ON THE ANGULAR CORRELATIONS IN RADIATIVE MUON CAPTURE

By Z. Oziewicz and A. Pikulski

Institute of Theoretical Physics, University of Wrocław*

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The angular correlation in the reaction

$$\mu^- + (A, Z) \rightarrow (A, Z-1)^* + \nu_\mu \rightarrow (A, Z-1) + \gamma + \nu_\mu$$

between the polarization pseudovector of the muon, the direction of the neutrino momentum, the direction of the gamma quantum momentum and its circular polarization are calculated. The exact formulas are given for the allowed transitions and the first and second forbidden transitions. It is shown that for the unique transitions some correlation constants depend very strongly on the induced pseudoscalar term and are almost independent of the nuclear structure. The results of numerical calculations are presented for some unique transitions.

1. *Introduction*

One of the main questions in the theory of nuclear muon capture process is to determine for this reaction the weak interaction constants. The experimental data admit a very range of values [1] for some of them. The total capture rate is rather insensitive to the relations between the constants because the character of elementary processes is disturbed by the nuclear effects [2]. However, we can get more information with the aid of angular correlation effects in partial transitions. The transitions depending weakly on nuclear structure are favoured. The theory of partial transitions was given by Morita and Fujii [3] where the classification of the forbiddenness of muon capture reaction is presented. Morita and Greenberg [4] obtained the formula for the angular distribution of recoil nuclei for unique $0 \xrightarrow{\mu^-} \mathrm{I}$ transitions. Korenman and Eramzyan [5] considered also the nonunique $0 \xrightarrow{\mu^-} \mathrm{I}$ transitions.

Popov [6] was the first who proposed to study the gamma-neutrino correlation in radiative muon capture in order to determine the induced pseudoscalar constant.

Lately, Bukhvostov and Popov [7] obtained a general expression for the angular correlations considered below for any partial transition by taking into account mesoatom superfine structure levels. But this general expression is not convenient for comparison with experiment.

* Address: Uniwersytet Wrocławski, Instytut Fizyki Teoretycznej, Wrocław, Polska.

Those correlations have not yet been experimentally examined, but it would be interesting to prepare the theroretical conclusions, bearing in mind possible experiments in the future. In this paper we give a modified form of Popov's formula (section 2). We get the exact formulas for the allowed and the first and second forbidden transitions without taking into consideration the circular polarization of the gamma quantum in section 3. In other sections we study the dependence of some interesting correlations (the gamma-neutrino correlations, correlations with the circular polarization of the gamma quantum, etc.) on the induced pseudoscalar term.

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2. *General expression*

We consider the following complex process

$$\mu^- + (A, Z) \to (A, Z-1)^* + \nu_\mu \to (A, Z-1) + \gamma + \nu_\mu$$

The changes of nuclear spins are the following

$$j_0 \xrightarrow{\mu^-} j_1 \xrightarrow{\gamma} j_2$$

where j_0, j_1, j_2 denote the spins of the initial, excited and final nuclei, respectively. We consider angular correlations between the following vectors: the polarization pseudovector of the muon on the K-orbit of the mesoatom

$$\boldsymbol{P} = P\boldsymbol{\sigma}$$

($\boldsymbol{\sigma}$ is a unit vector), the unit vector in the direction of neutrino momentum q, the unit vector in the direction of the momentum of the gamma quantum k and its circular polarization. The complicated Popov's formula for angular correlations [6] is transformed to the following simpler and more convenient form

$$W \propto \sum i^{I-I'}(-1)^{J'-\frac{1}{2}} C^{f0}_{J\frac{1}{2}J'-\frac{1}{2}} X\left((JJ'fII'\,S\,\frac{1}{2}\,\frac{1}{2}\,r\right) \times \Lambda^{II'}_{S,\eta} A_r B^{IJ} B^{I'J'*} S_{rfs}(\boldsymbol{\sigma q k}) \tag{1}$$

where we sum over all quantum numbers. We want to stress that J (and J') mean the total angular momentum of the neutrino. Here I and I' denote the total angular momentum of the neutrino-muon system.

Of course, we have

$$|j_1 - j_0| \leqslant I \leqslant |j_1 + j_0| \tag{2}$$

The quantities $\Lambda^{II'}_{S,\eta}$ describe the multipolarity L and the circular polarization $\eta = \pm 1$ of the gamma quantum. For the pure electromagnetic transitions of character 2^L we have

$$\Lambda^{II'}_{S,\eta} = (2j_1+1)[(2S+1)(2L+1)]^{\frac{1}{2}} C^{L\eta}_{L\eta S0} W(j_0 j_1 I S I' j_1) W(j_2\, L j_1 S j_1 L). \tag{3}$$

$C^{a\alpha}_{b\beta c\gamma}$, $W(abcdef)$, $X(abcdefghi)$ are Clebsch-Gordan, Racah and Fano coefficients, respectively. $\Lambda^{II'}_{S,\eta}$ is connected with the F-coefficients defined by the formula (96) in [8]:

$$\Lambda^{II'}_{S,-1} = (2j_1+1)^{\frac{1}{2}} W(j_0 j_1\, I S I' j_1)\, F_S(LLj_2 j_1). \tag{4}$$

Our coefficient satisfies

$$\Lambda^{II'}_{S,1} = (-1)^S \Lambda^{II'}_{S,-1} \tag{5}$$

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For the case of mixed ML and $E\ (L+1)$ radiations we have to make a substitution using the formula (99) from the above mentioned work.

