DIFFRACTION OF ELECTROMAGNETIC DIPOLE RADIATION BY AN IDEALLY-CONDUCTING WEDGE

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The paper presents a solution to the problem of diffraction of electromagnetic waves from an electric dipole by an ideally-conducting wedge. This problem has already been studied by Y. Nomura (1952) and R. Teisseyre (1955). The method used in this paper was developed by J. Petykiewicz (1967) who was the first to apply it in solving the problem of diffraction of scalar multipole radiation by a wedge. Compared with the methods applied by the two authors first mentioned, it is much simpler and leads directly to the solution. Taking the wedge divergence angle $\chi = 2\pi$ gives the solution to the problem of diffraction of electromagnetic waves in the case of an ideally-conducting half-plane.

1. *Introduction*

To describe the problem we use the cylindrical system of coordinates in which the z-axis is the diffracting edge and the half-planes $\varphi = 0$ and $\varphi = \chi$ are the surfaces limiting the wedge. We assume that at the point $L(\varrho, \varphi_0, z_0)$ there is a radiating dipole, and point $P(r, \varphi, z)$ is the point of observation (Fig. 1). Sometimes we shall write the components of the vectors in the Cartesian system, with the axes directed as shown in Fig. 1.

In unlimited space we assume the Hertz vector for the dipole in the form:

$$\boldsymbol{U} = \boldsymbol{u}\ \frac{e^{-i(kR-wt)}}{kR}$$

where $\boldsymbol{u}$ is the unit vector along the direction of the dipole axis, and $R = \sqrt{r^2+\varrho^2+(2-z_0)^2-2r\varrho\cos(\varphi-\varphi_0)}$ is the distance between the points L and P. The electric and magnetic fields are expressed in the familar way:

$$\boldsymbol{E} = \mathrm{grad}_p\ \mathrm{div}_p\ \boldsymbol{U}+k^2\boldsymbol{u} = B(R)(\boldsymbol{uR})+\boldsymbol{u}[A(R)+k^2f(R)].$$

$$\boldsymbol{H} = ik\ \mathrm{curl}_p\ \boldsymbol{U} = ik\ [\boldsymbol{R}\times\boldsymbol{u}]\ A(R) \tag{1.1}$$

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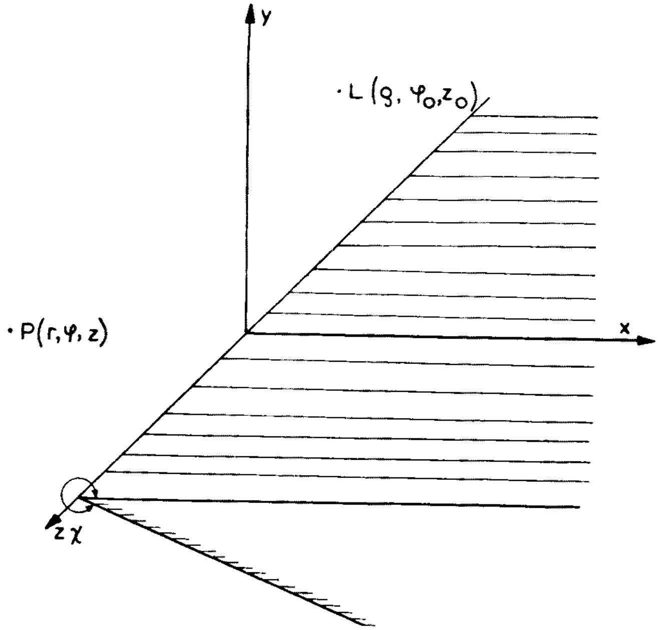

Fig. 1.

where

$$R_x = r\cos\varphi - \varrho\cos\varphi_0,\ R_y = r\sin\varphi - \varrho\sin\varphi_0,\ R_z = z - z_0$$

$$f(R) = \frac{e^{-ikR}}{kR}, \qquad A(R) = \frac{1}{R}\frac{\partial}{\partial R} f(R), \qquad B(R) = \frac{1}{R}\frac{\partial}{\partial R} A(R),$$

The time factor $e^{i\omega t}$ has been omitted everywhere.

The components of the field in the cylindrical coordinate system are expressed as follows:

$$\begin{aligned}
E_r &= (\boldsymbol{uR})\, B(R)\, [r - \varrho\cos(\varphi - \varphi_0)] + [A(R) + k^2 f(R)]\,(u_x\cos\varphi + u_y\sin\varphi),\\
E_\varphi &= (\boldsymbol{uR})\, B(R)\, \varrho\sin(\varphi - \varphi_0) + [A(R) + k^2 f(R)]\,(u_y\cos\varphi - u_x\sin\varphi),\\
E_z &= (\boldsymbol{uR})\, B(R)\,(z - z_0) + u_z[A(R) + k^2 f(R)].\\
H_r &= ik\, A(R)\, [u_z\varrho\sin(\varphi - \varphi_0) - (z - z_0)\,(u_y\cos\varphi - u_x\sin\varphi)],\\
H_\varphi &= -ik\, A(R)\, \{u_z\, [r - \varrho\cos(\varphi - \varphi_0)] - (z - z_0)\,(u_x\cos\varphi + u_y\sin\varphi)\},\\
H_z &= ik\, A(R)\, [u_y(r\cos\varphi - \varrho\cos\varphi_0) - u_x\,(r\sin\varphi - \varrho\sin\varphi_0)].
\end{aligned} \tag{1.2}$$

In the Carslaw-Sommerfeld method, commonly applied in solving the scalar Sommerfeld problem, the procedure is as follows:

a. Instead of ordinary three-dimensional space we introduce an infinite number of diplicates of Riemann space in which the z-axis becomes the branching line, wherein the region $0 < \varphi < \chi$, $-\infty < z < +\infty$ is known as the physical region.

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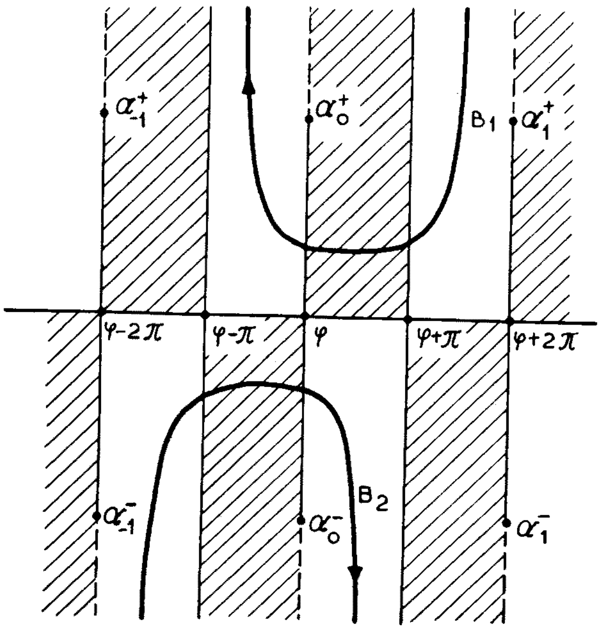

Fig. 2. The α-plane, α_o^+, α_o^- are the branch points, $R_\alpha = \sqrt{r^2+\varrho^2+(z-z_0)^2-2r\varrho\cos(\varphi-\alpha)}$. The cross-hatched regions correspond to the 1-st quarter on the R_α-plane in the first sheet (the 3-rd in the second). The plain regions correspond to the 4-th quarter (2-nd in the second sheet)

b. In the expressions of the type in Eq. (1.2), the angle φ_0 is substituted by α and they are then multiplied by the function $\Phi_{\pi/\chi}(\alpha-\varphi_0)$ possessing poles at the points $\alpha = \varphi_0+2m\chi$ $(m = 0, \pm 1, \pm 2 \ldots)$, and integrated over the paths B_1 and B_2 which are on the first of the double-sheeted Riemann surface of the complex variable α (Fig. 2). In this way we find a new solution, which we write as $u_L(r, \varphi, z, \varrho, \varphi_0, z_0, \chi)$.

