THE INFLUENCE OF GRAVITATIONAL FIELDS ON THE PROPAGATION OF LIGHT

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The gravitational wave accompanying a light pulse is shown to interact with a stationary mass causing the mass to be displaced in the direction of the light path and endowing the mass with a constant velocity perpendicular to and toward the light path. Requiring the center of inertia of the mass plus light pulse to remain fixed, necessitates that the light pulse be delayed the amount predicted by Shapiro's fourth test of general relativity; in addition, and for the same reason, the path of the light pulse is bent an amount predicted by the second test of general relativity.

Two so called crucial experimental tests of General Relativity are shown to be a direct consequence of the linearized theory of gravitational radiation. The tests treated here are: (1) the time delay in transmitting a light pulse passing the sun on its way towards a planet [1] and (2) the bending of starlight as it passes the sun [2].

The optical phenomena are discussed with reference to a coordinate system centered on the emitter (see Fig. 1). The emitter, the sun and the absorber lie in the x^1, x^2 plane, the sun being a perpendicular distance d from the x^1 axis, and the absorber being a distance l from the emitter. The unperturbed pulses of light follows the x^1 coordinate with velocity c equal to the velocity of light in vacuo. In so moving the light pulse emits a gravitational wave [3] which causes the sun to move with velocity v^α. The requirement that the center of mass of the sun and light pulse remain fixed leads to a perturbed light velocity given by

$$c^\alpha = c\delta_{\alpha 1} - mc\,p^{-1}v^\alpha \tag{1}$$

with m the mass of the sun and p the momentum of the light pulse. The perturbed light pulse follows the dashed curve in Fig. 1. The velocity of the sun is found from the equation of motion (Eq. (4.3) of Ref. [3])

$$\frac{1}{c}\frac{\partial v^a}{\partial x^0} + 1/2\,\frac{\partial h_{00}}{\partial x^a} - \frac{\partial h_{0a}}{\partial x^0} \tag{2}$$

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in which the metric is $g_{ik} = n_{ik} + h_{ik}$, with η_{ik} the Galilean metric viz:

$$\eta_{\alpha\beta} = \delta_{\alpha\beta}, \quad \eta_{00} = -1, \quad \eta_{0\alpha} = 0.$$

When the emitter and the absorber are sufficiently far removed from the sun, the gravitational radiation from them can be neglected. In this case $x_E \gg d$ and $x_p \gg d$ and Eq. (4.4) of Ref. [3] Jields

$$h_{01} = -\psi_e, \quad h_{00} = \psi_e \quad \text{and} \quad h_{02} = 0 \tag{3}$$

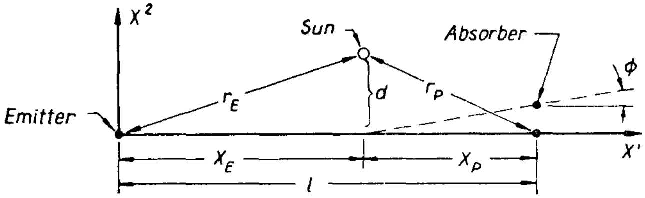

Fig. 1. Geometry of path of light pulse

in which ψ_e is the solution of the wave equation

$$\Box\, \psi_e = -\, 16\pi c^{-4} kpc[\theta(x^1)\delta(x^0 - x^1)\delta(x^2)\delta(x^3)$$
$$-\theta(x^1 - l)\delta(x^0 - x^1)\delta(x^2)\delta(x^3)]. \tag{4}$$

The solution to the above equation can be obtained from Ref. [3] and it turns out to be the sum of Eq. (3.6) of that reference plus the same equation modified in the manner indicated in the paragraph following Eqs (3.9) of Ref. [3], namely:

$$\psi_e = 4pkc^{-3}(x^0 - x_E^1)^{-1}[\theta(x^0 - r_E) - \theta(x^0 - r_p - l)]. \tag{5}$$

Utilizing Eqs (2), (3) and (5), it is a straightforward process to show that the two components of the sun's velocity are given by

$$v^1 = 2kpc^{-2}(x^0 - x_E^1)^{-1}[\theta(x^0 - r_E) - \theta(x^0 - r_p - \lambda)] \tag{6}$$

and

$$v^2 = -2kpc^{-2}d[r_E^{-1}(r_E - x_E^1)^{-1}\theta(x^0 - r_E) - r_p^{-1}(r_p + x_p^1)^{-1}\theta(x^0 - r_p - \lambda)]. \tag{7}$$

The travel time of the light pulse would normally be given by the integral

$$T = \int \frac{dx^1}{c^1}, \tag{8}$$

thus

$$T = \int \frac{dx^1}{c - mcp^{-1}v'} \approx \int \frac{dx}{c} + mp^{-1}c^{-1} \int v^1 dx. \tag{9}$$

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However, we wish to be sure the sun has reached a steady state when making the momentum balance so we transform the second term in Eq. (9) to an integral over time and chose for the upper limit a time after which the x^1 component of the sun's velocity vanishes. This occurs for $x^0 \geqslant r_p+\lambda$. It turns out that v^2 is constant for all $x^0 < r_p+\lambda$.