We also use the following notation

$$A_S^{II'} = \frac{1}{2} \sum_{\eta} A_{S,\eta}^{II'}. \tag{6}$$

Some interesting values of the $A_{S,\eta}^{II'}$ coefficients for the unique transitions are given in Table 1.

The angle-dependent term $S_{rfs}\ (\boldsymbol{\sigma q k})$ in formula (1) is given in Appendix. The quantity A_r has the form

$$A_r = \delta_{r0} + P\delta_{r1}. \tag{7}$$

The dynamics of weak interaction in the muon capture process is contained in the quantity B^{IJ}, which can be presented in the form

$$B^{IJ} = X_I \delta_{J,I+\frac{1}{2}} - Y_I \delta_{J,I-\frac{1}{2}} \tag{8}$$

TABLE I

Some values of $A_{S,1}^{II}$ for "pure" gamma transitions

Spin sequence I	$A_{S,1}^{II}$				Examples of unique muon capture		
	$S=1$	$S=2$	$S=4$	$S=6$	Capturing nucleus	E_γ [MeV]	Multipolarity
0 1 0 1	$\sqrt{1/2}$	$1/\sqrt{6}$					
1 2 0 1	$\frac{1}{2}\sqrt{1/2}$	$-\frac{1}{12}\sqrt{6}$			N^{14}	7.03[1]	$E2$
3 2 0 1	$-\frac{1}{3}\sqrt{1/2}$	$-\frac{1}{42}\sqrt{6}$			B^{10}	3.37	$E2$
0 2 0 2	$\frac{1}{\sqrt{10}}$	$-1/\sqrt{14}$	$-\frac{2}{35}\sqrt{70}$				
0 2 1 2	$\frac{3}{2\sqrt{10}}$	$\frac{1}{20}\sqrt{14}$	0		C^{12}	1.67[2]	$E1$
3 1 0 2	$-\frac{3}{5\sqrt{10}}$	$\frac{1}{70}\sqrt{14}$	0		B^{10}	5.96	$E1$
1 3 0 2	$\frac{2}{3\sqrt{10}}$	$-\frac{3}{35}\sqrt{14}$	$3/\sqrt{70}$		N^{14}		
0 3 0 3	$\frac{1}{2\sqrt{7}}$	$-\frac{1}{14}\sqrt{21}$	$1/\sqrt{154}$	$\frac{5}{154}\sqrt{231}$			

[1] C. P. Swann, *Phys. Rev.*, **148**, 1119 (1966).

[2] A. Gallmann, F. Hobou, P. Fintz, *Phys. Rev.*, **138**, B560 (1965).

where we introduce by definition

$$x_{\mathrm{I}} = \left(\frac{I+1}{2I+3}\right)^{\frac{1}{2}} (V_{\mathrm{I}}-A_{\mathrm{I}})+i\,\frac{I+1}{(2I+1^{\frac{1}{2}})}\,(M_{\mathrm{I}}-P_{\mathrm{I}}),$$

$$y_I = \left(\frac{I+1}{2I+3}\right)^{\frac{1}{2}} \left(\frac{I}{I+1}\,V_{\mathrm{I}}+A_{\mathrm{I}}\right) -i\,\frac{I+1}{(2I+1)^{\frac{1}{2}}}\left(M_{\mathrm{I}}+\frac{I}{I+1}\,P_{\mathrm{I}}\right). \tag{9}$$

We used above four combinations of nuclear matrix elements and weak formfactors characteristic for this process

$$A_{\mathrm{I}} = -\,[I(2I+3)]^{\frac{1}{2}}\Big\{C_A[1\,II]-\sqrt{3}\,C_V\,\frac{q}{2M}\,\frac{[I(I+1)]^{\frac{1}{2}}}{2I+1}\,([0\,II+]-[0\,II-])-$$

$$-\,C_V(1+\mu_p-\mu_n)\,\frac{q}{2M}\,\frac{I+1}{2I+1}\left([1\,II+]+\frac{I}{I+1}\,[1II-]\right)+$$

$$+\,\frac{C_V}{M}\,\frac{1}{(2I+1)^{\frac{1}{2}}}\,(\sqrt{I+1}\,[1I-\sqrt{1}Ip]-\sqrt{I}[1I+1Ip])\Big\},$$

$$V_{\mathrm{I}} = [3(I+1)(2I+3)]^{\frac{1}{2}}\Big\{C_V[0\,II]+C_V\,\frac{q}{2M}\,\frac{I+1}{2I+1}\left(\frac{I}{I+1}\,[0\,II+]+[0\,II-]\right.+$$

