DIFFRACTION OF ELECTROMAGNETIC QUADRUPOLE RADIATION ON PERFECTLY CONDUCTING WEDGE

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The solution of the problem of diffraction of electromagnetic waves emitted by an electric dipole on a perfectly conducting wedge has been given in a previous paper. The present paper gives the solution of the electromagnetic diffraction problem for waves generated by an electric quadrupole. The method used has been proposed by Petykiewicz who has applied it for the first time in the problem of diffraction of scalar multipole radiation on a wedge.

§1. Solution of the Sommerfeld problem for electromagnetic quadrupole radiation

We shall make use of Cartesian and cylindrical coordinate systems in which the z-axis coincides with the edge of the wedge while the $\varphi = 0$ and $\varphi = \chi$, planes are its boundary surfaces (Fig. 1). The coordinates of the radiation source L are denoted by x_{ol} ($l = 1, 2, 3$) or ϱ, p_0 z_0 while those of the observation point P by x_l or r, φ, z.

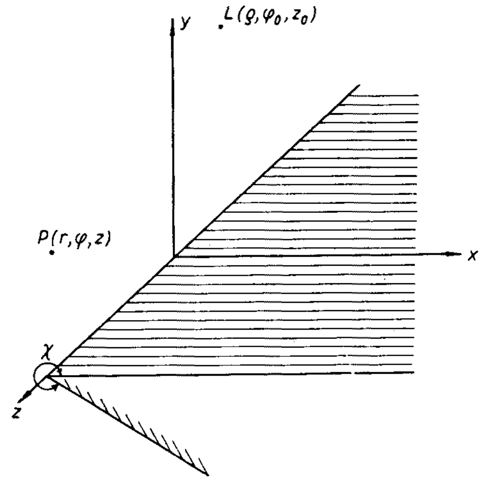

Fig. 1. Cartesian coordinate system in which z axis coincides with edge of the wedge; $\varphi = 0$ and $\varphi = \chi$ planes are boundary surfaces of the wedge

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In the unbounded space we accept for the electric quadrupole the Hertz vector with the Cartesian-components Z_a $(a = 1, 2, 3)$ of the form

$$Z_a = Q_{al}(x_l - x_{ol})\, A(R)\, e^{i\omega t} \tag{1.1}$$

where:

$$A(R) = \frac{1}{R} \frac{\partial}{\partial R} \frac{e^{-ikR}}{R}$$

and

$$R = [(x_l - x_{ol})^2]^{\frac{1}{2}} = [r^2 + \varrho^2 + (z - z_0)^2 - 2r\, \varrho \cos(\varphi - \varphi_0)]^{\frac{1}{2}}$$

is the distance between the points L and P, and Q_{al} is the symmetric quadrupole moment tensor.

We assume the summation convention for repeated indices.

The components of the vector (1.1) can be written in cylindrical coordinates as follows:

$$Z_r = A(R)\, [(r \cos\varphi - \varrho \cos\varphi_0)(Q_{11} \cos\varphi + Q_{12} \sin\varphi) +$$
$$+ (r \sin\varphi - \varrho \sin\varphi_0)(Q_{12} \cos\varphi + Q_{22} \sin\varphi) + (z - z_0)(Q_{13} \cos\varphi + Q_{23} \sin\varphi)]$$
$$Z_\varphi = A(R)\, [(r \cos\varphi - \varrho \cos\varphi_0)(Q_{12} \cos\varphi - Q_{11} \sin\varphi) +$$
$$+ (r \sin\varphi - \varrho \sin\varphi_0)(Q_{22} \cos\varphi - Q_{12} \sin\varphi) + (z - z_0)(Q_{23} \cos\varphi - Q_{13} \sin\varphi)]$$
$$Z_z = A(R)\, [Q_{13}(r \cos\varphi - \varrho \cos\varphi_0) + Q_{23}(r \sin\varphi - \varrho \sin\varphi_0) + Q_{33}(z - z_0)]. \tag{1.2}$$

The phase factor $\exp(i\omega t)$ has been ommitted in these formulae.

Instead of normal three-dimensional space we use the infinite-sheet Riemann space for which the z-axis is the branch line and the region $0 \leqslant \varphi \leqslant \chi$, $-\infty < z < \infty$ is physical space. Let the function Z_r^α, Z_φ^α, Z_z^α obtained from (1.2) by substituting $\varphi \to \varphi + \varphi_0 - \alpha$ be defined on a double-sheet Riemann surface of the complex variable (Petykiewicz 1967). The branch points of the α-plane obtained from the condition of vanishing function:

$$R_\alpha = [r^2 + \varrho^2 + (z - z_0)^2 - 2r\varrho \cos(\varphi - \alpha)]^{\frac{1}{2}}$$

are given by the equations:

$$\alpha_n = \varphi + 2n\pi \pm ib_0 \qquad (n = 0, \pm 1, \pm 2 \ldots)$$

where b_0 is the real and positive root of the equation (*cf.* Rubinowicz 1966):

$$\cosh b = \frac{r^2 + \varrho^2 + (z - z_0)^2}{2r\varrho} \tag{1.3}$$

We now define a new Hertz vector $\boldsymbol{Z}'$ multiplying the components Z_r^α, Z_φ^α, Z_z^α by the factor:

$$\Phi_{\pi/\chi}(\alpha - \varphi_0) = \frac{1}{2\chi} \frac{1}{1 - e^{i\pi(\varphi_0 - \alpha)/\chi}}$$