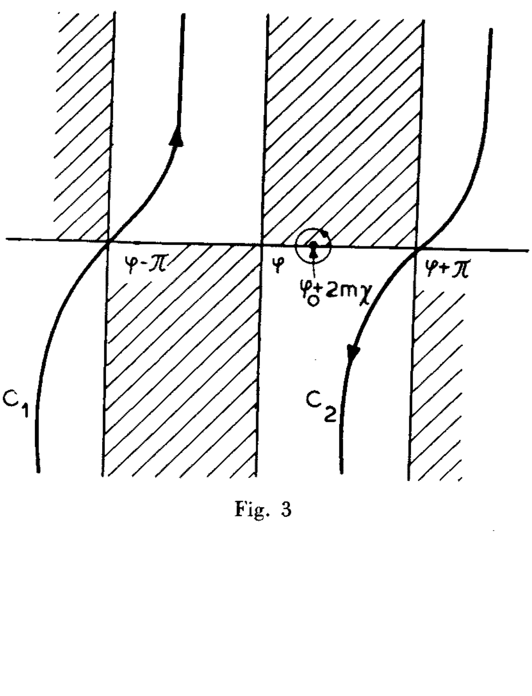

Fig. 3

c. The integration path B_1+B_2 can be transformed in a continuous manner into the paths C_1 and C_2 and circles around the poles $\varphi_0+2m\chi$, if they are within the region $\varphi-\pi \leqslant \varphi_0+2m\chi \leqslant \varphi+\pi$ (Fig. 3). Integration over C_1 and C_2 gives the diffraction field whereas the circles around the poles provide contributions, which should be expected in accordance with the laws of geometrical optics, from the sources $L_m(\varrho, \varphi_0+2m\chi, z_0)$, of which only L_0 is in the physical space.

d. From the solution $u_L(r, \varphi, z, \varrho, \varphi_0, z_0, \chi)$ obtained in this manner (in the scalar problem this is the solution of the equation of vibrations), we subtract the solution $u_{L'}(r, \varphi, z, \varrho, -\varphi_0, z_0, \chi)$, obtained identically, but representing the radiation from the source L' which is the image of L in the half-plane $\varphi = 0$. This difference is the solution to the diffraction problem in the case of the boundary condition $u_L - u_{L'} = 0$ for $\varphi = 0$ and $\varphi = \chi$.

In our case this procedure would give a solution to Maxwell's equations, but it would not satisfy the fundamental Meixner condition for diffraction problems. The method just outlined must be modified somewhat in order to be able to give the correct solution.

2. *Solution of the problem of diffraction of electromagnetic dipole radiation by a wedge*

The method of Petykiewicz, which we shall apply for finding the solution to Maxwell's equations, differs from the Carslaw-Sommerfeld method only by the use of the substitution $\varphi \to \varphi+\varphi_0-\alpha$ instead of the transformation $\varphi_0 \to \alpha$. In the scalar problem, when we are dealing with an isotropic point source, the two methods give identical results. In the case of vector fields the character of the substitution dictates the use of the field components in the cylindrical reference system. The new solution to Maxwell's equations obtained from Eq. (1.2), therefore, will have the form:

$$e_r = \frac{1}{2\chi} \int\limits_{B_1+B_2} \{(\boldsymbol{uR}_\alpha)B_\alpha(R_\alpha)[r-\varrho\cos(\varphi-\alpha)]+$$

$$+[A(R_\alpha)+k^2f(R_\alpha)]\,[u_x\cos(\varphi+\varphi_0-\alpha)+u_y\sin(\varphi+\varphi_0-\alpha)]\}\,\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$e_\varphi = \frac{1}{2\chi} \int\limits_{B_1+B_2} \{(\boldsymbol{uR}_\alpha)B(R_\alpha)\varrho\sin(\varphi-\alpha)+$$

$$[A(R_\alpha)+k^2f(R_\alpha)]\,[u_y\cos(\varphi+\varphi_0-\alpha)-u_x\sin(\varphi+\varphi_0-\alpha)]\}\,\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$e_z = \frac{1}{2\chi} \int\limits_{B_1+B_2} \{(\boldsymbol{uR}_\alpha)B(R_\alpha)(z-z_0)+u_z[A(R_\alpha)+k^2f(R_\alpha)]\}\,\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha \tag{2.1}$$

$$h_r = \frac{ik}{2\chi} \int\limits_{B_1+B_2} A(R_\alpha)\{u_z\varrho\sin(\varphi-\alpha)-$$

$$-(z-z_0)[u_y\cos(\varphi+\varphi_0-\alpha)-u_x\sin(\varphi+\varphi_0-\alpha)]\}\,\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$h_\varphi = -\frac{ik}{2\chi} \int\limits_{B_1+B_2} A(R_\alpha)\{u_z[r-\varrho\cos(\varphi-\alpha)]-$$

$$-(z-z_0)[u_x\cos(\varphi+\varphi_0-\alpha)+u_y\sin(\varphi+\varphi_0-\alpha)]\}\,\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$h_z = \frac{ik}{2\chi} \int\limits_{B_1+B_2} A(R_\alpha)\{u_y[r\cos(\varphi+\varphi_0-\alpha)-\varrho\cos\varphi_0]-$$
$$-u_x[r\sin(\varphi+\varphi_0-\alpha)-\varrho\sin\varphi_0]\}\,\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

where

$$R_\alpha = \sqrt{r^2+\varrho^2+(z-z_0)^2-2r\varrho\cos(\varphi-\alpha)}$$
$$\Phi_{\pi/\chi}(\alpha-\varphi_0) = \frac{1}{1-e^{i\frac{\pi}{\chi}(\varphi_0-\alpha)}}$$

and the components of the vector $\boldsymbol{R}_\alpha$ in the Cartesian system of coordinates are:

$$R_{\alpha x} = r\cos(\varphi+\varphi_0-\alpha)-\varrho\cos\varphi_0,\; R_{\alpha y} = r\sin(\varphi+\varphi_0-\alpha)-\varrho\sin\varphi_0,\; R_{\alpha z} = z-z_0$$

To have our solution (2.1) in the simplest possible form we now perform some transformations. First, we rewrite the e_r component, Eq. (2.1), for example, in a somewhat changed form:

$$e_r = \frac{1}{2\chi}\int\limits_{B_1+B_2} (\{u_\varrho[r\cos(\varphi-\alpha)-\varrho]+u_{\varphi_0}r\sin(\varphi-\alpha)+u_z(z-z_0)\}B(R_\alpha)[r-\varrho\cos(\varphi-\alpha)]+$$
$$+[A(R_\alpha)+k^2f(R_\alpha)][u_\varrho\cos(\varphi-\alpha)+u_{\varphi_0}\sin(\varphi-\alpha)])\,\Phi_{\pi/\chi}(a-\varphi_0)d\alpha$$

where:

$$u_\varrho = u_x\cos\varphi_0+u_y\sin\varphi_0,\; u_{\varphi_0} = -u_x\sin\varphi_0+u_y\cos\varphi_0.$$

Let us also note that:

$$\boldsymbol{u}\,\mathrm{grad}_L \int\limits_{B_1+B_2} f(R_\alpha)\Phi_{\pi/\chi}(a-\varphi_0)d\alpha = \int\limits_{B_1+B_2} u_\varrho A(R_\alpha)[\varrho-r\cos(\varphi-\alpha)])\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha+$$
$$+\int\limits_{B_1+B_2}\frac{u_{\varphi_0}}{\varrho}f(R_\alpha)\frac{\partial}{\partial\varphi_0}\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha-\int\limits_{B_1+B_2}u_zA(R_\alpha)(z-z_0)\,\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha.$$

We now make use of the relation $\partial/\partial\varphi_0\,\Phi_{\pi/\chi}(\alpha-\varphi_0) = -\partial/\partial\alpha\,\Phi_{\pi/\chi}(\alpha-\varphi_0)$ and integrate the second term of the right-hand side by parts, and after considering that the expression $u_{\varphi_0}f(R_\alpha)\,\Phi_{\pi/\chi}(\alpha-\varphi_0)/\varrho$ vanishes at the ends of the paths B_1 and B_2, being in infinity, we get:

$$\boldsymbol{u}\,\mathrm{grad}_L \int\limits_{B_1+B_2} f(R_\alpha)\,\Phi_{\pi/\chi}(\alpha-\varphi_0)\,d\alpha$$
$$= \int\limits_{B_1+B_2} A(R_\alpha)\,\{u_\varrho[\varrho-r\cos(\varphi-\alpha)]-u_{\varphi_0}r\sin(\varphi-\alpha)-u_z(z-z_0)\}\,\Phi_{\pi/\chi}(\alpha-\varphi_0)\,d\alpha \quad (2.2)$$

