PION-NUCLEON COUPLING CONSTANT FROM SPIN-FLIP DISPERSION RELATION

By A. M. Harun-ar Rashid and Md. Mozaharul Islam

Atomic Energy Centre, Dacca*

(Received June 25, 1967)

Single variable dispersion relation for the spin-flip part of the pion-nucleon scattering amplitude is used to obtain a value of the pion-nucleon coupling constant. The method has the advantage of the using accurately known phase-shifts from the recent analyses.

Introduction

Since the time the dispersion relations were first introduced, a great deal of work has been done on the application of these relations to elementary particle processes. It is the purpose of this paper to use a particular set of dispersion relations for pion-nucleon scattering to derive a value of the coupling constant using the recent phase-shift analyses.

The analysis of scattering in terms of phase-shifts is the traditional meeting ground between experimental data and the various theoretical schemes. The usual form of this analysis is in terms of sets of phase-shifts at single energies but this is successful only at low energies where only elastic scattering processes are present. In the last couple of years, extensive phase-shift analyses of pion-nucleon scattering have been performed at Livermore [1], Yale [2], Hawaii [3], London [4] and Chilton [5]. The work of the London group which is probably the most accurate is based on applying dispersion relations criteria to set limits on the phase-shifts. The second type of analysis uses specific functional forms for the phase-shifts as functions of the centre of mass energy.

In view of the existence of such accurate phase-shift analyses, it is of some interest to re-examine some of the earlier applications of dispersions relations. For example, it is well-known that one of the early triumphs of dispersion relations has been to resolve the ambiguity between the Fermi set and the Yang set of phase-shifts. Davidon and Goldberger [6] utilized for this purpose the spin-flip dispersion relations and showed quite unambiguously why from the theoretical point of view the Fermi set is to be preferred. We have used their method without however making any approximations and have derived a value of the coupling constant.

The most recent and accurate determination of the coupling constant has been done by Samarnayake & Woolcock [7]. A previous determination of Hamilton and Woolcock [8]

* Address: Atomic Energy Centre, Dacca, East Pakistan.

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 2

using truncated Taylor series expansion as well as the new determination both give the value of the coupling constant as

$$f^2/_{4\pi} = 0.081 \pm 0.003$$

Our object is to make an evaluation of this quantity using for this purpose the spin-flip forward dispersion relations. The advantage of using this particular relation is that it does not involve the poorly known S-wave phase-shifts but only the rather accurately known δ_{33} and δ_{31} phase-shifts. Another advantage is that the contribution of the high energy part of the dispersion integral is explicitly separated out and is used as a parameter of the theory. This obviates the necessity of introducing high energy assumptions like Regge poles. This form of the dispersion relations has been used by Hüper & Draxler [9] to decide on the various proposed sets of phase-shifts and they have shown that the Hawaii analysis is not consistent with these dispersion relations. We have found also that the parameterisa-

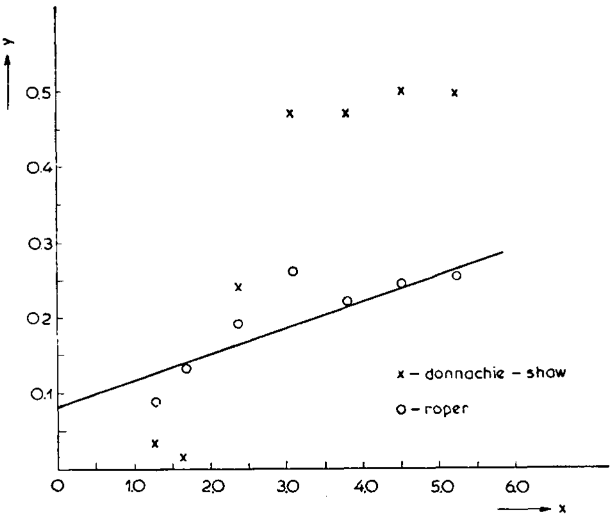

Fig. 1

tion of δ_{33} phase-shift given by Donnachie & Shaw [10] is inconsistent with the spin-flip dispersion relation in the sense that it leads to a much lower value of the coupling constant.

In the next section we give a brief derivation of the dispersion relation used and in Sect. 3 we discuss the numerical methods employed. In the final section we give a discussion of our results.