$$+\,\frac{1}{\sqrt{3}}\,C_V(1+\mu_p-\mu_n)\,\frac{q}{2M}\,\frac{[I(I+1)]^{\frac{1}{2}}}{2I+1}\,([1II+]-[1II-])-$$

$$-\,\frac{1}{\sqrt{3}}\,\frac{C_V}{M}\,\frac{1}{(2I+1)^{\frac{1}{2}}}\,(\sqrt{I}\,[1I-1Ip]+\sqrt{I+1}\,[1I+1Ip])\Big\},$$

$$M_{\mathrm{I}} = \left(\frac{I}{I+1}\right)^{\frac{1}{2}}\Big\{C_A\,(\sqrt{I+1}\,[1I-1I]-\sqrt{I}\,[1I+1I])+$$

$$+\,C_V(1+\mu_p-\mu_n)\,\frac{q}{2M}\,(\sqrt{I}\,[1I+1I+]-\sqrt{I+1}[1I-1I-]-$$

$$-\,\frac{C_V}{M}\,(2I+1)^{\frac{1}{2}}\,[1IIp]\Big\},$$

$$P_{\mathrm{I}} = C_A\,(\sqrt{I}\,[1I-1I]+\sqrt{I+1}\,[1I+1I])+(C_P-C_A)\,\frac{q}{2M}\,(\sqrt{I}\,[1I-1I-]+$$

$$+\,\sqrt{I+1}\,[1I+1I+])+[3(2I+1)]^{\frac{1}{2}}\,\frac{C_A}{M}\,[0\,IIP], \tag{10}$$

where $[kwI]$ etc. denote nuclear matrix elements defined by Morita and Fujii [3], C_i denote the weak interaction constants, μ_p and μ_n are anomalous magnetic moments of proton and neutron respectively, q is the energy of the neurtino, M the nucleon mass.

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The quantities A_I and V_I are different from zero in the case 1 when the change of parity of the nuclear levels j_0 and j_1 is $(-1)^I$ only. For M_I and P_I the allowed parity change is $(-1)^{I+1}$. According to the Morita and Fujii [3] classification the spin $I(2)$ and parity change in muon capture can assume the values $(0^+, 1^+)$, $(0^-, 1^-, 2^-)$, $(N(-)^N, N+1\,(-)^N)$ for the allowed transition ($N=0$) and the first and N-th forbidden transitions, respectively. Then for the N-th forbidden transitions the general expression (1) is simpler because we can write x_I and y_I (9) in the form

$$x_I = \sqrt{N+1}\,(V_N - A_N)\,\delta_{I,N} + i(N+2)(M_{N+1} - P_{N+1})\,\delta_{I,N+1} - i\sqrt{5}\,P_0\delta_{N1}\delta_{I0}$$

$$y_I = \sqrt{N+1}\left(\frac{N}{N+1}\,V_N + A_N\right)\delta_{IN} - i\,[(N+2)\,M_{N+1} + (N+1)\,P_{N+1}]\,\delta_{I,N+1}\,. \quad (11)$$

The unique transitions are defined by $I = I' = N+1$. Then the formulas become simpler.

We want to stress that formula (1) describes a real situation only in those cases when the superfine structure of the mesoatom does not play any role [7] (*i. e.* $j_0 = 0$ or the statistical occupation of the superfine levels for the unpolarized muons).

3. *Exact formulas for partial transitions*

The form of correlations easily compared with the experiment for the N-th forbidden transitions is as follows

$$W^N = 1 + \left(\alpha^N - \frac{1}{3}\,C_1^N\right)\boldsymbol{P}\cdot\boldsymbol{q} + \sum_{l=1}^{2N+2}(a_l^N + b_l^N\boldsymbol{P}\cdot\boldsymbol{q})\,P_l(\boldsymbol{k}\cdot\boldsymbol{q}) +$$

$$+\sum_{l=0}^{2N+1} c_l^N\boldsymbol{P}\cdot\boldsymbol{k} + d_l^N\boldsymbol{P}\Lambda\boldsymbol{q}\cdot\boldsymbol{k})P_l(\boldsymbol{k}\cdot\boldsymbol{q}), \quad (12)$$

where α^N, a_l^N, b_l^N, c_l^N and d_l^N are the correlation constants depending on the weak interaction form factors, the structure of nuclei and kinematic effects. The quantities $P_l(\boldsymbol{k}\cdot\boldsymbol{q})$ are the Legendre polynomials. When we do not measure the correlations with the circular polarization of the gamma quantum, k appears in even powers only. Then, because of (5) and (6), we have

$$a_{2i+1}^N = b_{2i+1}^N = c_{2i}^N = d_{2i}^N = 0, \;\; (i = 0, 1, \dots N).$$

The additional restrictions give us, as usually, the gamma quantum multipolarity and the spin of the intermediate nucleus j_1

$$a_l^N = b_l^N = c_{l-1}^N = d_{l-1}^N = 0 \text{ for all } l,$$

$$l > \begin{cases} \min\{2j_1, 2L\} & \text{in the case of pure radiations,} \\ \min\{2j_1, 2L+2\} & \text{in the case of mixed radiations.} \end{cases}$$

The correlations $\boldsymbol{P}\Lambda\boldsymbol{q}\cdot\boldsymbol{k}P_l(\boldsymbol{k}\cdot\boldsymbol{q})$ are not time-reversal invariant. They are considered in paper [9].