On the basis of Eq. (2.2) we easily get for e_r:

$$e_r = -\frac{1}{2\chi}\frac{\partial}{\partial r}\left(\boldsymbol{u}\,\mathrm{grad}_L\int\limits_{B_1+B_2} f(R_\alpha)\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha\right)+$$
$$+\frac{k^2}{2\chi}\int\limits_{B_1+B_2} f(R_\alpha)\,[u_\varrho\cos(\varphi-\alpha)+u_{\varphi_0}\sin(\varphi-\alpha)]\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha.$$

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Similar transformation in the expressions for the other field components yield:

$$\boldsymbol{e}(\boldsymbol{u}, r, \varphi, z, \varrho, \varphi_0, z_0, \chi) = -\frac{1}{2\chi}\,\mathrm{grad}_\varphi\left(\boldsymbol{u}\,\mathrm{grad}_L \int\limits_{B_1+B_2} f(R_\alpha)\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha\right) +$$

$$+\frac{k^2}{2\chi}\int\limits_{B_1+B_2} \boldsymbol{C}f(R_\alpha)\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$\boldsymbol{h}(\boldsymbol{u}, r, \varphi, z, \varrho, \varphi_0, z_0, \chi) = \frac{ik}{2\chi}\,\mathrm{curl}_p \int\limits_{B_1+B_2} \boldsymbol{C}f(R_\alpha)\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha. \tag{2.3}$$

where the components of the vector $\boldsymbol{C}$ in the chosen cylindrical and Cartesian systems of coordinates will have the form:

$$C_r = u_\varrho \cos(\varphi-\alpha)+u_{\varphi_0}\sin(\varphi-\alpha),\; C_\varphi = u_{\varphi_0}\cos(\varphi-\alpha)-u_\varrho \sin(\varphi-\alpha),\; C_z = u_z$$

$$C_x = u_x \cos(\varphi_0-\alpha)+u_y \sin(\varphi_0-\alpha)\; C_y = u_y\cos(\varphi_0-\alpha)-u_x\sin(\varphi_0-\alpha),\; C_z = u_z$$

We can get another form of the solution (2.3) which is simpler to interpret if we notice that:

$$\mathrm{div}_p \int\limits_{B_1+B_2} \boldsymbol{C}f(R_\alpha)\,\Phi_{\pi/\chi}(\alpha-\varphi_0)\,d\alpha$$

$$= \int\limits_{B_1+B_2} A(R_\alpha)\,\{u_\varrho[r\cos(\varphi-\alpha)-\varrho]+u_{\varphi_0} r\sin(\varphi-\alpha)+u_z(z-z_0)\}\;\Phi_{\pi/\chi}(\alpha-\varphi_0)\,d\alpha$$

Comparing this with Eq. (2.2) we immediately find

$$\boldsymbol{e}(\boldsymbol{u}, r, \varphi, z, \varrho, \varphi_0, z_0, \chi) = \frac{1}{2\chi}\,(\mathrm{grad}_p\,\mathrm{div}_p + k^2)\int\limits_{B_1+B_2} \boldsymbol{C}f(R_\alpha)\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$\boldsymbol{h}(\boldsymbol{u}, r, \varphi, z, \varrho, \varphi_0, z_0, \chi) = \frac{ik}{2\chi}\,\mathrm{curl}_p \int\limits_{B_1+B_2} \boldsymbol{C}f(R_\alpha)\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha \tag{2.4}$$

From Eqs (2.4) we see that we could find the solution (2.1) by introducing the Hertz vector:

$$\boldsymbol{Z} = \frac{1}{2\chi}\int\limits_{B_1+B_2} \boldsymbol{C}f(R_\alpha)\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha \tag{2.5}$$

and differentiating it with respect to the coordinates of the observation point according to Eq. (1.1).

In accordance with what has been said in Sec 1, (c), the solution of Eqs (2.1), (2.3) and (2.4) can be separated into two terms correspondingly representing the geometrical and diffraction fields, if the integration paths B_1 and B_2 are transformed into circles around the poles $\varphi_0+2m\chi$ $(m = 0, \pm 1, \pm 2, \ldots)$ and the paths C_1 and C_2. Then:

$$\boldsymbol{e} = \boldsymbol{e}^g + \boldsymbol{e}^d \tag{2.6}$$

$$\boldsymbol{h} = \boldsymbol{h}^g + \boldsymbol{h}^d$$

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where the geometrical field is presented by the relationships:

$$\boldsymbol{e}^g = (\mathrm{grad}_p\, \mathrm{div}_p + k^2) \sum_{m=-\infty}^{+\infty} \boldsymbol{C}^m \frac{e^{-ikR_m}}{kR_m} v_m(\varphi), \tag{2.7}$$

$$\boldsymbol{h}^g = ik\, \mathrm{curl}_p \sum_{m=-\infty}^{+\infty} \boldsymbol{C}^m \frac{e^{-ikR_m}}{kR_m} v_m(\varphi),$$

$$R_m = \sqrt{r^2+\varrho^2+(z-z_0)^2-2r\,\varrho\cos(\varphi-\varphi_0-2m\chi)}$$

$$C_r^m = u_\varrho \cos(\varphi-\varphi_0-2m\chi) + u_{\varphi_0} \sin(\varphi-\varphi_0-2m\chi),$$

$$C_\varphi^m = u_{\varphi_0} \cos(\varphi-\varphi_0-2m\chi) - u_\varrho \sin(\varphi-\varphi_0-2m\chi),$$

$$C_z^m = u_z$$

$$v_m(\varphi) = \begin{cases} 1 & \text{for } \varphi_0+2m\chi-\pi \leqslant \varphi_0 \leqslant \varphi_0+2m\chi+\pi \\ 0 & \text{for } \varphi < \varphi_0+2m\chi-\pi \text{ or } \varphi > \varphi_0+2m\chi+\pi \end{cases}$$

The diffraction field is presented by the formulae:

$$\boldsymbol{e}^d = \frac{1}{2\chi} (\mathrm{grad}_p\, \mathrm{div}_p + k^2) \int\limits_{C_1+C_2} \boldsymbol{C}(R_\alpha) \Phi_{\pi/\chi}(\alpha-\varphi_0) d\alpha,$$

$$\boldsymbol{h}^d = \frac{ik}{2\chi} \mathrm{curl}_p \int\limits_{C_1+C_2} \boldsymbol{C} f(R_\alpha) \Phi_{\pi/\chi}(\alpha-\varphi_0) d\alpha. \tag{2.8}$$

The solutions (2.3) and (2.4) of Maxwell's equations are still not solutions of the diffraction problem. The latter, by virtue of Sec. 1, (d), will have the form:

$$\vec{\varepsilon} = \boldsymbol{e}(\boldsymbol{u}, r, \varphi, z, \varrho, \varphi_0, z_0, \chi) - \boldsymbol{e}(\boldsymbol{u}', r, \varphi, z, \varrho, -\varphi_0, z_0, \chi),$$

$$\boldsymbol{H} = \boldsymbol{h}(\boldsymbol{u}, r, \varphi, z, \varrho, \varphi_0, z_0, \chi) - \boldsymbol{h}(\boldsymbol{u}', r, \varphi, z, \varrho, -\varphi_0, z_0, \chi). \tag{2.9}$$

where the second term on the right-hand side represents the field from the dipole $\boldsymbol{u}'$, which is the image of $\boldsymbol{u}$ in the half-plane $\varphi = 0$.

The obtained solution (2.9) has all the properties that should be required of such solutions, *viz.*

1. it is generally a regular solution of Maxwell's equations everywhere in multiple-sheeted Riemann space;

2. when the observation point P approaches the light source L it indicates singularity of the required order;

3. in infinity it satisfies the electromagnetic emission conditions and finiteness of Sommerfeld;

4. at both surfaces of the wedge, as it is an ideal conductor, the field components ε_r, ε_φ and H_φ vanish;

5. the Meixner edge condition is satisfied.

