POLARIZATION PARAMETERS AND RESONANCE DECAY DISTRIBUTIONS IN HIGH-ENERGY TWO-BODY PROCESSES. I. BARYON-BARYON COLLISIONS

By Z. Gołąb and A. Kotański

Institute of Physics, Jagellonian University, Cracow*

(Received May 8, 1967)

The cross sections, polarization parameters and resonance decay distributions for the simplest types of two-body reactions are expressed in terms of independent transversity amplitudes. Crossing relations are explicitly written down for each type of process. Implications of discrete symmetries are taken into account. The case of a polarized target is also considered. Finally, the discussion of the number of independent parameters, which can be found by performing various measurements, is presented for each reaction.

1. *Introduction*

In the present paper we review the simplest types of two-body high-energy reactions and try to express various measurable quantities (as *e. g.* cross sections, polarization parameters, and resonance decay distributions) in terms of independent spin amplitudes. Possible experiments with the polarized target are also discussed. We think that this paper may be regarded as the first step in the systematic phenomenological analysis of these processes.

We use the transversity amplitudes throughout the work (*cf.* Kotański 1966a and b, quoted respectively as *A* and *B*) and we find it rather convenient as the relations implied by parity conservation, the transformation law from the centre-of-mass to the laboratory frame, and especially the crossing relations are simple in the transversity basis. On the other hand, we do not need the partial-wave expansion which seems to be rather complicated for transversity amplitudes.

In Sec. 2 we compile the general formulae to be used in next sections. Further on we discuss the following types of reactions (the numbers are the spins of particles involved, all parities are taken positive)

$$\frac{1}{2}+\frac{1}{2}\rightarrow\frac{1}{2}+\frac{1}{2}, \tag{1.1}$$

* Address: Instytut Fizyki UJ, Kraków 16, Reymonta 4, Polska.

$$\frac{1}{2}+\frac{1}{2}\rightarrow\frac{1}{2}+\frac{3}{2}, \tag{1.2}$$

$$\frac{1}{2}+\frac{1}{2}\rightarrow\frac{3}{2}+\frac{3}{2}. \tag{1.3}$$

The discussion of meson-baryon two-body reactions will be deal with in another paper.

For each reaction we write down

(*i*) the independent amplitudes which remain after taking parity conservation into account,

(*ii*) the *s*-*t* and *s*-*u* crossing relations,

(*iii*) relations between the amplitudes due to discrete symmetries other than parity (if any exist),

(*iv*) spin density matrices (simple and joint) for produced resonances,

(*v*) polarization parameters of spin-1/2 particles,

(*vi*) quantities enumerated under (*iv*) and (*v*), but when the target is polarized, unless the detailed expressions are too lengthy.

Finally, the number of independent real parameters (related to complex amplitudes) which can be found by performing various measurements is given in each case. Unfortunately many of interesting polarization measurements cannot be performed as yet.

2. *Basic formulae and notation*

We present here the general formulae which will be necessary in subsequent calculations. Some of them were derived in our previous papers (*A* and *B*).

The transversity amplitudes for a two-body process $a+c\rightarrow b+d$ are related to the helicity amplitudes by the formula

$$\mathcal{T}_{bd,ac}=\sum_{\text{primed}}u^*(s_b)_{bb'}u^*(s_d)_{dd'}\mathcal{H}_{b'd'a'c'}u(s_a)_{a'a}u(s_c)_{c'c} \tag{2.1}$$

where a, b, c, d denote particle transversities, primed indices are helicities, s_x is the spin of particle x, and matrices $u(s)$ are defined as

$$u(s)_{ab}=D^s\left(\frac{\pi}{2},\frac{\pi}{2},-\frac{\pi}{2}\right)_{ab}. \tag{2.2}$$

Explicit forms of these matrices together with their symmetry properties can be found in *A* or *B*.

We found it convenient to project the spins of antiparticles in the opposite direction to those particles (*cf.* *B*).

Therefore if there are any antiparticles in our process, we use in Eq. (2.1) $u^*(s)$ if the antiparticle s comes in or $u(s)$ if it goes out. Since

$$u^*(s)_{a,b}=e^{-i\pi s}u(s)_{a,-b}, \tag{2.3}$$

this means changing the sign of transversity of the antiparticle and a possible change of an over-all phase-factor.[1]

---

[1] Even for bosons identical with their antiparticles the crossing relations are somewhat simplified if we use the "antiparticle" convention for the transversity sign and over-all phase of the crossed particle.

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1

The spin-density matrices in the two bases are connected by the relation

$$\varrho^T = u^* \varrho^H u \tag{2.4}$$

for a particle and, according to our convention, by the relation

$$\varrho^T = u \varrho^H u^* \tag{2.5}$$

for antiparticles. This for instance the density matrices of a polarized spin-1/2 particle are

$$\varrho^H = \frac{1}{2}\begin{pmatrix} 1+P^h_z, & P^h_x - iP^h_g \\ P^h_x + iP^h_y, & 1-P^h_z \end{pmatrix} \text{ (helicity)} \tag{2.6}$$

and

$$\varrho^T = \frac{1}{2}\begin{pmatrix} 1+P^h_y, & P^h_x + iP^h_z \\ P^h_x - iP^h_z, & 1-P^h_y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1+P_n, & iP_p e^{-i\alpha} \\ -iP_p e^{i\alpha}, & 1-P_n \end{pmatrix} \text{ (transversity).} \tag{2.7}$$

Here P_n is the target polarization along the normal to the reaction plane, P_p is its polarization in the scattering plane, and α is the angle between $\vec{P}_p$ and the beam direction.