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The total probability of the N-th forbidden muon capture is proportional to the quantity

$$\Lambda_N = \frac{N+1}{N} A_N^2 + V_N^2 + (N+1)\, P_{N+1}^2 + (N+2) M_{N+1}^2 + 10\, P_0^2 \delta_{N,1}$$

with $\left(\frac{A_N^2}{N}\right)_{\text{for } N=0} \equiv 0$ (See formula (10)).

We give below the correlation constants for the allowed transitions and first and second forbidden transitions calculated from expression (1), where for simplicity we do not take into account the circular polarization of the gamma quantum.

I. Allowed transitions $N = 0$, $\Delta j = |j_0 - j_1| = 0,\ 1$ (no),

$$\alpha^0 \Lambda_0 = -V_0^2 + 2M_1^2 - P_1^2,$$

$$a_2^0 \Lambda_0 = \sqrt{6}\Lambda_2^{11}(M_1^2 - P_1^2),$$

$$b_2^0 \Lambda_0 = \sqrt{6}\, \Lambda_2^{11}(M_1 - P_1)^2,$$

$$(c_1^0 + i d_1^0)\, \Lambda_0 = 3\sqrt{6}\Lambda_2^{11} M_1 P_1^*. \tag{13}$$

Here and below for convenience we do not take into consideration the imaginary parts of the correlation constants invariant under time reversal, with the exception of the correlation constants C_l^N. As a particular case, we can obtain Popov's formulas [6] for a small induced pseudoscalar.

II. First forbidden transitions $N = 1$, $\Delta j = 0, 1, 2$ (yes),

$$\alpha^1 \Lambda_1 = -10P_0^2 + 2A_1^2 - V_1^2 + 3M_2^2 - 2P_2^2,$$

$$a_2^1 \Lambda_1 = \sqrt{6}\Lambda_2^{11}(A_1^2 - V_1^2) - 20\Lambda_2^{02} P_0 P_2 - 6\sqrt{5}\, \Lambda_2^{12} A_1 M_2 - \frac{5}{14}\sqrt{14}\, \Lambda_2^{22}(3M_2^2 + 4P_2^2),$$

$$a_4^1 \Lambda_1 = \frac{6}{7}\sqrt{70}\, \Lambda_4^{22}(P_2^2 - M_2^2),$$

$$b_2^1 \Lambda_1 = 20\, \Lambda_2^{02}\, P_0\,(P_2 - M_2) + \sqrt{6}\, \Lambda_2^{11}(A_1 - V_1)^2 +$$

$$+ 2\sqrt{5}\Lambda_2^{12}(-3A_1 M_2 + V_1 M_2 + 2A_1 P_2) +$$

$$+ \frac{5}{14}\sqrt{14}\, \Lambda_2^{22}(-3M_2^2 - 4M_2 P_2 + 4P_2^2) - \frac{5}{7}\, C_3^1 \Lambda_1,$$

$$b_4^1 \Lambda_1 = -\frac{6}{7}\sqrt{70}\, \Lambda_4^{22}\,(M_2 - P_2)^2,$$

$$(c_1^1 + i d_1^1)\, \Lambda_1 = 30\, \Lambda_2^{02} P_0^* M_2 + 3\sqrt{6}\, \Lambda_2^{11} A_1^* V_1 -$$

$$- 3\sqrt{5}\Lambda_2^{11}(V_1^* M_2 + 2A_1 P_2^*) + 15\sqrt{\frac{2}{7}}\, \Lambda_2^{22} P_2^* M_2 + \frac{3}{7}\,(c_3^1 + i d_3^1)\, \Lambda_1,$$

$$(c_3^1 + i d_3^1)\, \Lambda_1 = -3\sqrt{70}\, \Lambda_4^{22} M_2 P_2^*. \tag{14}$$

III. Second forbidden transitions $N=2$, $\Delta j=2$, 3 (no)

$$\alpha^2\Lambda_2=\frac{3}{2}A_2^2-V_2^2+4M_3^2-3P_3^2,$$

$$a^2\Lambda_2=-\frac{5}{\sqrt{14}}\Lambda_2^{22}\left(\frac{3}{2}A_2^2+2V_2^2\right)-2\sqrt{30}\,\Lambda_2^{23}A_2M_3-2\sqrt{21}\,\Lambda_2^{33}(M_3^2+P_3^2),$$

$$a_4^2\Lambda_2=-\frac{3}{7}\sqrt{70}\,\Lambda_4^{22}(A_2^2-V_2^2)+10\sqrt{3}\,\Lambda_4^{23}A_2M_3+\frac{1}{11}\sqrt{154}\,\Lambda_4^{33}(2M_3^2+9P_3^2),$$