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If we turn to Eq. (2.4), which represents the first term of the solution (2.9), then proving that Maxwell's equations are fulfilled is reduced to proving that the components of the Hertz vector, Eq. (2.5), satisfy the wave equation. This fact is obvious, however, for in Eq. (2.5) only $f(R_\alpha)$ depends on the coordinates of the observation point. Regularity of the solution is ensured in particular by the factor e^{-ikR_α} in the integrand of the integral in Eq. (2.1) and by the position of the part of the integration path B_1+B_2 which could give singularities, in the fourth quarter of the R_α-plane, where the imaginary part of R_α is negative. A singularity can appear only when the branch points α_0^+ and α_0^- lie on the real axis α and, moreover, if at this same point there is a pole $\varphi_0+2m\chi$. However, since (*cf.* Rubinowicz 1966),

$$\alpha_0^{\pm} = \varphi \pm i \operatorname{arc\,cosh} \frac{\varrho^2+r^2+(z-z_0)^2}{2r\varrho}.$$

this can happen only when $r \to \varrho$, $z \to z_0$ and $\varphi \to \varphi_0+2m\chi$, that is, when the observation point P approaches the source $L_m(\varrho, \varphi_0+2m\chi, z_0)$. We have shown, therefore, that the condition (2) for Eq. (2.4) is satisfied and, hence, it is also so for Eq. (2.9). Proof of the condition (3) is presented in Appendix I with the use of the asymptotic expression for $\boldsymbol{e}^d$ and $\boldsymbol{h}^d$ near to and far from the shadow boundary, obtained in Sec. 3. Appendices II and III give the proofs of the conditions (4) and (5).

3. *Asymptotic expressions for the diffraction wave*

For conciseness we shall consider only the first term of the general solution (2.9). Taking advantage of Eq. (2.1) we shall present, for example, the component e_r^d of the diffraction wave in the form:

$$e_r^d = \frac{1}{2\chi k} \int\limits_{C_2+C_1} \{(\boldsymbol{u}\boldsymbol{R}_\alpha)B^*(R_\alpha)[r-\varrho\cos(\varphi-\alpha)+$$

$$+[A^*(R_\alpha)+k^2 f^*(R_\alpha)]\,[u_x \cos(\varphi+\varphi_0-\alpha)+u_y \sin(\varphi+\varphi_0-\alpha)]\} e^{ikR_\alpha} \Phi_{\pi/\chi}(\alpha-\varphi_0) d\alpha,$$

where:

$$f^*(R_\alpha) = \frac{1}{R_\alpha}, \quad A^*(R_\alpha) = -\left(\frac{1}{R_\alpha^3}+\frac{ik}{R_\alpha^2}\right), \quad B^*(R_\alpha) = \frac{3}{R_\alpha^5}+\frac{3ik}{R_\alpha^4}-\frac{k^2}{R_\alpha^3}$$

We shall look for the asymptotic expression for e_r^d by the saddle-point method. Hence, we write e_r^d in the form:

$$e_r^d = \frac{1}{2\chi k} \int\limits_{C_1+C_2} e^{g(\alpha)} d\alpha$$

where:

$$g(\alpha) = -ikR_\alpha + \ln\left(\{(\boldsymbol{u}\boldsymbol{R}_\alpha)\,B^*(R_\alpha)\,[r-\varrho\cos(\varphi-\alpha)]+\right.$$

$$\left.+[A^*(R_\alpha)+k^2 f^*(R_\alpha)]\,[u_x\cos(\varphi+\varphi_0-\alpha)+u_y\sin(\varphi+\varphi_0-\alpha)]\}\,\Phi_{\frac{\pi}{\chi}}(\alpha-\varphi_0)\right).$$

In the function $g(\alpha)$, owing to large k, the largest contribution to the integral will be from the term $-ikR_\alpha$, hence, we substitute the condition $\partial g(\alpha)/\partial\alpha = 0$, from which the saddle-points are to be found, by the condition $\partial R_\alpha/\partial\alpha = 0$. They are then given by the equation $\alpha = \varphi \mp \pi n$ ($n = 0, \pm 1, \pm 2, \ldots$). Proceeding similarly as in the scalar problem of an isotropic spherical wave (Rubinowicz 1966), we find that the paths of steepest decrease are determined by the equations $\alpha(s) = \varphi \mp \pi + s e^{i\frac{\pi}{4}}$ and taking the saddle-point values of all the functions of α except $-ikR_\alpha$ that appear in $g(\alpha)$, yields for e_r^d the asymptotic expression:

$$e_r^d \cong \frac{e^{i\frac{3}{4}\pi}\sqrt{2\pi}\,\sin\dfrac{\pi^2}{\chi}}{2\chi k\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\,\frac{e^{ikR_0}}{\sqrt{R_0}\,\sqrt{kr\varrho}}\,\{(\boldsymbol{uR}_0)B^{**}(R_0)(r+\varrho)-$$
$$-[A^{**}(R_0)+k^2]\,[u_x\cos\varphi_0+u_y\sin\varphi_0]\},$$

where:

$$R_0 = \sqrt{(r+\varrho)^2+(z-z_0)^2}\cdot \boldsymbol{R}_0 = [-(r+\varrho)\cos\varphi_0, -(r+\varrho)\sin\varphi_0, z-z_0]$$

$$A^{**}(R_0) = -\left(\frac{1}{R_0^2}+\frac{ik}{R_0}\right),\qquad B^{**}(R_0) = \frac{3}{R_0^4}+\frac{3ik}{R_9^3}-\frac{k^2}{R_0^2},$$

and for the other components:

$$e_\varphi^d \cong \frac{e^{i\frac{3}{4}\pi}\sqrt{2\pi}\,\sin\dfrac{\pi^2}{\chi}}{2\chi k\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\,\frac{e^{-ikR_0}}{\sqrt{R_0}\,\sqrt{kr\varrho}}\,[A^{**}(R_0)+k^2][u_x\sin\varphi_0-u_y\cos\varphi_0],$$

$$e_z^d \cong \frac{e^{i\frac{3}{4}\pi}\sqrt{2\pi}\,\sin\dfrac{\pi^2}{\chi}}{2\chi k\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\,\frac{e^{-ikR_0}}{\sqrt{R_0}\,\sqrt{kr\varrho}}\,\{(\boldsymbol{uR}_0)B^{**}(R_0)(z-z_0)+u_z[A^{**}(R_0)+k^2]\}.$$

$$h_r^d \cong \frac{e^{i\frac{5}{4}\pi}\sqrt{2\pi}\,\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\,\frac{e^{-ikR_0}}{\sqrt{R_0}\,\sqrt{kr\varrho}}\,A^{**}(R_0)\,(z-z_0)\,(u_y\cos\varphi_0-u_x\sin\varphi_0) \tag{3.1}$$

$$h_\varphi^d \cong -\frac{e^{i\frac{5}{4}\pi}\sqrt{2\pi}\,\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\,\frac{e^{-ikR_0}}{\sqrt{R_0}\,\sqrt{kr\varrho}}\,A^{**}(R_0)[u_z(r+\varrho)+$$
$$+(z-z_0)(u_x\cos\varphi_0+u_y\sin\varphi_0)]$$

$$h_z^d \cong \frac{e^{i\frac{5}{4}\pi}\sqrt{2\pi}\,\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\,\frac{e^{-ikR_0}}{\sqrt{R_0}\,\sqrt{kr\varrho}}\,A^{**}(R_0)(u_x\sin\varphi_0-u_y\cos\varphi_0)$$

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As is seen, Eqs (3.1) for the field components assume infinite values at points placed on any one of the shadow boundaries $\varphi = \varphi_0 + 2m\chi \pm \pi$. This stems from the discontinuity of the diffracted wave at the shadow boundary. They are correct, therefore, only for points distant from the shadow boundary. For regions near these boundaries we must seek other asymptotic approximations. We then introduce (Rubinowicz *loc. cit*) auxiliary function by the operation

$$e_r'^d = \frac{\partial}{\partial k}\,(e^{ikR_m}\,e_r^d)$$

where:

$$R_m = \sqrt{r^2+\varrho^2+(z-z_0)^2-2r\varrho\cos(\varphi-\varphi_0-2m\chi)}$$

Application of the saddle-point method for $e_r'^d$, integration within the limits $+\infty$ to k, and multiplication of the obtained expression by e^{-ikR_m}, yields for e_r^d near the shadow boundary:

$$e_r^d = \frac{\pi}{\chi k}\,\frac{e^{i\frac{\pi}{4}}\sin\frac{\pi^2}{\chi}\sqrt{R_0-R_m}\,e^{-ikR_m}}{\sqrt{R_0 r\varrho}\left[\cos\frac{\pi}{\chi}(\varphi-\varphi_0)-\cos\frac{\pi^2}{\chi}\right]}\,[(\boldsymbol{u}\boldsymbol{R}_0)B^{**}(R_0)\,(r+\varrho)-$$

$$-(A^{**}(R_0)+k^2)\,(u_x\cos\varphi_0+u_y\sin\varphi_0)]\int\limits_{+\infty}^{\sqrt{\frac{2k}{\pi}(R_0-R_m)}} e^{-i\frac{\pi}{2}v^2}\,dv.$$