Usually the helicity frame of references for a particle taking part in a two-body process is chosen in the following way: the z-axis is parallel to the three-momentum of the particle (or to the reversed three-momentum of the centre of mass system if the particle is at rest) and the y-axis is parallel to the normal to the reaction plane (or exactly to $\vec{p}_1 \times \vec{p}_3$ for the reaction $1+2 \to 3+4$). Formula (2.7) means that in our conventions the transversity frame is constructed in the following way:

$$\begin{aligned} z_t &= y_h \quad \text{— along the normal,} \\ y_t &= -z_h \quad \text{antiparallel to the momentum,} \\ x_t &= x_h \quad \text{— to form a right-handed system} \end{aligned} \tag{2.8}$$

(there is a mistake in B in this point). Similarly for an antiparticle we have

$$\bar{z}_t = -y_h, \; \bar{y}_t = z_h, \; \bar{x}_t = x_h. \tag{2.9}$$

These three frames of reference are displayed in Fig. 1. This detailed discussion was necessary as all angular distributions and all values of polarizations are referred below to the transversity frame.

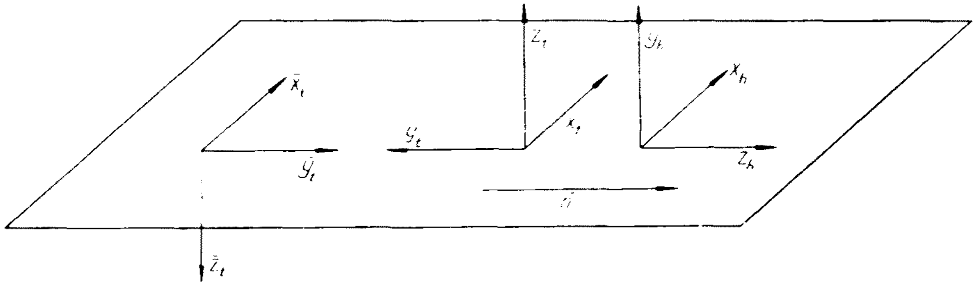

Fig. 1

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1

We present now our notation for the amplitudes

$$s\text{-channel}, \quad a+c \to b+d, \quad \text{amplitude} \quad G_{bd,ac'}, \tag{2.10a}$$

$$t\text{-channel}, \quad a+\bar{b} \to \bar{c}+d, \quad \text{amplitude} \quad F_{cd,ab'}, \tag{2.10b}$$

$$u\text{-channel}, \quad \bar{b}+c \to \bar{a}+d, \quad \text{amplitude} \quad H_{ad,bc'}. \tag{2.10c}$$

By convention, particles $\bar{b}$, $\bar{c}$ in the t-channel and particles $\bar{a}$, $\bar{b}$ in the u-channel are always treated as antiparticles — the convention of Eq. (2.3) is used. The crossing relations for our amplitudes are

$$G_{bd,ac} = (-1)^{b+c} e^{i(\psi_b b - \psi_d d - \psi_a a + \psi_c c)} F_{-cd,a-b}, \tag{2.11}$$

$$G_{bd,ac} = (-1)^{a+c} e^{i(\bar{\psi}_c c + \bar{\psi}_d d - \bar{\psi}_a a - \bar{\psi}_b b)} H_{-ad,-bc}. \tag{2.12}$$

In each case they are determined only to a transversity-independent phase-factor (*cf.* Svensson 1966). Formulae for the crossing angles ψ and $\bar{\psi}$ can be found in the papers by Trueman

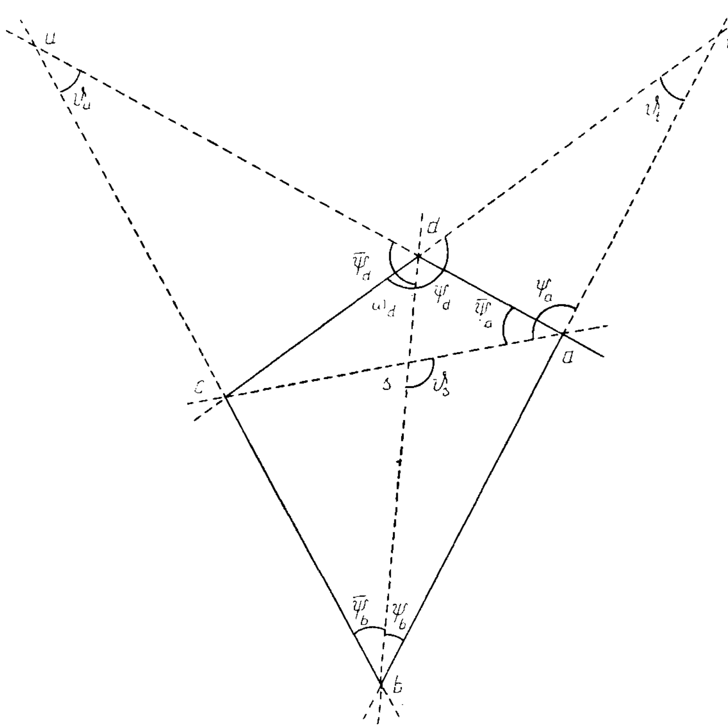

Fig. 2

and Wick (1964 — their notation in $\psi_1 = \psi_a, \psi_2 = \psi_b, \chi_1 = \psi_c, \chi_2 = \psi_d$) and by Białas and Svensson (1966). These angles are shown in the velocity diagram (Fig. 2).

Invariance properites of the interactions imply relations between amplitudes. Space-reflection invariance implies that in all processes considered here half of the transversity amplitudes vanish

$$G_{bd,ac} = \eta\,(-1)^{a+c-b-d} G_{bd,ac}. \tag{2.13}$$

Here $\eta = \eta_a \eta_b \eta_c \eta_d (-1)^{2\bar{s}}$ depends on the intrinsic parities η_x of the particles; s is the sum of the spins of all the antifermions involved. Note that for the majority of our reactions $\eta = +1$ and then the sum of transversities is conserved modulo 2. Besides, η has the same value for all of the reactin channels.