2. The dispersion relation

It is well-known [8] that the spin-flip forward scattering amplitude $B(\omega)$ as a function of the total energy of the pion in the laboratory system satisfies the dispersion relation

$$\mathrm{Re}\, B^{3/2}(\omega) = \frac{g^2/m}{\left(\frac{1}{2}\, m\right) - \omega} + \frac{P}{\pi} \int\limits_{1}^{\infty} \left[\frac{\mathrm{Im}\, B^{3/2}(\omega')}{\omega' - \omega} - \frac{\frac{2}{3}\, \mathrm{Im}\, B^{1/2}(\omega') + \frac{1}{3}\, \mathrm{Im}\, B^{3/2}(\omega')}{\omega' + \omega} \right] d\omega', \quad (1)$$

where the superscript denotes the isotopic spin state and m is the mass of the nucleon in units such that $\hbar = c =$ mass of pion $= 1$. Using the identity

$$\frac{1}{\omega' \pm \omega} = \frac{1}{\omega'} \mp \frac{\omega}{\omega'(\omega' \pm \omega)} \tag{2}$$

we can rewrite the relation in the form

$$\frac{1}{8\pi m} \operatorname{Re} B^{3/2}(\omega) = \frac{2 \cdot f^2/4\pi}{\left(\frac{1}{2m}\right) - \omega} + 2C + \omega \left[I_3(\omega) + \frac{1}{3} I_3(-\omega) + I_1(\omega) \right],$$

where

$$I_3(\omega) = \frac{P}{8m\pi^2} \int_1^\infty \frac{\operatorname{Im} B^{3/2}(\omega')}{\omega'(\omega' - \omega)} d\omega',$$

$$I_1(\omega) = \frac{1}{12m\pi^2} \int_1^\infty \frac{\operatorname{Im} B^{1/2}(\omega')}{\omega'(\omega' + \omega)} d\omega'$$

and

$$C = \frac{1}{24m\pi^2} \int_1^\infty \frac{d\omega'}{\omega'} [\operatorname{Im} B^{3/2}(\omega') - \operatorname{Im} B^{1/2}(\omega')].$$

We have introduced the pseudo-vector coupling constant

$$\frac{f^2}{4\pi} = \frac{g^2}{4\pi} \left(\frac{1}{2m}\right)^2.$$

The partial-wave expansion of the spin-flip amplitude $B(W)$ as a function of the total centre of mass energy W and scattering angle θ is given by

$$\frac{B(w, \theta)}{4\pi} = \frac{1}{E+m} \left[\sum_{l=0}^\infty f_{l+} P'_{l\pm} (\cos\theta) - \sum_{l=2}^\infty f_{l-} P'_{l-1}(\cos\theta) \right]$$

$$- \frac{1}{E-m} \sum_{l=1}^\infty [f_{l-} - f_{l+}] P'_l(\cos\theta), \tag{3}$$

where

$$f_{l\pm} = \frac{e^{i\delta_{l\pm}} \sin \delta_{l\pm}}{q}.$$

E is the energy of the nucleon and q the magnitude of the centre of mass momentum. Using this expansion and keeping only S and P waves we can then evaluate the integrals from the known phase-shifts. Writing

$$x = \omega / \left(1 + \frac{1}{2m\omega}\right)$$

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 2

and

$$y = \frac{x}{2}\left[\frac{1}{8\pi m}\ \mathrm{Re}\ B^{3/2}(\omega) - \omega\left\{I_3(\omega) + \frac{1}{3}\ I_3(-\omega) + I_1(\omega)\right\}\right]$$

we have finally

$$y = \frac{f^2}{4\pi} + Cx \tag{4}$$

and thus the intercept of the straight-line on the y-axis gives the value of $f^2/4\pi$.