Introduction on the angle of diffraction $\psi = \varphi-(\varphi_0+2m\chi+\pi)$ for the m-th shadow boundary $\varphi = \varphi_0+2m\chi+\pi$ and utilization of the fact that $\sqrt{R_0-R_m} = 2\,[\varrho r/(R_0+R_m)]^{1/2}$ $\left|\sin\frac{\psi}{2}\right|$, with the assumption that for small ψ we can write $\left|\sin\frac{\psi}{2}\right| = \left|\frac{\psi}{2}\right|$ and $R_m = R_0$, gives

$$e_r^d \cong \frac{e^{-ikR_0}}{2kR_0}\,\mathrm{sgn}\,\psi[(\boldsymbol{u}\boldsymbol{R}_0)B^{**}(R_0)(r+\varrho)-[A^{**}(R_0)+k^2]\,[u_x\cos\varphi_0+u_y\sin\varphi_0]\}$$

Similarly,

$$e_\varphi^d \cong \frac{e^{-ikR_0}}{2kR_0}\,\mathrm{sgn}\,\psi\,\{A^{**}(R_0)+k^2\}\,\{u_y\cos\varphi_0-u_x\sin\varphi_0\}$$

$$e_z^d \cong \frac{e^{-ikR_0}}{2kR_0}\,\mathrm{sgn}\,\psi\,\{(\boldsymbol{u}\boldsymbol{R}_0)B^{**}(R_0)\,(z-z_0)+u_z[A^{**}(R_0)+k^2]\}$$

$$h_r^d \cong \frac{ie^{-ikR_0}}{2R_0}\,\mathrm{sgn}\,\psi\,A^{**}(R_0)\,(z-z_0)\,(u_y\cos\varphi_0-u_x\sin\varphi_0)$$

$$h_\varphi^d \cong -\frac{ie^{-ikR_0}}{2R_0}\,\mathrm{sgn}\,A^{**}(R_0)\,[u_z(r+\varrho)+(z-z_0)\,(u_x\cos\varphi_0+u_y\sin\varphi_0)]$$

$$h_z^d \cong \frac{ie^{-ikR_0}}{2R_0}\,\mathrm{sgn}\,\psi A^{**}(R_0)\,(r+\varrho)\,(u_x\sin\varphi_0-u_y\cos\varphi_0) \tag{3.2}$$

Formulae (3.2) shown that the diffraction wave near the m-th shadow boundary $\varphi = \varphi_0+2m\chi+\pi$ has a value equal to half of the geometrical wave coming from the source L_m and that its phase undergoes a jump equal π at this boundary. This ensures the continuity of the general solution in all points of space.

4. *Diffraction of dipole radiation by a half-plane*

The solution of the problem now under consideration can be obtained formally from Eq. (2.9) by taking $\chi = 2\pi$. With the use of Eq. (2.3) and the field components in the Cartesian system we have:

$$\vec{\mathcal{E}} = -\frac{1}{4\pi}\operatorname{grad}_P\left(\boldsymbol{u}\operatorname{grad}_L\int\limits_{B_1+B_2} f(R_\alpha)\,\Phi_0(\alpha-\varphi_0)d\alpha\right)+\frac{k^2}{4\pi}\int\limits_{B_1+B_2}\boldsymbol{a}f(R_\alpha)\Phi(\alpha-\varphi_0)d\alpha+$$

$$+\frac{1}{4\pi}\operatorname{grad}_P\left(\boldsymbol{u}'\operatorname{grad}_{L'}\int\limits_{B_1+B_2} f(R_\alpha)\,\Phi_0(\alpha+\varphi_0)d\alpha\right)-\frac{k^2}{4\pi}\int\limits_{B_1+B_2}\boldsymbol{a}'f(R_\alpha)\,\Phi(\alpha+\varphi_0)d\alpha \tag{4.1}$$

$$\vec{\mathcal{H}} = \frac{ik}{4\pi}\operatorname{curl}_P\int\limits_{B_1+B_2}\boldsymbol{a}f(R_\alpha)\Phi(\alpha-\varphi_0)d\alpha-\frac{ik}{4\pi}\operatorname{curl}_P\int\limits_{B_1+B_2}\boldsymbol{a}'f(R_\alpha)\Phi_0(\alpha+\varphi_0)d\alpha$$

where:

$$\Phi_0(a\mp\varphi_0) = \frac{1}{1-e^{-i(\alpha\mp\varphi_0)/2}}$$

$\boldsymbol{u}' = [u_x', u_y', u_z'] = [u_x, -u_y, u_z]$, as the image of vector $\boldsymbol{u}$ in the half-plane $\varphi = 0$,

$$\boldsymbol{a} = [a_x, a_y, a_z] = [u_x\cos(\varphi_0-\alpha)+u_y\sin(\varphi_0-\alpha),\ u_y\cos(\varphi_0-\alpha)-u_x\sin(\varphi_0-\alpha),\ u_2]$$

$$\boldsymbol{a}' = (a_x, a_y, a_z] = ([u_x'\cos(\varphi_0+\alpha)-u_y'\sin(\varphi_0+\alpha),\ u_y'\cos(\varphi_0+\alpha)+u_x'\sin(\varphi_0+\alpha),\ u_z']$$

We now want to write Eq. (4.1) in a somewhat different form.
For this, let us notice that:

$$\frac{1}{4\pi}\int\limits_{B_1+B_2}\boldsymbol{a}f(R_\alpha)\Phi_0(\alpha-\varphi_0)d\alpha-\frac{1}{4\pi}\int\limits_{B_1+B_2}\boldsymbol{a}'f(R_\alpha)\Phi(\alpha+\varphi_0)\,d\alpha$$

$$=\frac{\boldsymbol{u}}{4\pi}\int\limits_{B_1+B_2}f(R_\alpha)\Phi_0(\alpha-\varphi_0)d\alpha-\frac{\boldsymbol{u}'}{4\pi}\int\limits_{B_1+B_2}f(R)\Phi(\alpha+\varphi_0)d\alpha+\boldsymbol{W}$$

where the components of the vector $\boldsymbol{W}$ are given by the relations:

$$W_x = \frac{1}{4\pi}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{kR_\alpha}\left\{\frac{u_x[\cos(\varphi_0-\alpha)-1]+u_y\sin(\varphi_0-\alpha)}{1-e^{i(\varphi_0-\alpha)/2}}-\right.$$

$$\left.-\frac{u_x[\cos(\varphi_0+\alpha)-1]+u_y\sin(\varphi_0+\alpha)}{1-e^{-i(\varphi_0+\alpha)/2}}\right\}d\alpha$$

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$$W_y = \frac{1}{4\pi} \int\limits_{B_1+B_2} \frac{e^{-ikR_\alpha}}{kR_\alpha} \left\{ \frac{u_y[\cos(\varphi_0-\alpha)-1]-u_x \sin(\varphi_0-\alpha)}{1-e^{i(\varphi_0-\alpha)/2}} - \right.$$

$$\left. - \frac{u_y[1-\cos(\varphi_0+\alpha)]+u_x \sin(\varphi_0+\alpha)}{1-e^{-i(\varphi_0+\beta)/2}} \right\} d\alpha$$

$$W_z = 0$$

Simple trigonometrical conversions bring the component W_x to the form:

$$W_x = -\frac{i}{4\pi} \int\limits_{B_1+B_2} \frac{e^{-ikR_\alpha}}{kR_\alpha} \left[u_x \left(\sin\varphi_0 e^{i\alpha} + 2 \sin\frac{\varphi_0}{2} \cos\frac{\alpha}{2} \right) - \right.$$

$$\left. - u_y \left(\cos\varphi_0 \, e^{i\alpha} + 2\cos\frac{\varphi_0}{2} \cos\frac{\alpha}{2} - 1 \right) \right] d\alpha$$