In a similar way, from time-reversal invariance we obtain

$$G_{bd,ac} = G^T_{ac,bd} \tag{2.14}$$

and from invariance under charge conjugation

$$G_{bd,ac} = G^C_{-b-d,-a-c}\,. \tag{2.15}$$

Here G^T is the amplitude for the time-reversed process and G^C — for the charge-conjugated process. This means that transversity does not change under time-reversal but changes its sign under charge conjugation.

By simply exchanging both incoming and outgoing particles we obtain the relation

$$G_{bd,ac} = G^E_{db,ca}\,. \tag{2.16}$$

For n-p scattering we also need the relation obtained by using an isospin rotation I which changes proton into neutron and neutron into proton

$$G_{bd,ac} = G^I_{bd,ac}\,. \tag{2.17}$$

Our Eqs (2.13)—(2.17) can also be rewritten for the t- and u-channel amplitudes. All these relations will be used to obtain independent amplitudes.

The spin density matrices of produced resonances can be expressed by the amplitudes. We quote here only two formulae: the simplest one for a resonance produced by scattering on an unpolarized target

$$\varrho_{dd'} = N_1 \sum_{abc} G_{bd,ac} G^*_{bd',ac} \tag{2.18}$$

and the most complicated for the joint density matrix of two resonances produced on a polarized target

$$\varrho_{bd,b'd'} = N_2 \sum_{acc'} G_{bd,ac} \varrho_{cc'} G_{b'd',ac'}\,. \tag{2.19}$$

Coefficients N_1 and N_2 are to be found from the condition Tr $\varrho = 1$. $\varrho_{cc'}$ is a density matrix which describes the target.

If a resonance is produced in a parity-conserving reaction with unpolarized initial particles, its transversity density matrix satisfies the equation

$$\varrho_{mm'} = (-1)^{m-m'} \varrho_{mm'} \tag{2.20}$$

i. e. its every second element vanishes. This is also valid in the case of a target polarized along the normal to the reaction plane. The joint density matrices have an analogous property

$$\varrho_{mn,m'n'} = (-1)^{m+n,-m'-n'} \varrho_{mn,m'n'}\,. \tag{2.21}$$

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1

The angular distribution of a resonance two-body decay can be determined by using the following formula (*cf.* Gottfried and Jackson 1964, also *B*, Eq. 3.1)

$$W(\vartheta,\varphi)=N\sum_{\alpha,\beta,m,m'}|M(\alpha,\beta)|^2\,e^{i(m-m')\varphi}d^s(\vartheta)_{m,\alpha-\beta}\,d^s(\vartheta)_{m',\alpha-\beta}\varrho_{mm'}\tag{2.22}$$

where angles ϑ, φ are measured in the transversity frame, $M(\alpha,\beta)$ are decay coupling constants which fulfil the relation

$$|M(\alpha,\beta)|=|M(-\alpha-\beta)|\tag{2.23}$$

if parity is conserved in the decay, and α, β are helicities of the decay products. N is a normalization coefficient which can be found from the condition

$$\int d\varphi d\,(\cos\vartheta)\,W(\vartheta,\varphi)=1\tag{2.24}$$

and therefore

$$N\sum_{\alpha,\beta}|M(\alpha,\beta)|^2=\frac{2s+1}{4\pi}\,.\tag{2.25}$$

The explicit formulae for the angular distributions for several decays are given in *B* (there are some errors in this reference — all the terms with $\sin n\varphi$ should have the reversed signs). A formula similar to Eq. (2.26) is also valid for the joint decay distribution (*cf.* Pilkuhn and Svensson 1965).

It is easily seen that if Eq. (2.20) is satisfied then the decay distribution (2.22) does not depend on $\varrho_{mm'}-\varrho_{-m',-m}$ but only on $\varrho_{m,m'}+\varrho_{-m',-m}$. This is another way of expressing the familiar fact that one cannot measure all the elements of a density matrix merely from the two-body decay distributions. This is true even if Eq. (2.20) is not fulfilled (target polarized not along the normal).
One can find then $\varrho_{m,m'}+\varrho_{-m',-m}$ for even $m-m'$ and $\varrho_{m,m'}-\varrho_{-m',-m}$ for odd $m-m'$.

Note that if the helicity amplitudes have equal phases (as is the case one-particle or one Regge-pole exchange, *cf.* Białas and Kotański 1966) then the density matrix in the transversity basis has the property

$$\varrho_{m,m'}=\varrho_{-m',-m}\tag{2.26}$$

(with target unpolarized) because then the transversity amplitudes satisfy the relation (cf. B)

$$G_{-b-d,-a-c}=(-1)^{2\bar{s}+s_a+s_c-s_b-s_d}G_{bd,ac}\tag{2.27}$$

where $\bar{s}$ is the sum of the spins of all antiparticles taking part in the reaction.

The transformation from CMS to LAB is fairly simple for the transversity density matrices. It has the form

$$\varrho^{\mathrm{LAB}}_{dd'}=e^{i\omega_d(d-d')}\varrho_{dd'}\tag{2.28}$$

and, for the particle b

$$\varrho^{\mathrm{LAB}}_{bb'}=e^{i\omega_b(b-b')}\varrho_{bb'}\tag{2.29}$$

The Wigner angles ω_b and ω_d are connected with the crossing angles (*cf.* Fig. 2)

$$\omega_b = \overline{\psi}_b, \qquad \omega_d = \pi - \psi_d. \tag{2.30}$$

Finally, the differential cross section (for an unpolarized target) in the s-channel is in our conventions

$$\frac{dt}{d\sigma} = \frac{1}{16\pi\lambda(s, m_a^2, m_c^2)} \frac{1}{(2s_a+1)(2s_c+1)} \sum_{abcd} |G_{bd,ab}|^2 \tag{2.31}$$

where

$$\lambda(a, b, c) = a^2+b^2+c^2-2(ab+bc+ac). \tag{2.32}$$

3. *Reactions of the type* $1/2^+ + 1/2^+ \to 1/2^+ + 1/2^+$

Apart from the nucleon-nucleon scattering we have here also the production of the spin 1/2 isobars. We shall consider in detail the nucleon-nucleon scattering which is more complicated as it is subject to more symmetries and, on the other hand, simpler because the equality of masses implies simple crossing relations.

