ON SOME IDENTITIES FOR GREEN'S FUNCTIONS IN THE HYDRODYNAMICAL APPROXIMATION. II. SUPERFLUID

By Z. K. Petru

Institute of Theoretical Physics University of Wrocław*

(*Received January 30, 1968*)

After the calculation of Green's function, with the help of the hydrodynamical equations for the Bose-superfluod, it is demonstrated that they are connected by formulas, derived earlier by means of the gauge transformation.

The aim of the present paper is to derive some connections for the Green's functions *GF*-s in the superfluid Bose-system.

The method for the calculation of the *GF*-s in the hydrodynamical approximation was given by Bogoliubov [1] for the ordinary fluid and the superfluid and applied in Ref. [2]–[5] for the viscous case.

Moreover in [6] the relations between *GF*-s were obtained by aplying the gauge transformation to the Hamiltonian and the field operators. On the other hand it was remarked in [4] that these relations for the *GF*-s can be obtained directly from the hydrodynamical equations. These relations for the ordinary fluid were derived in [7] .Now we wish to calculat them for the case of the superfluid.

In the paper [8] we considered the Bose-system with the Hamiltonian

$$\hat{H}[\delta U; \delta \boldsymbol{A}; \delta\eta] = \hat{H}^0 + \delta\hat{H}_t^1[\delta U(r, t); \delta \boldsymbol{A}(r, t); \delta\eta(r, t)] \tag{1}$$

where the therm $\delta\hat{H}_t^1$ described the adiabatically introduced, time dependent perturbation, which was the function of the "density sources" $\delta\eta$ and the infinitesimal scalar and vectorial potentials of the external field, δU and $\delta \boldsymbol{A}$ respectively.

With the help of the Hamiltonian (1) one can obtain the linearized hydrodynamical equations (see [8]). The solutions of these equations, for the variations of density of particles $\delta\varrho$ and the normal components or superfluid components of the velocity, δv_n^α and δv^α respectively, are linear in δU, δA^α, $\delta\eta$. In the Fourier representation we can write

$$\delta\varrho_k = R^U\delta U_k + \boldsymbol{R}^A\delta \boldsymbol{A}_k + R^\eta\delta\eta_k + R^{\eta^*}\delta\eta^*_{-k}$$

$$\boldsymbol{k}\delta v_{s,n}(k) = V^U_{s,n}\delta U_k + \boldsymbol{V}^A_{s,n}\delta \boldsymbol{A}_k + V^\eta\delta\eta_k + V^{\eta^*}\delta\eta^*_{-k}$$

---

* Address: Instytut Fizyki Teoretycznej, Uniwersytet Wrocławski, Wrocław, ul. Cybulskiego 36, Polska.

We are now interested in the derivatives with respect to δU_k and δA_k^α. After some calculation we have

$$\frac{\delta \varrho_k}{\delta U_k} = R^U = \frac{k^2 \varrho D}{m\Omega} \tag{2}$$

$$\frac{\delta \varrho_k}{\delta A_k^\alpha} = R^{A^\alpha} = -\frac{\omega \varrho k^\alpha D}{m\Omega} \tag{3}$$

Using the expression for the total current

$$\delta j_k^\alpha = m\varrho_s \delta v_s^\alpha + m\varrho_n \delta v_n^\alpha,$$

where $\varrho_s + \varrho_n = \varrho$, we have

$$\boldsymbol{k} \frac{\delta \boldsymbol{j}^k}{\delta U^k} = m\varrho_s V_s^U + m\varrho_n V_n^U = \frac{\omega k^2 \varrho D}{\Omega} \tag{4}$$

$$\boldsymbol{k} \frac{\delta \boldsymbol{j}_k}{\delta A_k^\alpha} = m\varrho_s V_s^{A\alpha} + m\varrho_n V_n^{A\alpha} = -\frac{\omega^2 \varrho k^\alpha D}{\Omega} \tag{5}$$

where D and Ω are the polinomials in k, ω linear in the dessipative coefficients: $\varkappa$, ζ_1, ζ_2 ζ_3, ζ_4, η, *e.g.*

$$D = \omega^2 - k^2 c_2^2 - i\omega k^2 \left[\frac{\varrho}{m\varrho\varrho_n} \left(\frac{4}{3}\eta - \varrho\zeta_1 + \zeta_2 + \varrho^2\zeta_3 - \varrho\zeta_4 \right) + \frac{\varkappa}{\varrho c_V} \right].$$

If we compare the right hand sides of (3)–(6) we find

$$\omega \frac{\delta \varrho_k}{\delta U_k} + \sum_\alpha k^\alpha \frac{\delta \varrho_k}{\delta A_k} = 0$$

$$\omega \boldsymbol{k} \frac{\delta \boldsymbol{j}_k}{\delta U_k} + \sum_{\alpha,\beta} k^\alpha k^\beta \frac{\delta j_k^\alpha}{\delta A_k^\beta} = 0$$

$$m\omega^2 \frac{\delta \varrho_k}{\delta U_k} + \sum_{\alpha,\beta} k^\alpha k^\beta \frac{\delta j_k^\alpha}{\delta A_k^\beta} = 0. \tag{6}$$