Since the integrand does not have any poles now, the paths of integration B_1 and B_2 can be substituted by the paths C_1 and C_2, while these, in turn, by the straight lines $\alpha = \varphi-\pi+iv$ and $\alpha = \varphi+\pi+iv$. Then,

$$W_x = -\frac{1}{\pi} \sin\frac{\varphi}{2} \left(u_y \cos\frac{\varphi_0}{2} - u_x \sin\frac{\varphi_0}{2} \right) \int\limits_{-\infty}^{+\infty} \frac{e^{-ik\sqrt{r^2+\varrho^2+(z-z_0)^2+2r\varrho\cosh v}}}{k\sqrt{r^2+\varrho^2+(z-z_0)^2+2r\varrho\cosh v}} \cosh\frac{v}{2}\, dv$$

$$= \frac{i \sin\frac{\varphi}{2} \left(u_y \cos\frac{\varphi_0}{2} - u_x \sin\frac{\varphi_0}{2} \right)}{k\sqrt{r\varrho}} H_0^{(2)} \left(k\sqrt{(r+\varrho)^2+(z-z_0)^2}\right.$$

where $H_0^{(2)}$ denotes the Hankel function of the second kind. Proceeding in like manner with W_y and taking advantage of the fact that $W_z = 0$, we get for the vector $\boldsymbol{W}$ the expression:

$$\boldsymbol{W} = \boldsymbol{w}_0 \frac{i\left(u_y \cos\frac{\varphi_0}{2} - u_x \sin\frac{\varphi_0}{2} \right)}{k\sqrt{r\varrho}} H_0^{(2)} \left(k\sqrt{(r+\varrho)^2+(z-z_0)^2}\right)$$

where:

$$\boldsymbol{w}_0 = \left[\sin\frac{\varphi}{2}, \, -\cos\frac{\varphi}{2}, 0 \right]$$

Solution (4.1) now takes the form:

$$\vec{\mathcal{E}} = -\frac{1}{4\pi} \operatorname{grad}_P \left(\boldsymbol{u} \operatorname{grad}_L \int\limits_{B_1+B_2} f(R_\alpha)\Phi_0(\alpha-\varphi_0)\, d\alpha \right) + \frac{k^2\boldsymbol{u}}{4\pi} \int\limits_{B_1+B_2} f(R_\alpha)\, \Phi_0\, (\alpha-\varphi_0) d\alpha +$$

$$+ \frac{1}{4\pi} \operatorname{grad}_P \left(\boldsymbol{u}' \operatorname{grad}_{L'} \int\limits_{B_1+B_2} f(R_\alpha)\, \Phi_0(\alpha+\varphi_0) d\alpha \right) - \frac{k^2\boldsymbol{u}'}{4\pi} \int\limits_{B_1+B_2} f(R_\alpha) \Phi_0(\alpha+\varphi_0) d\alpha + \boldsymbol{W} \tag{4.2}$$

$$\vec{\mathcal{H}} = \frac{\mathrm{ik}}{4\pi} \operatorname{curl}_P \left\{ \boldsymbol{u} \int\limits_{B_1+B_2} f(R_\alpha)\Phi_0(\alpha-\varphi_0) d\alpha - \boldsymbol{u}' \int\limits_{B_1+B_2} f(R_\alpha)\Phi_0(\alpha+\varphi_0)\, d\alpha + 4\pi\boldsymbol{W} \right\}$$

The form of the solution (4.2) is identical with the solutions given previously by T. B. A. Senior (1953) and B. D. Woods (1957). The main difficulty that these authors had to overcome was to find such a solution which would satisfy the Maixner edge condition. For this purpose, Senior expands the Hertz vector $\boldsymbol{Z} = \boldsymbol{u}e^{-ikR}/kR$ for the dipole into plane waves and using the known Sommerfeld solutions for the plane waves he received by means of rather complicated calculus a result which satisfied the necessary stipulations. Here, the edge condition is satisfied because the solution for the plane wave satisfies it. B. D. Woods uses the branched solution of the equation of vibrations for a Hertz vector, identical to the one Senior used, and by differentiating it she obtains expressions for the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields which, however, do not satisfy the edge condition if the vector $\boldsymbol{u}$ is not parallel to the edge. In this case, the authoress must look for a solution of Maxwell's equations which should be added to the previous one for the aggregate to have the required singularities at the edge. Using Senior's method, R. Teisseyre received a solution to the problem of dipole radiation diffraction by a wedge.

As is seen, Petykiewicz's method gives the possibility of obtaining results in agreement with the above mentioned authors in a manner which is direct and mathematically quite simple.

This paper had already been written when Professor W. Rubinowicz turned the author's attention to the paper by G. D. Maluzhinets and A. A. Tuzhilin (1962), which also presents a solution to the problem of diffraction of electromagnetic dipole radiation by a wedge. Very briefly, the method that they used consists in postulating the branching of the solution for a Hertz vector with an unknown vector function $\boldsymbol{\pi}(\alpha)$, corresponding in our solution to the function $\boldsymbol{C}\Phi_{\pi/\chi}(\alpha \pm \varphi_0)$. The form of this function is obtained by solving three integral equations.

The author expresses his gratitude to Professor W. Rubinowicz for comments and discussion during work on this problem, and to Dr J. Petykiewicz also for making the results of his work available.

APPENDIX I

Proof of fulfilment of Sommerfeld's condition

Sommerfeld's condition of finiteness stipulates that

$$\lim_{r\to\infty} r\,\boldsymbol{E} = \text{finite} \tag{I.1}$$

$$\lim_{r\to\infty} r\,\boldsymbol{H} = \text{finite}$$

On the other hand, the emission conditions necessitates:

$$\lim_{r\to\infty} r\,(\boldsymbol{E}+r^*\times\boldsymbol{H}) = 0 \tag{I.2}$$

$$\lim_{r\to\infty} r\,(\boldsymbol{H}-r^*\times\boldsymbol{E}) = 0$$

where $\boldsymbol{r}^*$ denotes the propagation direction of the wave in infinity.

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We shall show that Eq. (2.4) and, hence, Eq. (2.9) fulfill these conditions. For this, let us notice first that the so-called geometrical wave, Eq. (2.7), is ordinary dipole radiation from the sources $L_m(\varrho, \varphi_0+2m\chi, z_0)$ and as such it must satisfy Sommerfeld's conditions. All that remains to be proved, therefore, is that the diffracting field, Eq. (2.8), possesses the same property. From the formula (3.1) for points distant from the shadow boundary we have:

$$\lim_{r\to\infty} r\boldsymbol{e}^d = \frac{ke^{i\frac{3}{4}\pi}\sqrt{2\pi}\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\frac{e^{-ikR_0}}{\sqrt{k\varrho}}\boldsymbol{c}$$

$$\lim_{r\to\infty} r\boldsymbol{h}^d = \frac{ke^{i\frac{3}{4}\pi}\sqrt{2\pi}\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\frac{e^{-ikR_0}}{\sqrt{k\varrho}}\boldsymbol{d}$$

where:

$$c_r = 0,\ c_\varphi = u_x\sin\varphi_0-u_y\cos\varphi_0,\ c_z = u_z$$

$$d_r = 0,\ d_\varphi = -u_z,\ d_z = u_x\sin\varphi_0-u_y\cos\varphi_0$$

and, from Eq. (3.2), for points near the shadow boundary:

$$\lim_{r\to\infty} r\boldsymbol{e}^d = \frac{ke^{ikR_0}}{2}\operatorname{sgn}\psi\boldsymbol{c}$$

$$\lim_{r\to\infty} r\boldsymbol{h}^d = \frac{ke^{-ikR_0}}{2}\operatorname{sgn}\psi\boldsymbol{d}$$

Hence, the condition of finiteness is satisfied and, moreover, we see that at infinity the wave travels in the direction of the unit vector $\boldsymbol{i}_r$. The condition (I.2) will thus assume the form:

$$\lim r(\boldsymbol{e}^d+\boldsymbol{i}_r\times\boldsymbol{h}^d) = 0$$

$$\lim r(\boldsymbol{h}^d-\boldsymbol{i}_r\times\boldsymbol{e}^d) = 0$$

and, because $\boldsymbol{i}_r\times\boldsymbol{d} = -\boldsymbol{c}$ and $\boldsymbol{i}_r\times\boldsymbol{c} = \boldsymbol{d}$, it is satisfied both near to and far from the shadow boundary.