In the general case we have eight independent amplitudes in the s-channel which may be labelled $G(bd, ac) = G_k$ where

$$G_1 = G(++,++),\ G_2 = G(++,--),\ G_3 = G(+-,+-),\ G_4 = G(+-,-+),$$
$$G_5 = G(-+,+-),\ G_6 = G(-+,-+),\ G_7 = G(--,++),\ G_8 = G(--,--). \tag{3.1}$$

For the proton-proton and neutron-proton scattering, there are additional relations between the amplitudes and only five independent ones remain. In the table below the symmetries in question are written down. The amplitudes in all channels are labelled as in (3.1) (Table I). Notation: T — time reversal, E — exchange of two particles, C — charge conjugation, I — isospin rotation exchanging the proton with the neutron (*cf.* Eqs (2.14—2.17)).

TABLE I

Channel	Symmetries applied for		Relations (same for both reactions)
	$pp \to pp$	$np \to np$	
s	T, E	T, IE	$G_2 = G_7$, $G_4 = G_5$; $G_3 = G_6$
t	CT, CE	CTI, CE	$F_1 = F_8$, $F_3 = F_6$; $F_2 = F_7$
u	T, CE	T, CEI	$H_2 = H_7$, $H_4 = H_5$, $H_1 = H_8$

In the equal-mass case the crossing relations are remarkably simple

$$G_1 = e^{-2i\psi} F_5 = e^{2i\psi} H_6$$
$$G_2 = F_2 = H_2$$

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1

$$G_3 = F_3 = H_8$$
$$G_4 = F_8 = H_4$$
$$G_5 = F_1 = H_5$$
$$G_6 = F_6 = H_1$$
$$G_7 = F_7 = H_7$$
$$G_8 = e^{2i\psi} F_4 = e^{-2i\bar{\psi}} H_3 \tag{3.2}$$

with

$$\psi = \psi_a = \pi - \psi_b = \pi - \psi_c = \psi_d \tag{3.3}$$

and

$$\bar{\psi} = \pi - \bar{\psi}_a = \pi - \bar{\psi}_b = \bar{\psi}_c = \bar{\psi}_d. \tag{3.4}$$

The crossing angle in this case is

$$\cos\psi = -(-ts)^{1/2}(s-4m^2)^{-1/2}(4m^2-t)^{-1/2},$$
$$\sin\psi = 2m(t+s-4m^2)^{1/2}(s-4m^2)^{-1/2}(4m^2-t)^{1/2} \tag{3.5}$$

and the similar formulae for $\bar{\psi}$ can be obtained from Eq. (3.6) by substituting $t \to u^2$.

We shall discuss now how one can measure all the parameters, *e. g.* in the reaction $np \to np$.

The moduli of the five amplitudes can be found from the differential cross section and the polarizations of both the final particles measured with the target particle first unpolarized and then when polarized along the normal to the scattering plane. In order to get the values of the relative phases between the amplitudes one must measure the polarizations of the final particles when the target is polarized in the scattering plane. All these measurements should be performed for at least two values of the target polarization in each direction. Of course, such experiments seem to be rather difficult at present.

For an unpolarized target we have:

$$\frac{dt}{d\sigma} = \frac{1}{64\,\pi\lambda(s, m_a^2, m_c^2)} (|G_1|^2 + 2|G_2|^2 + 2|G_3|^2 + 2|G_4|^2 + |G_8|^2) \tag{3.6}$$

and the final particle polarization (equal for both particles and directed along the normal) is:

$$P = \frac{1}{(|G_1|^2 + 2|G_2|^2 + 2|G_3|^2 + 2(G_4|^2 + |G_8|^2)} (|G_1|^2 - |G_8|^2). \tag{3.7}$$

For the target polarized along the normal (P_n)

$$\left.\frac{dt}{d\sigma}\right|_n = \frac{1}{64\pi\lambda(s, m_a^2, m_c^2)} [|G_1|^2(1+P_n) + 2|G_2|^2 + 2|G_3|^2 + 2|G_4|^2 + |G_8|^2(1-P_n)] \tag{3.8}$$

---

[2] Incidentally, we see that the crossing relations are comparible with the discrete symmetries (*cf.* the Table I).

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1

and the polarizations of the final particles (still along the normal) are:

$$\left.\frac{d\sigma}{dt}\right|_n P(b)_n = \frac{1}{64\pi\lambda(s, m_a^2, m_c^2)} \{|G_1|^2(1+P_n)-2P_n(|G_2|^2+|G_3|^2-|G_4|^2)$$

$$-|G_8|^2(1-P_n)\}, \tag{3.9}$$

and

$$\left.\frac{d\sigma}{dt}\right|_n P(d)_n = \frac{1}{64\pi\lambda(s, m_a^2, m_c^2)} \{|G_1|^2(1+P_n)-2P_n(|G_2|^2+|G_3|^2-|G_4|^2)-|G_8|(1-P_n)\} \tag{3.10}$$

We see that the moduli of the amplitudes can ben obtained from Eqs (3.6)—(3.10). For the target polarized in an arbitrary direction, *i. e.* with the density matrix (2.7)

$$\varrho_{cc'} = \frac{1}{2}\begin{bmatrix} 1+P_n & iP_p e^{-i\alpha} \\ -iP_p e^{i\alpha} & 1-P_n \end{bmatrix} \tag{3.11}$$

the differential cross section and the normal polarization of the final particles are still given by Eqs (3.8)—(3.10) and their measurement cannot provide any new information.