ACTA PHYSICA POLONICA Vol. XXXIII (1968) Fasc. 2

3. Comparison with experiment

Except for δ_{11} and δ_{33} we have used the phase-shift parametrisation of Roper, Wright & Feld [1]:

$$\delta^{2T,l\pm} = q^{2l+1}\sum_{n=0}^{l_m-l}[a_{l\pm}^{2T}]_n\, q^n.$$

The resonant δ_{33} phase-shifts are also taken from this work which includes a non-resonance background to the Layson form of the $3-3$ amplitude. For convenience we have taken δ_{11} from an earlier reference of Roper in the form

$$\delta_{11} = \tan^{-1}\left[-\frac{q_{0z}-q_0}{q_{0r}-q_0}\ (a_1q^3 + a_2q^4 + a_3q^5 + a_4q^6)\right]$$

with the values $q_{0z} = 1.73$, $q_{0r} = 3.242$, $a_1 = 0.335$, $a_2 = -0.229$, $a_3 = 0.0544$ and $a_4 = -0.00449$. Here q_0 is the c. m. pion energy $\sqrt{1+q^2}$.

In all such calculations with dispersion relations, the important point in the accurate determination of the principal value integral. In the present work, we have tested our computer programme for principal value integration by three different methods. One method — the subtracting out of the singularity — is rather well-known. The second method is due to Davidon and Goldberger and is based on the approximation of the function in the neighbourhood of a pole by a sequence of straight lines. The third method is a slight generalization of the second in that the function is here approximated by a parabolic interpolation in the vicinity of the pole. All three methods give the same results.

E (lab) [MeV]	x	y
50	1.29	0.088
100	1.65	0.135
200	2.36	0.193
300	3.08	0.263
400	3.79	0.218
500	4.51	0.242
600	5.22	0.253

As seen in the figure, an extrapolation of the straight line gives

$$\frac{f^2}{4\pi} = 0.082$$

From the slope of the straight-line we can also find the value of C which comes out to be 0.03 whereas a direct numerical integration gives 0.026.

3. Discussion

The value of the pion-nucleon coupling constant obtained from the dispersion relation for the spin-flip amplitude using Roper's phase-shifts is thus consistent with that obtained from the very accurate analysis of Samarnayake & Woolcock. However the parameterisation of the δ_{33} phase-shift given by Donnachie and Shaw does not seem to be consistent with this relation. Since the latter authors use their approximations only for the photoproduction process, such discrepancies may not be of any significance to their final conclusions.

Our aim in this work has not however been to decide on such phase-shift solutions but rather to see how the old calculations of Davidon & Goldberger stand up to the new accumulation of data. For this purpose we have used the dispersion relation for the spin--flip amplitude without making any approximation at any stage. Davidon & Goldberger had for example neglected the S-wave and the small P-wave phase-shifts. We have indeed found that the use of these phase-shifts from Roper's analysis does not produce any significant effect. Indeed one of our objectives was to see how the resonant δ_{11} phase-shift affects the Davidon & Goldberger straight-line. Since this resonance is at a much higher energy, it does not indeed affect the coupling constant calculation.

The method of Davidon & Godberger is therefore very suitable for the determination of the hadron coupling constant, although of course it is difficult to estimate the error. With the small computing machine at our disposal, we have not found it possible to give the error limits to the coupling constant.

We are grateful to Mr Hanifuddin Miah and other members of the computing staff for help in the running of the machine.

REFERENCES

[1] L. D. Roper, *Phys. Rev. Letters*, **12,** 340 (1964).
L. D. Roper & R. M. Wright, *UCRL* — 7846.
L. D. Roper, R. M. Wright & B. T. Feld, *Phys. Rev.*, **138,** B203 (1965).
[2] M. H. Hull & F. C. Lin, *Phys. Rev.*, **139,** B630 (1965).
[3] R. C. Cence, *Phys. Letters.*, **20,** 306 (1966).
[4] P. Auvil, A. Donnachie, A. T. Lea & C. Lovelau, *Phys. Letters* **12,** 76 (1964).
[5] B. H. Bransdeu, P. J. O'Donnel& R. G. Moorhouse, *Phys. Letters*, **11,** 339 (1964).
[6] W. C. Davidon & M. L. Goldberger, *Phys. Rev.*, **104,** 1119 (1956).
[7] V. Samarnayake & W. S. Woolcock. *Phys. Rev. Letters*, **15,** 936 (1965).
[8] J. Hamilton & W. S. Woolcock, *Rev. Mod. Phys.*, **35,** 737 (1963).
[9] R. Huper & K. Draxler, *Phys. Letters*, **20,** 199 (1966).
[10] A. Donnachie & G. Shaw, *Ann. Phys.*, **37,** 333 (1966).