APPENDIX II

Proof of fulfilment of boundary conditions

The boundary condition states that the components $\mathcal{E}_r$, $\mathcal{E}_z$ and $\mathcal{H}_\varphi$ of the general solution (2.9) must vanish at both surfaces of the wedge, $\varphi = 0$ and $\varphi = \chi$. We shall prove this for the component $\mathcal{E}_r$. Using Eq. (2.1) we have:

$$\mathcal{E}_r = \frac{1}{2\chi}\int\limits_{B_1+B_2}(\{u_\varrho[r\cos(\varphi-\alpha)-\varrho]+u_{\varphi_0}r\sin(\varphi-\alpha)+u_z(z-z_0)\}B(R_\alpha)[r-\varrho\cos(\varphi-\alpha)]+$$
$$+[A(R_\alpha)+k^2f(R_\alpha)][u_\varrho\cos(\varphi-\alpha)+u_{\varphi_0}\sin(\varphi-\alpha)])\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha-$$

$$-\frac{1}{2\pi}\int\limits_{B_1+B_2}(\{u'_\varrho[r\cos(\varphi-\alpha)-\varrho]+u'_{\varphi_0}r\sin(\varphi-\alpha)+u_z(z-z_0)\}B(R_\alpha)[r-\varrho\cos(\varphi-\alpha)]+$$
$$+[A(R_\alpha)+k^2f(R_\alpha)][u'_\varrho\cos(\varphi-\alpha)+u_{\varphi_0}\sin(\varphi-\alpha)])\Phi_{\pi/\chi}(\alpha+\varphi_0)d\alpha$$

For the half-plane $\varphi = 0$ we have:

$$\mathcal{E}_r|_{\varphi=0} = \frac{1}{2\pi}\int\limits_{B_1+B_2}(\{u_\varrho[r\cos\alpha-\varrho]-u_{\varphi_0}r\sin\alpha+u(z-z_0)\}B(R^0_\alpha)[r-\varrho\cos\alpha]+$$
$$+[A(R^0_\alpha)+k^2f(R^0_\alpha)]\;(u_\varrho\cos\alpha-u_{\varphi_0}\sin\alpha))\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha-$$
$$-\frac{1}{2\pi}\int\limits_{B_1+B_2}(\{u'_\varrho[r\cos\alpha-\varrho]-u'_{\varphi_0}r\,\mathrm{cin}\,\alpha+u'_z(z-z_0)\}B(R^0_\alpha)[r-\varrho\cos\alpha]+$$
$$+[A(R^0_\alpha)+k^2f(R^0_\alpha)]\;(u'_\varrho\cos\alpha-u'_{\varphi_0}\sin\alpha))\Phi_{\pi/\chi}(\alpha+\varphi_0)d\alpha \tag{II 1}$$

where

$$R^0_\alpha = \sqrt{r^2+\varrho^2+(z-z_0)^2-2r\varrho\cos\alpha}$$

Let us notice above all that:

$$u'_\varrho = u'_x\cos(+\varphi_0)+u'_y\sin(-\varphi_0) = u_x\cos\varphi_0+u_y\sin\varphi_0 = u_\varrho \tag{II.2}$$
$$u'_{\varrho_0} = -u'_x\sin(-\varphi_0)+u'_y\cos(-\varphi_0) = u_x\sin\varphi_0-u_y\cos\varphi_0 = -u_{\varphi_0}$$
$$u'_z = u_z$$

If in the second integral of Eq. (II.1) we now make the substitution $\alpha = -\alpha'$, then at the same time the path of integration B, will convert into B_2 and vice versa, as follows from Fig. 2. Moreover, making use of Eq. (II.2) we receive:

$$\mathcal{E}_r|_{\varphi=0} = \frac{1}{2\chi}\int\limits_{B_1+B_2}(\{u_\varrho[r\cos\alpha-\varrho]-u_{\varphi_0}r\sin\alpha+u_z(z-z_0)\}B(R^0_\alpha)[r-\varrho\cos\alpha]+$$
$$+[A(R^0_\alpha)+k^2f(R^0_\alpha)]\;(u_\varrho\cos\alpha-u_{\varphi_0}\sin\alpha))\;[\Phi_{\pi/\chi}(\alpha-\varphi_0)+\Phi_{\pi/\chi}(-\alpha+\varphi_0)]d\alpha$$

where:

$$\Phi_{\pi/\chi}(\alpha-\varphi_0)+\Phi_{\pi/\chi}(-\alpha+\varphi_0) = \frac{1}{1-e^{i\pi(\varphi_0-\alpha)/\chi}}+\frac{1}{1-e^{-i\pi(\varphi_0-\alpha)/\chi}} = 1$$

Hence, the integrand now does not have any poles and, therefore, the path B_1+B_2 can be exchanged by C_1+C_2:

$$\mathcal{E}_r|_{\varphi=0} = \frac{1}{2\chi}\int\limits_{C_1+C_2}\{[u_\varrho(r\cos\alpha-\varrho)-u_{\varphi_0}r\sin\alpha+u_z(z-z_0)]B(R^0_\alpha)[r-\varrho\cos\alpha]+$$
$$+[A^0_\alpha(R^0_\alpha)+k^2f(R^0_\alpha)]\;[u_\varrho\cos\alpha-u_{\varphi_0}\sin\alpha]\}\;d\alpha$$

This integral vanishes, however, for all the function of α appearing in the integrand have a period of 2π, whereas the paths C_1 and C_2 are shifted with respect to each other along the real axis by 2π and integration along them runs in opposite directions.

In order to prove that $\mathcal{E}_r$ vanishes at the half-plane $\varphi = \chi$ suffice it to make the substitution $\alpha' = -\alpha+2\chi$ in the second term of the expression $\mathcal{E}_r|_{\varphi=\chi}$, analogous to Eq. (II.1), and the proof proceeds as in the case just outlined. The proof that the components $\mathcal{E}_z$ and $\mathcal{H}_\varphi$ vanish at both wedge surfaces is identical.

APPENDIX III

Proof of fulfilment of the edge condition

In order to see how the field components behave near the wedge edge, we shall find their asymptotic approximations for small r with the use of the method applied by Franz in the scalar problem of an isotropic spherical wave (*cf.* Rubinowicz 1966). Let us notice that the branch points of the α-plane, determined by the zeros of the function $R_\alpha = \sqrt{r^2+\varrho^2+(z-z_0)^2-2r\varrho\cos(\varphi-\alpha)}$ are defined by:

$$\alpha_n^{\pm} = \varphi+2n\pi\pm ib_0 \; (n = 0, \pm1, \pm2, \ldots)$$

where b_0 is a real and positive root of the equation

$$\cosh b = \frac{r^2+\varrho^2+(z-z_0)^2}{2r\varrho}$$

These branch points shift towards infinity along the imaginary axis α when $r\to 0$. Each of the integration paths B_1 or B_2 can be deformed into a segment of length 2π, parallel to the real axis α and two half-rays, parallel to the imaginary axis, emerging from the ends of that segment. They will pass to infinity when $r\to 0$ along the imaginary axis α, together with the points $\alpha_n^{\pm}$. Moreover, we can then assume that:

along the path B_1:

$$\cos(\varphi-\alpha) = \frac{1}{2}\left\{e^{i(\varphi-\alpha_r)-\alpha_i}+e^{-i(\varphi-\alpha_r)-\alpha_i}\right\} \simeq \frac{1}{2}\, e^{i(\varphi-\alpha_r)+\alpha_i}$$

$$\sin(\varphi-\alpha) = \frac{1}{2i}\, e^{i(\varphi-\alpha)}$$

along the path B_2:

$$\cos(\varphi-\alpha) = \frac{1}{2}\, e^{-i(\varphi-\alpha)}$$

$$\sin(\varphi-\alpha) = -\frac{1}{2i}\, e^{-i(\varphi-\alpha)} \tag{III.1}$$

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We then find the asymptotic approximation for small r for the component e_r from Eqs (2.1) and (III.1)

$$e_r \cong \frac{1}{2\chi}\int_{B_1}\left(\left\{u_\varrho\left[\frac{re^{i(\varphi-\alpha)}}{2}-\varrho\right]+u_{\varphi_0}\frac{re^{i(\varphi-\alpha)}}{2i}+u_2(z-z_0)\right\}B(R_\alpha^*)\left[r-\frac{\varrho e^{i(\varphi-\alpha)}}{2}\right]+\right.$$
$$\left.+\,[A(R_\alpha^*)+k^2f(R_\alpha^*)](u_\varrho-iu_{\varphi_0})\frac{e_{i(\varphi-\alpha)}}{2}\right)\frac{1}{1-e^{i\frac{\pi}{\chi}(\varphi_0-\alpha)}e^{i\frac{\pi}{\chi}(\varphi-\alpha)}}\,d\alpha+$$
$$+\frac{1}{2\chi}\int_{B_2}\left(\left\{u_\varrho\left[\frac{re^{-i(\varphi-\alpha)}}{2}-\varrho\right]-u_{\varphi_0}\frac{re^{i-(\varphi-\alpha)}}{2i}+u_2(z-z_0)\right\}(R_\alpha^{**})\left[r-\frac{\varrho e^{i-(\varphi-\alpha)}}{2}\right]+\right.$$
$$\left.+[A(R_\alpha^{**})+k^2f(R_\alpha^{**})](u_\sigma+iu_{\varphi_0})\frac{e^{-(\varphi-\alpha)}}{2}\right)\frac{1}{1-e^{i\frac{\pi}{\chi}(\varphi_0-\varphi)}e^{i\frac{\pi}{\chi}(\varphi-\alpha)}}\,d\alpha$$

where:

$$R_\alpha^* = \sqrt{r^2+\varrho^2+(z-z_0)^2-r\varrho\, e^{i(\varphi-\alpha)}}$$
$$R_\alpha^{**} = \sqrt{r^2+\varrho^2+(z-z_0)^2-r\varrho e^{-i(\varphi-\alpha)}}$$