However the outgoing particles have then a coplanar polarization given by

$$\left.\frac{d\sigma}{dt}\right|_n P_p(b)e^{-i\alpha(b)} = \frac{1}{64\pi\lambda(s, m_a^2, m_c^2)} \{(G_1G_4^*+G_4G_8)e^{-i\alpha}P_p-(G_2G_3^*+G_3G_2^*)e^{i\alpha}P_p\} \tag{3.12}$$

and

$$\left.\frac{d\sigma}{dt}\right|_n P_p(d)e^{-i\alpha(d)} = \frac{1}{64\pi\lambda(s, m_a^2, m_c^2)} \{(G_1G_3^*+G_3G_8)e^{-i\alpha}P_p-(G_2G_4^*+G_4G_2^*)P_pe^{i\alpha}\} \tag{3.13}$$

Subscripts p and n refer to the transversity frame (labelled t in Fig. 1).

We see that from quantities (3.12) and (3.13) (measured for at least two values of α) one can find all relative phases between the five amplitudes.

4. *Type* $1/2^+ + 1/2^+ \to 1/2^+ + 3/2^+$

An example of this type is the spin-3/2 isobar production in nucleon-nucleon collision. There are 16 independent amplitudes, after taking account of parity conservation. We shall label them with numbers. They fulfil the crossing relations:

1. $G(+{}^3/_2,+-) = \exp[i(\psi_b-3\psi_d-\psi_a-\psi_c)/2]\,F(+{}^3/_2,+-)$
$= -\exp[i(-\bar{\psi}_b+3\bar{\psi}_d-\bar{\psi}_a-\bar{\psi}_b)/2]\,H(-{}^3/_2,--)$

2. $G(+{}^3/_2,-+) = -\exp[i(\psi_b-3\psi_d+\psi_a+\psi_c)/2]\,F(-{}^3/_2,--)$
$= \exp[i(-\bar{\psi}_b+3\bar{\psi}_d+\bar{\psi}_a+\bar{\psi}_c)/2]\,H(+{}^3/_2,-+)$

3. $G(++,++) = -\exp[i(\psi_b-\psi_d-\psi_a+\psi_c)/2]\,F(-+,+-)$
$= -\exp[i(-\bar{\psi}_b+\bar{\psi}_d-\bar{\psi}_a+\bar{\psi}_c)/2]\;H(-+.-+)$

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1

$$
\begin{aligned}
&4.\ G(++,--) &&= \exp\,[i(\psi_b-\psi_d+\psi_a-\psi_c)/2]\ F(++,--)\\
& &&= \exp\,[i(-\bar{\psi}_b+\bar{\psi}_d+\bar{\psi}_a-\bar{\psi}_c)/2]\ H(++,--)\\
&5.\ G\ (+-,+-) &&= \exp\,[i(\psi_b+\psi_d-\psi_a-\psi_c)/2]\ F(+-,+-)\\
& &&= -\exp\,[i(-\bar{\psi}_b-\bar{\psi}_d-\bar{\psi}_a-\bar{\psi}_c)/2]\ H(+-,+-)\\
&6.\ (G(+-,-+) &&= -\exp\,[i(\psi_b+\psi_d+\psi_a+\psi_c)/2]\ F(--,--)\\
& &&= \exp\,[i(-\bar{\psi}_b-\bar{\psi}_d+\bar{\psi}_a+\bar{\psi}_c)/2]\ H(+-,-+)\\
&7.\ G(+-{}^3/_2,++) &&= -\exp\,[i(\psi_b+3\psi_d-\psi_a+\psi_c)/2]\ F(--{}^3/_2,+-)\\
& &&= -\exp\,[i(-\bar{\psi}_b-3\bar{\psi}_d-\bar{\psi}_a+\bar{\psi}_c)/2]\ H(--{}^3/_2,-+)\\
&8.\ G(+-{}^3/_2,--) &&= \exp\,[i(\psi_b+3\psi_d+\psi_a-\psi_c)/2]\ F(+-{}^3/_2,--)\\
& &&= \exp\,[i(-\bar{\psi}_b-3\bar{\psi}_d+\bar{\psi}_a-\bar{\psi}_c)/2]\ H(+-{}^3/_2,--)\\
&9.\ G(-{}^3/_2,++) &&= \exp\,[i(-\psi_b-3\psi_d-\psi_a+\psi_c)/2]\ F(-{}^3/_2,++)\\
& &&= \exp\,[i(\bar{\psi}_b+3\bar{\psi}_d-\bar{\psi}_a+\bar{\psi}_c)/2]\ H(-{}^3/_2,++)\\
&10.\ G(-{}^3/_2,--) &&= -\exp\,[i(-\psi_b-3\psi_d+\psi_a-\psi_c)/2]\ F(+{}^3/_2,-+)\\
& &&= -\exp\,[i(\bar{\psi}_b+3\bar{\psi}_d+\bar{\psi}_a-\bar{\psi}_c)/2]\ H(+{}^3/_2,+-)\\
&11.\ G(-+,+-) &&= -\exp\,[i(-\psi_b-\psi_d-\psi_a-\psi_c)/2]\ F(++,++)\\
& &&= \exp\,[i(\bar{\psi}_b+\bar{\psi}_d-\bar{\psi}_a-\bar{\psi}_c)/2]\ H(-+,+-)\\
&12.\ G(-+,-+) &&= \exp\,[i(-\psi_b-\psi_d+\psi_a+\psi_c)/2]\ F(-+,-+)\\
& &&= -\exp\,[i(\bar{\psi}_b+\bar{\psi}_d+\bar{\psi}_a+\bar{\psi}_c)/2]\ H(++,++)\\
&13.\ G(--,++) &&= \exp\,[i(-\psi_b+\psi_d-\psi_a+\psi_c)/2]\ F(--,++)\\
& &&= \exp\,[i(\bar{\psi}_b-\bar{\psi}_d-\bar{\psi}_a+\bar{\psi}_c)/2]\ H(--,++)\\
&14.\ G(--,--) &&= -\exp\,[i(-\psi_b+\psi_d+\psi_a-\psi_c)/2]\ F(+-,-+)\\
& &&= -\exp\,[i(\bar{\psi}_b-\bar{\psi}_d+\bar{\psi}_a-\bar{\psi}_c)/2]\ H(+-,+-)\\
&15.\ G(--{}^3/_2,+-) &&= -\exp\,[i(-\psi_b+3\psi_d-\psi_a-\psi_c)/2]\ F(+-{}^3/_2,++)\\
& &&= \exp\,[i(\bar{\psi}_b-3\bar{\psi}_d-\bar{\psi}_a-\bar{\psi}_c)/2]\ H(--{}^3/_2,+-)\\
&16.\ G(--{}^3/_2,-+) &&= \exp\,[i(-\psi_b+3\psi_d+\psi_a+\psi_c)/2]\ F(--{}^3/_2,-+)\\
& &&= -\exp\,[i(\bar{\psi}_b-3\bar{\psi}_d+\bar{\psi}_a+\bar{\psi}_c)/2]\ H(+-{}^3/_2,++)
\end{aligned}
\tag{4.1}
$$

