describe the correlatin tensors and which are analogous to Maxwell's equations. This leads to a formula identical with that obtained above. The reason for choosing the method presented in this paper was its somewhat greater conciseness.

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