On the other hand on the basis of the relations between the maen values and the retarded *GF*-s (see *e.g.* [9]) we find the formulae

$$\frac{\delta \varrho_k}{\delta U_k} = 2\pi \langle\langle \hat{\varrho}_k ; \hat{\varrho}_{-k} \rangle\rangle_\omega^r$$

$$\frac{\delta \varrho_k}{\delta A_k^\alpha} = -\frac{2\pi}{m} \langle\langle \hat{\varrho}_k ; \hat{j}_{-k}^{0\alpha} \rangle\rangle_\omega^r$$

$$\frac{\delta j_k^\alpha}{\delta U_k} = 2\pi \langle\langle \hat{j}_k^{0\alpha} ; \hat{\varrho}_{=-k} \rangle\rangle_\omega^r$$

$$\frac{\delta j_k^\beta}{\delta A_k^\alpha} + \varrho\delta_{\alpha\beta} = -\frac{2\pi}{m} \langle\langle \hat{j}_k^{0\beta} ; \hat{j}_{-k}^{0\alpha} \rangle\rangle_\omega^r \tag{7}$$

$$\left(\hat{\boldsymbol{j}}(r,t) = \frac{i}{2}(\nabla\psi^+\psi - \psi^+\nabla\psi) - \psi^+\psi\delta \boldsymbol{A} \equiv \hat{\boldsymbol{j}}^0(r,t) - \hat{\varrho}(r,t)\,\delta\hat{\boldsymbol{A}}\right).$$

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 1

Introducing (7) into (6) we obtain the following connections between the *FG*-s (see [6])

$$m\omega \ll \hat{\varrho}_k;\ \hat{\varrho}_{-k}\gg \frac{r}{\omega} - \boldsymbol{k} \ll \hat{\varrho}_k;\ \hat{\boldsymbol{j}}^0_{-k}\gg^r_\omega = 0$$

$$m\omega\ \boldsymbol{k} \ll \hat{\boldsymbol{j}}^0_k;\ \hat{\varrho}_{-k}\gg \frac{r}{\omega} - \ll \boldsymbol{k}\hat{\boldsymbol{j}}^0_k;\ \boldsymbol{k}\hat{\boldsymbol{j}}^0_{-k}\gg^r_\omega = \frac{1}{2\pi} m\varrho\, k^2$$

$$m^2\omega^2 \ll \hat{\varrho}_k;\ \hat{\varrho}_{-k}\gg^r_\omega - \ll \boldsymbol{k}\hat{\boldsymbol{j}}^0_k;\ \boldsymbol{k}\hat{\boldsymbol{j}}^0_{-k}\gg^r_\omega = \frac{1}{2\pi} m\varrho k^2. \tag{8}$$

Taking into account (3) and (4) we have for the longitudinal potential ($\delta \boldsymbol{A}_k = (\delta \boldsymbol{A}_k)_{||} || \boldsymbol{k}$ and then $\delta \boldsymbol{j}_k = (\delta \boldsymbol{j}_k)_{||}$)

$$\frac{\delta (j^\alpha_k)_{||}}{\delta U_k} + m\, \frac{\delta \varrho_k}{\delta (A^\alpha_k)_{||}} = 0. \tag{9}$$

If we insert (7) into (9) we obtain the equality

$$\ll (\hat{j}^{0\alpha}_k)_{||};\ \hat{\varrho}_{-k}\gg^r_\omega = \ll \hat{\varrho}_k;\ (\hat{j}^{0\alpha}_{-k})_{||}\gg^r_\omega. \tag{10}$$

Identities (8) and (10) have the very same form as in the case of the ordinary fluid (see [7]).

It is a pleasent duty to thank Professor Z. Galasiewicz for suggesting this problem and for his valuable discussions.

REFERENCES

[1] N. N. Bogoliubov, *On the Hydrodynamics of a Superfluid*, Dubna preprint, 1963, (in Russian).

[2] Z. Galasiewicz, *Asymptotic Calculation of the Green Functions in Viscous Liquid Approximation for Superfluid Bose-Systems*, Dubna preprint 1964, (in Russian).

[3] Z. Galasiewicz, *Acta Phys. Polon.*, **30**, 567 (1966); Z. Galasiewicz, *Bull. Acad. Polon. Sci Ser. Sci. Math. Astron. Phys.*, **15**, 191 (1967).

[4] Z. Galasiewicz, *Calculation of the Green's Functions for the Superfluid Bose-System*, in print in *the Materials of the* III *Winter School of Theoretical Physics* in Karpacz 1967.

[5] W. A. Krasnikov, *Dokl. Akad. Nauk SSSR*, **174**, 1037 (1967).

[6] Z. Galasiewicz, *Bull. Acad. Polon. Sci. Ser. Math. Astron. Phys.*, **15**, 185 (1967), see *erratum*, *ibid.*

[7] Z. K. Petru, *ibid.*, in print. **16**, 67 (1968).

[8] Z. K. Petru, *Acta Phys. Polon.*, **35**, 49 (1969).

[9] D. W. Zubarev, *Uspekhi Fiz. Nauk* USSR), **71**, 71 (1961).

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 1