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1

The isobar density matrix (s channel, unpolarized target) is

$$\varrho(\Delta) = \begin{Vmatrix} |G_1|^2+|G_2|^2+|G_9|^2+|G_{10}|^2, & 0, & G_1G_5^*+G_2G_6^*+G_9G_{13}^*+G_{10}G_{14}^* & 0 \\ 0, & |G_3|^2+|G_4|^2+|G_{11}|^2+|G_{12}|^2, & 0, & G_3G_7^*+G_4G_8^*+G_{11}G_{15}^*+G_{12}G_{16}^* \\ G_5G_1^*+G_6G_2^*+G_{13}G_9^*+G_{14}G_{10}^*, & 0, & |G_5|^2+|G_6|^2+|G_{13}|^2+|G_{14}|^2, & 0, \\ 0, & G_7G_3^*+G_8G_4^*+G_{15}G_{11}^*+G_{16}G_{12}^*, & 0, & |G_7|^2+|G_8|^2+|G_{15}|^2+|G_{16}|^2 \end{Vmatrix} \tag{4.2}$$

where

$$N = 1/\sum_{i=1}^{16} |G_i|^2. \tag{4.2a}$$

The corresponding angular distribution of the decay into two particles of spin 1/2 ans 0 is (*cf. B.* Eq. C. 28)

$$W(\vartheta, \varphi) = \frac{1}{2\pi R}\left[\left(\frac{1}{2}+\sum\right)+3\left(\frac{1}{2}-\sum\right)\cos^2\vartheta - \sqrt{3}\,\sin^2\vartheta\,\mathrm{Re}(Se^{2i\varphi})\right. \tag{4.3}$$

with

$$\sum = |G_1|^2+|G_2|^2+|G_7|^2+|G_8|^2+|G_9|^2+|G_{10}|^2+|G_{15}|^2+|G_{16}|^2, \tag{4.3a}$$

$$R = \sum_{i=1}^{16}|G_i|^2 \tag{4.3b}$$

and

$$S = G_1G_5^*+G_2G_6^*+G_9G_{13}^*+G_{10}G_{14}^*+G_3G_7^*+G_4G_8^*+G_{11}G_{15}^*+G_{12}G_{16}^*. \tag{4.3c}$$

The recoil nucleon polarization is given by:

$$P = N\left(\sum_{n=1}^{8}|G_n|^2-\sum_{n=9}^{16}|G_n|^2\right. . \tag{4.4}$$

One can also measure correlations between isobar and nucleon polarizations, namely the dependence of the recoil-nucleon polarization on the isobar decay angles. It is expressed by the joint density matix

$$\varrho_{bd,b'd'} = N\sum_{ac} G_{bd,ac}G^*_{b'd',ac} \tag{4.5}$$

in the following way:

$$P(\vartheta, \varphi) = \frac{1}{\pi}\sum_{dd'}(\varrho_{\frac{1}{2}d,\frac{1}{2}d'}-\varrho_{-\frac{1}{2}d,-\frac{1}{2}d'})\sum_{m=-\frac{1}{2},\frac{1}{2}} D^{3/2}(\varphi, \vartheta, 0)_{d,m}D^{3/2*}(\varphi, \vartheta, 0)_{d',m}. \tag{4.6}$$