The time delay is

$$T = \int_0^l \frac{dx}{c} + mp^{-1} \int_0^{c^{-1}(r_p+l)} v^1 dt. \tag{10}$$

The second term in Eq. (10) represents the general relativistic correction to the one-way time delay and it may be expressed in terms of ξ^1, the displacement of the sun in the x' direction

$$T = lc^{-1} + (\delta t)_{\text{out}},$$

with

$$(\delta t)_{\text{out}} = mp^{-1}\xi^1. \tag{11}$$

The displacement ξ^1 is the integral of Eq. (6) evaluated at any time after the sun stops moving in the x^1 direction

$$\xi^1 = \int_0^{c^{-1}(r_p+l)} v^1 dt = 2pkc^{-3}\{[(ln(x^0-x_E^1)-(ln(r_E-x_E^1)]\theta(x^0-r_E)$$

$$-[ln(x^0-x_E^1)-ln(r_p+l-x_E')]\theta(x^0-r_p-\lambda)\}\Big|_{x^0=0}^{x^0\geqslant r_p+l},$$

which, upon putting in the limits, becomes

$$\xi^1 = 2pkc^{-3} ln \frac{r_p+x_p^1}{r_E-x_E^1}. \tag{12}$$

Combining Eq. (11) and Eq. (12) and introducing the gravitational radius $r_0 = kmc^{-2}$ yields

$$(\delta t)_{\text{out}} = 2r_0 c^{-1} ln \frac{r_p+x_p^1}{r_E-x_E^1}, \tag{13}$$

in complete agreement with Eq. 6–103 of Ref. [4] when the earth and planet are on opposite sides of the sun. The correction $(\delta t)_{\text{out}}$ corresponding to the case when the earth and planet are on the same side of the sun is easily shown to be

$$(\delta t)_{\text{out}} = 2r_0 c^{-1} ln \frac{r_p-x_p^1}{r_E-x_E^1}. \tag{14}$$

This one-way time delay is approximately doubled for a round trip [4].

The bending of the light pulse towards the sun is given directly in terms of the x^2 component of the sun's velocity. Thus the angular deflection φ is

$$\varphi = -mv^2p^{-1}. \tag{15}$$

Putting $\sin\beta = dr_E^{-1}$, $\cos\beta = x_E r_E^{-1}$, $\sin\gamma = dr_E^{-1}$ and $\cos\gamma = x_p r_p^{-1}$ into Eq. (7) and letting $x^0 \gg r_p + l$ one finds for the terminal velocity of the sun

$$v^2 = -2kpc^{-2}[r_E^{-1}(1-\cos\beta)^{-1}\sin\beta - r_p^{-1}(1+\cos\gamma)^{-1}\sin\gamma].$$

In general the second term in the square brackets is negligible with respect to the first term in which case v^2 becomes

$$v^2 = kpc^{-2}r_E^{-1}\sin\beta\,(\sin^2\beta/2)^{-1}. \quad (16)$$

Since the earth is far from the sun compared with the distance of closest approach of the light pulse to the sun, $r_E \gg d$ and Eq. (16) becomes

$$v^2 = -4kpc^{-2}d^{-1}. \quad (17)$$

Combining Eq. (15) and Eq. (17) yields the well known formula for the angle of deflection [2]

$$\varphi = 4kmc^{-2}d^{-1} = 4r_0/d. \quad (18)$$

In conclusion it appears that the two optical tests of general relativity are so intimately related to one another, that it may be meaningless to attribute a greater fundamental significance for example to this radar time delay test over the test for the bending of star light. This point has previously been made by Dicke and Peebles [5]. Our results can be interpreted as giving independent evidence for the physical reality of gravitational waves. This is also the case in our previous work [3] wherein the quadratic acceleration-dependent term in the cosmological red shift law was obtained on the basis of similar methods.

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