In the integrals over the paths B_1 and B_2 we now take the appropriate substitutions:

$$\sigma = r\varrho e^{i(\varphi-\alpha)} \qquad \text{(III.2a)}$$

$$\sigma = r\varrho e^{-i(\varphi-\alpha)} \qquad \text{(III.2b)}$$

The component e_r can then be written as

$$e_r = -\frac{1}{2\chi}\int_U\left(\left\{u_\sigma\left[\frac{\sigma}{2\varrho}-\varrho\right]+u_{\varphi_0}\frac{\sigma}{2i\varrho}+u_2(z-z_0)\right\}B(R_\varphi)\left[r-\frac{\sigma}{2r}\right]+\right.$$
$$\left.+[A(R_\sigma)+k^2(R_\sigma)](u_\varrho-iu_{\varphi_0})\frac{\sigma}{2r\varrho}\right)\frac{1}{1-e^{i\frac{\pi}{\chi}(\varphi_0-\varphi)}\sigma^{\frac{\pi}{\chi}}(r\varrho)^{-\frac{\pi}{\chi}}}\frac{d\sigma}{i\sigma}+$$
$$+\frac{1}{2\chi}\int_U\left(\left\{u_\sigma\left[\frac{\sigma}{2\varrho}-\varrho\right]-u_{\varphi_0}\frac{\sigma}{2i\varrho}+u_2(z-z_0)\right\}B(R_\sigma)\left[r-\frac{\sigma}{2r}\right]+\right.$$
$$\left.+[A(R_\sigma)+k^2f(R_\sigma)]\,(u_\varrho+iu_{\varphi_0})\frac{\sigma}{2r\varrho}\right)\frac{1}{1-e^{i\frac{\pi}{\chi}(\varphi-\varphi_0)}\sigma^{\frac{\pi}{\chi}}(r\varrho)^{\frac{\pi}{\chi}}}\frac{d\sigma}{i\sigma} \qquad \text{(III.3)}$$

where

$$R_\sigma = \sqrt{r^2+\varrho^2+(z-z_0)^2-\sigma}$$

With the substitutions (III.2a) or (III.2b) the branching half-rays in the α-plane, emerging from the points α_0^+ or α_0^-, respectively, are transformed in the σ -plane into a half-ray emerging from the point $\sigma_0 = r^2+\varrho^2+(z-z_0)^2$ and running to infinity along the positive real half-axis σ. To both paths B_1 and B_2 there corresponds a path U in the σ-plane composed of a circle around the point $\sigma=0$ and two half-rays running to infinity on both sides of the negative real half-axis σ (Fig. 4).

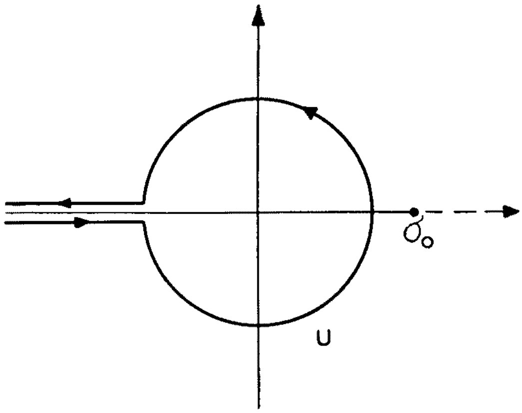

Fig. 4. The σ-plane, σ_0 is the branch point of R_σ, the dashed line in the branching half-ray

If we now consider that for small r we can assume

$$r-\frac{\sigma}{r} \cong -\frac{\sigma}{r}$$

$$1-e^{i\frac{\pi}{\chi}(\varphi-\varphi_0)}\sigma^{\frac{\pi}{\chi}}(r\varrho)^{-\frac{\pi}{\chi}} \cong -e^{i\frac{\pi}{\chi}(\varphi-\varphi)}\sigma^{\frac{\pi}{\chi}}(r\varrho)^{-\frac{\pi}{\chi}}$$

$$\frac{1}{1-e^{i\frac{\pi}{\chi}(\varphi_0-\varphi)}\sigma^{-\frac{\pi}{\chi}}(r\varrho)^{\frac{\pi}{\chi}}} \cong 1+e^{i\frac{\pi}{\chi}(\varphi_0-\varphi)}\sigma^{-\frac{\pi}{\chi}}(r\varrho)$$

we get for Eq. (III.3)

$$e_r \cong \frac{i}{4\chi}(r\varrho)^{\frac{\pi}{\chi}-1}\int\limits_{\dot{U}}\left(\left\{2u_\varrho\cos(\varphi-\varphi_0)\left(\frac{\sigma}{2\varrho}-\varrho\right)+\frac{u_{\varphi_0}\sin(\varphi-\varphi_0)}{\varrho}+\right.\right.$$

$$\left.\left.+2u_z\cos(\varphi-\varphi_0)\right\}\varrho B(R_\sigma)-[A(R_\sigma)+k^2f(R_\sigma)]\,[u_\varrho\cos(\varphi-\varphi_0)+u_{\varphi_0}\sin(\varphi-\varphi_0)]\right)\frac{d\sigma}{\sigma^{\frac{\pi}{\chi}}}+$$

$$+\frac{i}{4\chi r\varrho}\int\limits_{\dot{U}}\left(\left\{u_\varrho\left(\frac{\sigma}{2\varrho}-\varrho\right)-\frac{u_{\varphi_0}\sigma}{2i\varrho}+u_2(z-z_0)\right\}\varrho B(R_\varrho)-\right.$$

$$\left.+[A(R_\sigma)+k^2f(R_\sigma)]\,(u_\varrho+iu_{\varphi_0})\right)d\sigma$$

The first integral cannot be calculated because the path U has inside it a branch point of the factor $\sigma^{\frac{\pi}{\chi}}$, but it is independent of r. The integrand of the second integral does not have any branch points inside U and the integration path is reduced to the circle around

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the point $\sigma = 0$, as the contributions from the half-rays cancel out. Moreover, since there are no poles inside this circle, the integral vanishes in accordance with Cauchy's theorem.

We see thus that near the edge the component E_r, Eq. (2.9), will behave like $r^{\pi/\chi-1}$. Proceeding likewise with the remaining components we get

$$\mathcal{E}_r \sim r^{\frac{\pi}{\chi}-1} \qquad \mathcal{H}_r \sim r^{\frac{\pi}{\chi}-1}$$

$$\mathcal{E}_\varphi \sim r^{\frac{\pi}{\chi}-1} \qquad \mathcal{H}_\varphi \sim r^{\frac{\pi}{\chi}-1}$$

$$\mathcal{E}_z \sim r^{\frac{\pi}{\chi}} \qquad \mathcal{H}_z - (\mathcal{H}_z)_{r=0} \sim r^{\frac{\pi}{\chi}}$$

The Meixner condition stipulates that the field energy density $w = \frac{1}{8\pi}(\mathcal{E}^2 + \mathcal{H}^2)$ must be spatially integrable near the egde. The squares of the field components must have, therefore, singularities not stronger than $r^{-2\alpha}$, where $\alpha < 1$. In our case we have $w \sim r^{\frac{2\pi}{\chi}-2}$, which ensures satisfaction of the postulated condition.

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