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1

5. *Reaction* $1/2^+ + 1/2^+ \to 3/2^+ + 3/2^+$

Space reflection invariance leaves us in this case with 32 amplitudes in each of the three channels. They fulfil the crossing relations:

$$1.\quad G({}^{3}/_{2}\,{}^{3}/_{2},++) = \exp[-i(\psi+3\chi)] \qquad F(-{}^{3}/_{2},+-{}^{3}/_{2})$$
$$= \exp[i(\bar{\psi}+3\bar{\chi})] \qquad H(-{}^{3}/_{2},-{}^{3}/_{2}+)$$

$$2.\quad G({}^{3}/_{2}\,{}^{3}/_{2},--) = \exp[-i(-\psi+3\chi)] \qquad F(+{}^{3}/_{2},--{}^{3}/_{2})]$$
$$= \exp[i[-\bar{\psi}+3\bar{\chi})] \qquad H(+{}^{3}/_{2},-{}^{3}/_{2}-)$$

$$3.\quad G({}^{3}/_{2}\,{}^{1}/_{2},+-) = \exp[-2i\chi] \qquad F(+{}^{1}/_{2},+-{}^{3}/_{2})$$
$$= \exp[2i\bar{\chi}] \qquad H(-{}^{1}/_{2},-{}^{3}/_{2}-)$$

$$4.\quad G({}^{3}/_{2}\,{}^{1}/_{2},-+) = \exp[-2i\chi] \qquad F(-{}^{1}/_{2},--{}^{3}/_{2})$$
$$= \exp[2i\bar{\chi}] \qquad H(+{}^{1}/_{2},-{}^{3}/_{2}+)$$

$$5.\quad G({}^{3}/_{2}-{}^{1}/_{2},--) = \exp[-i(\psi+\chi)] \qquad F(--{}^{1}/_{2},+-{}^{3}/_{2})$$
$$= \exp[i(\bar{\psi}+\bar{\chi})] \qquad H(--{}^{1}/_{2},-{}^{3}/_{2}+)$$

$$6.\quad G({}^{3}/_{2}-{}^{1}/_{2},--) = \exp[-i(-\psi+\chi)] \qquad F(+-{}^{1}/_{2},--{}^{3}/_{2})$$
$$= \exp[i(-\bar{\psi}+\bar{\chi})] \qquad H(+-{}^{1}/_{2},-{}^{3}/_{2}-)$$

$$7.\quad G({}^{3}/_{2}-{}^{3}/_{2},+-) = \qquad F(+-{}^{3}/_{2},+-{}^{3}/_{2}) = \qquad H(--{}^{3}/_{2},-{}^{3}/_{2}-)$$

$$8.\quad G({}^{3}/_{2}-{}^{3}/_{2},-+) = \qquad F(--{}^{3}/_{2},--{}^{3}/_{2}) = \qquad H(+-{}^{3}/_{2},-{}^{3}/_{2}+)$$

$$9.\quad G({}^{1}/_{2}\,{}^{3}/_{2},+-) = \exp[-2i\chi] \qquad F(+{}^{3}/_{2},+-{}^{1}/_{2}$$
$$= \exp(2i\bar{\chi}) \qquad H(-{}^{3}/_{2},-{}^{1}/_{2}-)$$

$$10.\quad G({}^{1}/_{2}\,{}^{3}/_{2},-+) = \exp[-2i\chi] \qquad F(-{}^{3}/_{2},--{}^{1}/_{2})$$
$$= \exp[2i\bar{\chi}] \qquad H(+{}^{3}/_{2},-{}^{1}/_{2}+)$$

$$11.\quad G({}^{1}/_{2}\,{}^{1}/_{2},++) = \exp[-i(\psi+\chi] \qquad F(-{}^{1}/_{2},+-{}^{1}/_{2})$$
$$= \exp[i(\bar{\psi}+\bar{\chi})] \qquad H(-{}^{1}/_{2},{}^{1}/_{2}+)$$

$$12.\quad G({}^{1}/_{2}\,{}^{1}/_{2},--) = \exp[-i(-\psi+\chi)] \qquad F(+{}^{1}/_{2}\;--{}^{1}/_{2})$$
$$= \exp[i(-\bar{\psi}+\bar{\chi})] \qquad H(+{}^{1}/_{2},-{}^{1}/_{2}-)$$

$$13.\quad G({}^{1}/_{2}-{}^{1}/_{2},+-) = \qquad F(+-{}^{1}/_{2},+-{}^{1}/_{2}) = \qquad H(--{}^{1}/_{2},-{}^{1}/_{2}-)$$

$$14.\quad G({}^{1}/_{2}-{}^{1}/_{2},-+) = \qquad F(--{}^{1}/_{2},--{}^{1}/_{2}) = \qquad H(+-{}^{1}/_{2},-{}^{1}/_{2}+)$$

$$15.\quad G({}^{1}/_{2}-{}^{3}/_{2},++) = \exp[-i(\psi-\chi)] \qquad F(--{}^{3}/_{2},+-{}^{3}/_{2})$$
$$= \exp[i(\bar{\psi}-\bar{\chi})] \qquad H(--{}^{3}/_{2},-{}^{1}/_{2}+)$$

16. $G(-{}^1/_2-{}^3/_2,--) = \exp[i(\psi+\chi)]$ $\quad F(+-{}^3/_2,--{}^1/_2)$
$= \exp[-i(\bar\psi+\bar\chi)] \quad H(+-{}^3/_2,-{}^1/_2-)$

17. $G(-{}^1/_2\,{}^3/_2,++) = \exp[-i(\psi+\chi)]$ $\quad F(-{}^3/_2,+{}^1/_2)$
$= \exp[i(\bar\psi+\bar\chi)] \quad H(-{}^3/_2,\,{}^1/_2+)$

18. $G(-{}^1/_2\,{}^3/_2,--) = \exp[-i(-\psi+\chi)]$ $\quad F(+{}^3/_2,-{}^1/_2)$
$= \exp[i(-\psi+\chi)] \quad H(+{}^3/_2,\,{}^1/_2-)$

19. $G(-{}^1/_2\,{}^1/_2,+-) = \quad F(+{}^1/_2,+{}^1/_2) = H(-{}^1/_2,\,{}^1/_2-)$

20. $G(-{}^1/_2\,{}^1/_2,-+) = \quad F(-{}^1/_2,-{}^1/_2) = H(+{}^1/_2,{}^1/_2+)$