$$a_6^2\Lambda_1=\frac{10}{11}\sqrt{231}\,\Lambda_6^{33}(M_3^2-P_3^2),$$

$$b_2^2\Lambda_2=\frac{5}{14}\Lambda_2^{22}\left(-\frac{3}{2}A_2^2-2A_2V_2+2V_2^2\right)+\frac{2}{3}\sqrt{30}\,\Lambda_2^{23}(-3A_2M_3+3A_2P_3+2V_2M_3)+$$

$$+\frac{2}{3}\sqrt{21}\,\Lambda_2^{33}(-3M_3^2-2M_3P_3+3P_3^2)-\frac{5}{7}C_3^2\Lambda_2,$$

$$b_4^2\Lambda_2=-\frac{3}{7}\sqrt{70}\,\Lambda_4^{22}(A_2-V_2)^2-2\sqrt{3}\,\Lambda_4^{23}(-5A_2\cdot M_3+3A_2P_3+$$

$$+2V_2M_3)+\frac{1}{11}\sqrt{154}\,\Lambda_4^{33}(2M_3^2-12M_3P_3-9P_3^2),$$

$$b_6^2\Lambda_2=\frac{10}{11}\sqrt{231}\,\Lambda_6^{33}(M_3-P_3)^2,$$

$$(c_1^2+id_1^2)\,\Lambda_2=\frac{15}{7}\sqrt{\frac{7}{2}}\,\Lambda_2^{22}A_2V_2^*-\sqrt{30}\,\Lambda_2^{23}(2V_2^*M_3+3A_2P_3^*)+$$

$$+2\sqrt{21}\,\Lambda_2^{33}M_3P_3+\frac{3}{7}(c_3^2+id_3^2)\,\Lambda_2,$$

$$(c_3^2+id_3^2)\,\Lambda_2=-3\sqrt{\frac{35}{2}}\,\Lambda_4^{22}A_2V_2^*+\frac{7}{2}\sqrt{3}\,\Lambda_4^{23}(2V_2^*M_3+3A_2P_3^*)-$$

$$-\frac{21}{11}\sqrt{154}\,\Lambda_4^{33}M_3P_3^*+\frac{7}{11}(c_5^1+id_5^2)\,\Lambda_2,$$

$$(c_5^2+id_5^2)\,\Lambda_2=\frac{10}{3}\sqrt{231}\,\Lambda_6^{33}M_3P_3^*. \tag{15}$$

4. *Capture of unpolarized muons*

Because of muon depolarization in mesoatoms caused by spin-orbital interaction the magnitude of the vector $\boldsymbol{P}$ cannot exceed approximately one-sixth of the initial polarization [10]. The additional interaction with the nuclear spin in the case of muon capture by spin targets diminishes the magnitude of the polarization [11]. Then the importance of correlations independent of $\boldsymbol{P}$ grows. When muons are unpolarized we are dealing with gamma-neutrino correlations only. In the case of nonunique

$$0 \xrightarrow{\mu^-} j_1 \xrightarrow{\gamma} j_2$$

ransitions the correlation constants are the following

$$a_l^N \Lambda_N = (-1)^N \frac{1}{2N^2} \Lambda_{l,\eta}^{NN} \times$$

$$\times \{(2N+1)\, C_{N0N0}^{l0} \,[2N^2 V_N^2 + (2N(N+1) - l(l+1)\, A_N^2] -$$

$$- [l(l+1)(2N+1-l)\,(2N+l+2)]^{1/2}\,(2N+1)\, C_{N0N+10}^{l0}\, A_N^2\},$$

except for $O^{\pm} \xrightarrow{\mu^-} O^{\mp} \xrightarrow{\gamma} j_2$ transitions, for which

$$a_l^1 = 0.$$

For unique transitions we obtain correspondingly

$$a_l^N \Lambda^N = \frac{(-1)^{N+1}}{2(N+1)} \Lambda_{l,\eta}^{N+1,N+1} \times$$

$$\times \{(2N+3)\, C_{N+10\,N+10}^{l0}\,(2(N+1)^2 P_{N+1}^2 + [2(N+1)(N+2) - l(l+1)]\, M_{N+1}^2) -$$

$$- [l(l+1)(2N+3-l)(2N+4+l)]^{\frac{1}{2}}\,(2N+3)\, C_{N+10\,N+20}^{l0}\, M_{N+1}^2\}.$$

Now we restrict ourselves to the study of unique transitions

$$A_N = V_N = P_0 = 0$$

In this case almost all matrix elements are of the Gamov-Teller type. One of them [1 $NN+1$] is considerably greater than others [3]. Omitting in M_{N+1} and P_{N+1} the additional small terms we obtain the effective constants for the muon capture process.

$$P_{N+1} \propto P = C_A + \frac{q}{2M}(C_A - C_P)$$

$$M_{N+1} \propto M = C_A - \frac{q}{2M} C_V(1+\mu_p-\mu_n) \tag{16}$$

We want to stress that all a_{2i}^N (like α^N and Λ_N) give twofold values of C_P/C_A and for a unique interpretation of the experimental results it is necessary to postulate $C_P \lesssim 21\, C_A$. These quantities have one extremal point at $C_P \approx 21\, C_A$ and are symmetrical with regard to this point. In the N-th forbidden transitions only the correlation constant a_{2N+2}^N depends strongly on C_P/C_A. This strong dependence is caused by the partial compensation of the main Gamov-Teller term because

$$a_{2N+2}^N \Lambda_N \propto P^2 - M^2. \tag{17}$$

This correlation constant is determined by the $(N+1)$-th wave of the neutrino because the N-th wave vanishes. It is easy to see that the maximal value of all correlation constants (except for those not invariant under time reversal) does not dependent on the nuclear structure. The dependence $a_2^0\left(\frac{C_P}{C_A}\right)$ for $0^{\pm} \xrightarrow{\mu^-} 1^{\pm} \longrightarrow 0$ transitions is shown in Fig. 1. One can see that for the transitions $0 \rightarrow N+1 \rightarrow 0$ the correlation constants are always the largest.

We show the influence of the nuclear structure in Fig. 1. This influence is similar for other correlation constants. We assumed that the ratios between small main matrix ele-

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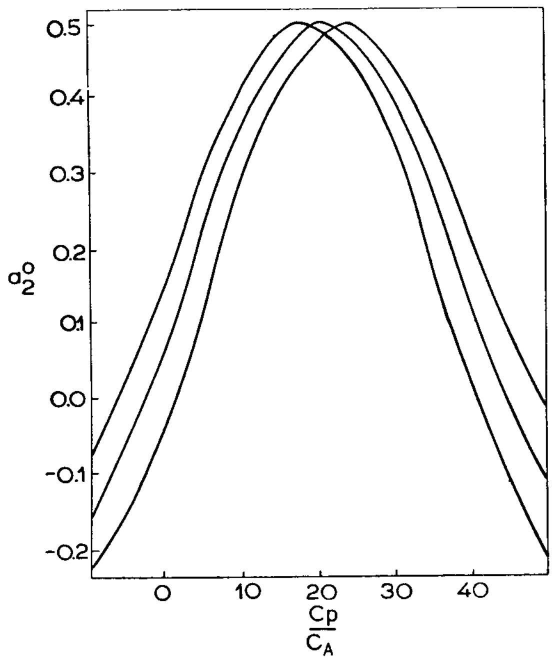

Fig. 1. The correlation constant a_2^0 for the $0^{\pm}\rightarrow 1^{\pm}\rightarrow 0$ transition $\left(\frac{C_V}{C_A} = -0.8,\ \mu_p-\mu_n = 3.7,\ \frac{q}{M} = 0.1\right)$. The influence of nuclear structure is marked

ments change in the range $(-0.04 \div +0.04)$ independently. The medium curve is drawn for the cases when the additional matrix elements are not taken into account. It refers also to other graphs. Two other curves express maximal deviation from it.

Instead of the experiment determining the angular distribution, the angular asymmetry measurement is often used. We define the asymmetry coefficient by the ratio of recoil nuclei in the directions perpendicular and parallel to the direction of the gamma quantum momentum. For $0^{\pm}\rightarrow 2^{\mp}\rightarrow 1$ transitions (for the exemplary muon capture by C^{12}) we have (without taking into account the gamma quantum polarization)

$$A = \frac{1-\frac{1}{2}\,a_2^1}{1+a_2^1}$$

This is shown in Fig. 2.

The correlation $\propto P_{2N+2}(\boldsymbol{k}\cdot\boldsymbol{q})$ can be used in order to determine the value of C_P more exactly.

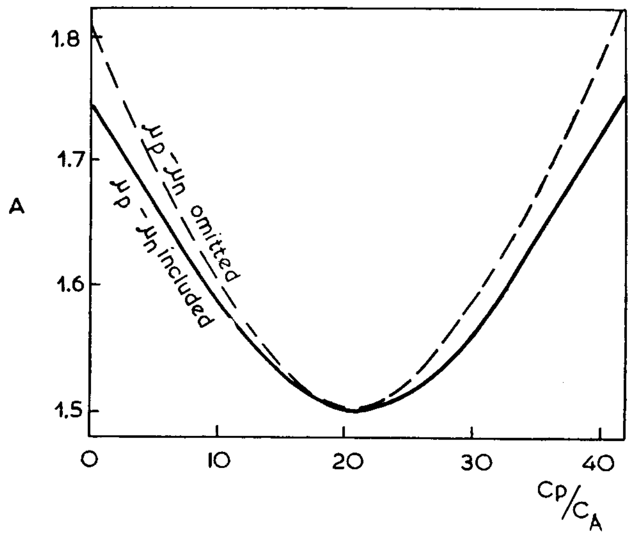

Fig. 2. The asymmetry coefficient A for $0^{\pm} \rightarrow 2^{\mp} \rightarrow 1$ transition

5. Other time invariant correlations

When we do not measure the gamma quantum angular distribution, then the formula (12 takes the usual form

$$W^N = 1+\alpha^N \boldsymbol{P}\cdot\boldsymbol{q}$$

where

$$\alpha^N A_N^2 = \frac{N+1}{N}\, A_N^2 - V_N^2 - (N+1)\, P_{N+1}^2 + (N+2) M_{N+1}^2 - 10\, P_0^2 \delta_{N1}.$$

The exact form for special cases of α^N was given in some papers [4], [5]. For muon capture by the protons the known result for the asymmetry of recoil neutrons

$$\alpha^0 = \frac{2M^2 - C_V^2 - P^2}{2M^2 + C_V^2 + P^2}$$

can be easily obtained.