21. $G(-{}^1/_2-{}^1/_2,++) = \exp[-i(\psi-\chi)]$ $\quad F(--{}^1/_2,+{}^1/_2)$
$= \exp[i(\bar\psi-\bar\chi)] \quad H(--{}^1/_2,\,{}^1/_2+)$

22. $G(-{}^1/_2-{}^1/_2,--) = \exp[i(2\chi+\psi)]$ $\quad F(+-{}^1/_2,-{}^1/_2)$
$= \exp[-i(\bar\psi+\bar\chi)] \quad H(+-{}^1/_2,\,{}^1/_2-)$

23. $G(-{}^1/_2-{}^3/_2,+-) = \exp[2i\chi]$ $\quad F(+-{}^3/_2,+{}^1/_2)$
$= \exp[-2i\bar\chi] \quad H(--{}^3/_2,\,{}^1/_2-)$

24. $G(-{}^1/_2-{}^3/_2,-+) = \exp[2i\chi]$ $\quad F(--{}^3/_2,-{}^1/_2)$
$= \exp[-2i\bar\chi] \quad H(+-{}^3/_2,\,{}^1/_2+)$

25. $G(-{}^3/_2\,{}^3/_2,+-) = \quad F(+{}^3/_2,+{}^3/_2) = H(-{}^3/_2,\,{}^3/_2-)$

26. $G(-{}^3/_2\,{}^3/_2,-+) = \quad F(-{}^3/_2,-{}^3/_2) = H(+{}^3/_2,\,{}^3/_2+)$

27. $G(-{}^3/_2\,{}^1/_2,++) = \exp[-i(\psi-\chi)]$ $\quad F(-{}^1/_2,+{}^3/_2)$
$= \exp[i(\bar\psi-\bar\chi)] \quad H(-{}^1/_2,\,{}^3/_2+)$

28. $G(-{}^3/_2\,{}^1/_2,--) = \exp[i(\psi+\chi)]$ $\quad F(+{}^1/_2,-{}^3/_2)$
$= \exp[-i(\bar\psi+\bar\chi)] \quad H(+{}^1/_2,\,{}^3/_2-)$

29. $G(-{}^3/_2-{}^1/_2,+-) = \exp[2i\chi]$ $\quad F(+-{}^1/_2,+{}^3/_2)$
$= \exp[-2i\bar\chi] \quad H(--{}^1/_2,\,{}^3/_2-)$

30. $G(-{}^3/_2-{}^1/_2,-+) = \exp[2i\chi]$ $\quad F(--{}^1/_2,-{}^3/_2)$
$= \exp[-2i\bar\chi] \quad H(+-{}^1/_2,\,{}^3/_2+)$

$$31.\ G(-{}^3/_2-{}^3/_2,++) = \exp\,[-i(\psi-3\psi)] \quad F(--{}^3/_2,{}^3/_2+)$$
$$= \exp\,[i(\overline{\psi}-3\overline{\chi})] \quad H(--{}^3/_2,{}^3/_2+)$$
$$32.\ G(-{}^3/_2-{}^3/_2,--) = \exp\,[i(\psi+3\chi)] \quad F(+-{}^3/_2,{}^3/_2-)$$
$$= \exp\,[-i(\overline{\psi}+3\overline{\chi})] \quad H(+-{}^3/_2,{}^3/_2-) \tag{5.1}$$

If the initial particles are identical, as well as the final particles, *e. g.* for the reaction $p+p \to N^*(1236)+N^*(1236)$ there are further relations between the amplitudes. The same can be said of processes in which the two pairs of particles are related to each other by isospin (as $p+n$) or charge conjugation (as $\overline{p}+\overline{p}$). *E. g.* for identical particles we have the relations: $G_3 = G_{10}$, $G_4 = G_9$, $G_5 = G_{17}$, $G_6 = G_{18}$, $G_7 = G_{20}$, $G_8 = G_{25}$, $G_{13} = G_{20}$, $G_{14} = G_{19}$, $G_{15} = G_{27}$, $G_{16} = G_{28}$, $G_{23} = G_{30}$, $G_{24} = G_{29}$. Therefore there remain 20 independent amplitudes. Similar relations are satisfied in the crossed channels where they can be independently derived from CT and CE invariance in the t and u channel, respectively.

The crossing relations were written down under the assumption that in both the initial and final states there are pairs of particles of equal masses. Then there are only two different crossing angles which were denoted:

$$\psi = \psi_a = \pi-\psi_c, \quad \chi = \psi_d = \pi-\psi_d \tag{5.2}$$

for s-t and

$$\overline{\psi} = \overline{\psi}_c = \pi-\overline{\psi}_a, \quad \overline{\chi} = \overline{\psi}_d = \pi-\overline{\psi}_b \tag{5.3}$$

One of the authors (A. K.) is indebted to Dr A. Białas for stimulating discussions.

REFERENCES

Białas, A., Kotański, A., *Acta Phys. Polon.*, **30,** 833 (1966).
Białas, A., Svensson, B. E. Y., *Nuovo Cimento*, **42,** 672 (1966).
Gottfried, K., Jackson, J. D., *Nuovo Cimento* **33,** 309 (1964).
Kotański, A., *Acta Phys. Polon.*, **29,** 699 (1966a), quoted as *A*.
Kotański, A., *Acta Phys. Polon.*, **30,** 629 (1966b), quoted as *B*.
Pilkuhn, H., Svensson, B. E. Y., *Nuovo Cimento*, **38,** 518 (1965).
Svensson, B. E. Y., *Ark. Fys.* (Sweden), **33** 93 (1966).
Trueman, T. L., Wick, G. C., *Ann. Phys.*, **26,** 322 (1964).

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 1