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In another simple case when the recoil nuclei are not registered we have

$$W^N = 1 + \left(C_0^N + \frac{1}{3} b_1^N \right) \boldsymbol{P} \cdot \boldsymbol{k}$$

where

$$(3C_0^N + b_1^N)\, \Lambda_N = \frac{[(2N+1)(N+1)]^{\frac{1}{2}}}{N^{3/2}} \Lambda_{1,\eta}^{NN} (\Lambda_N^2 + 2NV_N\Lambda_N) +$$

$$+ \left[\frac{[(2N+3)(N+2)]}{N+1} \right]^{\frac{1}{2}} \Lambda_{1,\eta}^{(N+1)(N+1)} [M_{N+1}^2 + 2(N+1)\, M_{N+1}\, P_{N+1}].$$

We show these correlation constants for some transitions with the right polarized gamma quantum in Fig. 3.

Among other correlation constants the C_{2i+1}^N are remarkable. For unique transitions they have the form

$$C_{2i+1}^N \Lambda_N \propto M_{N+1} P_{N+1}$$

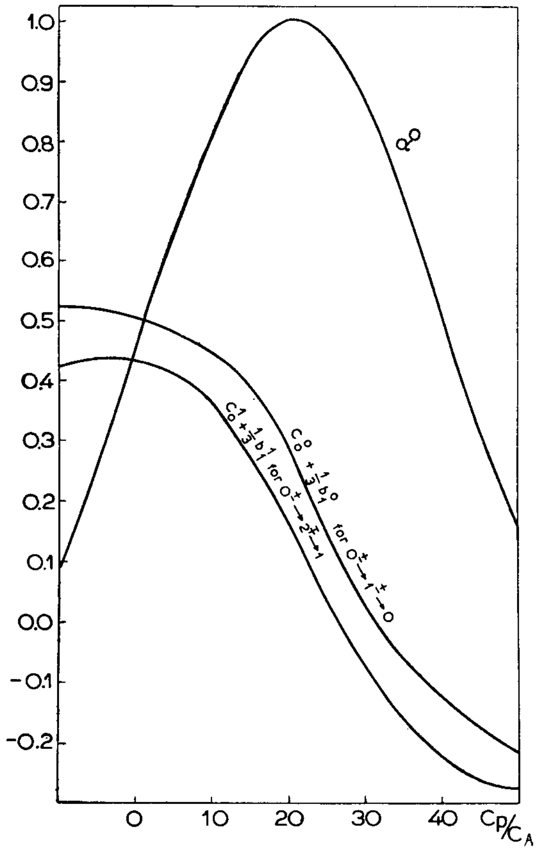

Fig. 3. The correlation constants α^0 for allowed transitions and $c_0^N + \dfrac{1}{3} b_1^N$

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They give a unique value of C_P and depend strongly on the pseudoscalar formfactor. They are large for small arguments, for $C_P = 21\ C_A$ they are equal to zero and antisymmetrical with respect to this point. The quantities C_1^0 and b_2^0 for $0^\pm \to 1^\pm \to 0$ transitions are shown in Fig. 4. The information about the sign of the constant C_1^0 is sufficient to remove the ambiguity of the value of C_P in all experiments where a twofold value is allowed.

It is noteworthy that nonunique $0 \xrightarrow{\mu^-} N$ transitions do not depend on C_P and can be helpful for a more exact estimation of the other interaction constants [12].

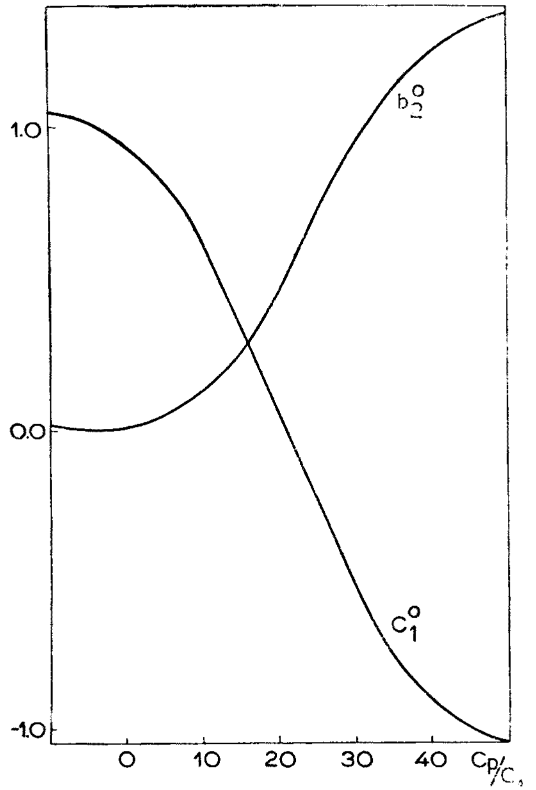

Fig. 4. The correlation constants b_2^0 and c_1^0 for $0^\pm \to 1^\pm \to 0$ transition

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APPENDIX

The functions $S_{rfs}(\boldsymbol{\sigma q k})$ are defined by

$$S_{rfs}(\boldsymbol{\sigma q k}) = (4\pi)^{3/2} \frac{1}{(2S+1)^{\frac{1}{2}}} \sum_{\eta_1\eta_2\eta_3} C^{s\eta_3}_{r\eta_1 f\eta_2}\, Y^*_{r\eta_1}(\Omega_\sigma)\, Y^*_{f\eta_2}(\Omega_q)\, Y_{s\eta_3}(\Omega_k)\,. \tag{A1}$$

These functions are invariant under rotations and can be expressed by invariants constructed with the aid of the vectors $\boldsymbol{\sigma}$, $\boldsymbol{q}$ and $\boldsymbol{k}$. We are interested in this paper in the cases when $r = 0.1$ only. Then we obtain

$$S_{0fS}(\boldsymbol{\sigma q k}) = \delta_{Sf}(2S+1)^{\frac{1}{2}}P_s(\boldsymbol{k}\cdot\boldsymbol{q}), \tag{A2}$$

$$\begin{gathered} S_{1fS}(\boldsymbol{\sigma q k}) = \sqrt{3}\,(2S+1)^{\frac{1}{2}}\times \\ \times\{[S(2S-1)]^{-\frac{1}{2}}\delta_{f,S-1}[\boldsymbol{\sigma}\cdot\boldsymbol{k}\,P'_S(\boldsymbol{k}\cdot\boldsymbol{q})-\boldsymbol{\sigma}\cdot\boldsymbol{q}\,P'_{S-1}(\boldsymbol{k}\cdot\boldsymbol{q})]+ \\ +[(S+1)\,(2S+3)]^{-\frac{1}{2}}\delta_{f,S+1}[\boldsymbol{\sigma}\cdot\boldsymbol{k}\,P'_S(\boldsymbol{k}\cdot\boldsymbol{q})-\boldsymbol{\sigma}\cdot\boldsymbol{q}\,P'_{S+1}(\boldsymbol{k}\cdot\boldsymbol{q})]+ \\ +i\delta_{f,S}[S(S+1)]^{-\frac{1}{2}}\boldsymbol{q}\Lambda\boldsymbol{\sigma}\cdot\boldsymbol{k}\,P'_S(\boldsymbol{k}\cdot\boldsymbol{q})\} \end{gathered} \tag{A3}$$

where

$$P'_l(x) = \frac{d}{dx}P_l(x) = \sum_{l=0}^{n}(2l-4i-1)\,P_{l-2i-1}(x) \tag{A4}$$

$$n = \begin{cases} \frac{1}{2}(l-2) & \text{for } l \text{ even} \\ \frac{1}{2}(l-1) & \text{for } l \text{ odd} \end{cases}$$

and $P_l(x)$ are the usual Legendre polynomials. The formulas (A3) can be easily obtained when we consider the functions $S_{1fS}(\boldsymbol{\sigma q k})$ in a coordinate system where $\boldsymbol{q}$ or $\boldsymbol{k}$ is parallel to z axis.

REFERENCES

[1] H. P. C. Rood, *Suppl. Nuovo Cimento*, **4**, 185 (1966).
L. L. Foldy, J. D. Walecka, *Phys. Rev.*, **140**, B 1339 (1965).
H. Ohtsubo, A. Fujii, *Nuovo Cimento*, **42**, 109 (1966).

[2] E. G. Primakoff, *Revs. Mod., Phys.* **31**, 802 (1959).

[3] M. Morita, A. Fujii, *Phys. Rev.*, **118**, 606 (1960).

[4] M. Morita, D. Greenberg, *Phys. Rev.*, **119**, 435 (1960).

[5] G. Ja. Korenman, R. A. Eramzyan, preprint P-2063, Dubna 1962.

[6] N. P. Popov, *Zh. Eksper. Teor. Fiz.*, **44**, 1679 (1963). G. M. Bukat, N. P. Popov, *Zh. Eksper. Teor. Fiz.*, **46**, 1782 (1964).

[7] A. P. Bukhvostov, N. P. Popov, *Yadernaya Fizika*, (in press).

[8] *Alpha-Beta-Gamma Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Co. Amsterdam 1965), vol. 2 p. 1031.

[9] Z. Oziewicz, *Acta Phys. Polon.*, **31**, 501 (1967).

[10] I. M. Smushkevitch, *Nuclear Phys.*, **11**, 419 (1959).

[11] A. P. Bukhvostov, N. P. Popov, *Zh. Eksper. Teor. Fiz.*, **46**, 1842 (1964). A. P. Bukvostov, *Yadernaya Fizyka*, **4**, 84 (1966).

[12] Z. Oziewicz, N. P. Popov, *Phys. Letters*, **15**, 273 (1965).

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