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and integrating over the path B_1+B_2 lying on the first sheet of the complex α plane (Fig. 2). The vector $\boldsymbol{Z}'$ is given by the following relations:

$$Z'_r = \frac{1}{2\chi}\int\limits_{B_1+B_2} A(R_\alpha)\{[r\cos(\varphi+\varphi_0-\alpha)-\varrho\cos\varphi_0][Q_{11}\cos(\varphi+\varphi_0-\alpha)+Q_{12}\sin(\varphi+\varphi_0-\alpha)]+$$
$$+[r\sin(\varphi+\varphi_0-\alpha)-\varrho\sin\varphi_0][Q_{12}\cos(\varphi+\varphi_0-\alpha)+Q_{22}\sin(\varphi+\varphi_0-\alpha)]+$$
$$+(z-z_0)[Q_{13}\cos(\varphi+\varphi_0-\alpha)+Q_{23}\sin(\varphi+\varphi_0-\alpha)]\}\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$Z'_\varphi = \frac{1}{2\chi}\int\limits_{B_1+B_2} A(R_\alpha)\{[r\cos(\varphi+\varphi_0-\alpha)-\varrho\cos\varphi_0][Q_{12}\cos(\varphi+\varphi_0-\alpha)-Q_{11}\sin(\varphi+\varphi_0-\alpha)]+$$
$$+[r\sin(\varphi+\varphi_0-\alpha)-\varrho\sin\varphi_0][Q_{22}\cos(\varphi+\varphi_0-\alpha)-Q_{12}\sin(\varphi+\varphi_0-\alpha)]+$$
$$+(z-z_0)[Q_{23}\cos(\varphi+\varphi_0-\alpha)-Q_{13}\sin(\varphi+\varphi_0-\alpha)]\}\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$Z'_z = \frac{1}{2\chi}\int\limits_{B_1+B_2} A(R_\alpha)\{Q_{13}[r\cos(\varphi+\varphi_0-\alpha)-\varrho\cos\varphi_0)]+Q_{23}[r\sin(\varphi+\varphi_0-\alpha)-\varrho\sin\varphi_0]+$$
$$+Q_{33}(z-z_0)\}\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha \qquad (1.4)$$

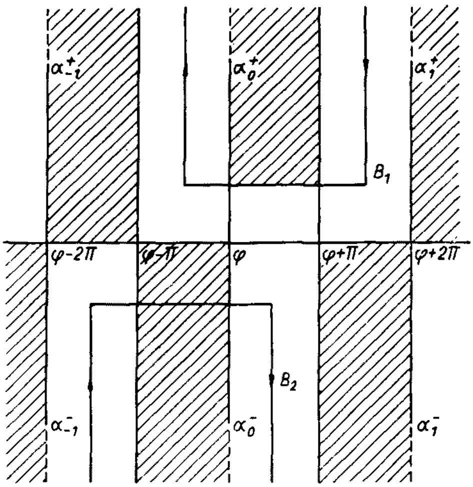

Fig. 2. α — plane. α_0^+, α_0^- are the branch points of R_α. The hatched region correspond to the 1st quadrant of the R_α — plane on the first sheet (3rd on the second sheet). Nonhatched regions correspond to the 4th quadrant (2nd on the second sheet)

The fact that the rectilinear parts of the paths B_1 and B_2 extending to infinity are in those regions of the α plane where the imaginary part of R_α — is negative ensures the convergence of the integrals (1.4). In addition this permits the vector $\boldsymbol{Z}'$ to be expressed in a much simpler form. For this purpose we first transform the components Z'_r by writting the integrand as the sum of the derivatives of the corresponding expressions with respect to the coordinates of the point source and some additional terms integrated

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by parts in further calculations. We make use of the convergence of (1.4) by putting the above-mentioned differentiation operators before the integral symbol. Thus we obtain for Z_r':

$$Z_r' = \frac{1}{2\chi} \int\limits_{B_1+B_2} A(R_\alpha) \left([r\cos(\varphi-\alpha)-\varrho]\{\cos\varphi_0[Q_{11}\cos(\varphi+\varphi_0-\alpha)+Q_{12}\sin(\varphi+\varphi_0-\alpha)] + \right.$$

$$+\sin\varphi_0[Q_{12}\cos(\varphi+\varphi_0-\alpha)+Q_{22}\sin(\varphi+\varphi_0-\alpha)]\}+$$

$$\left.+(z-z_0)[Q_{13}\cos(\varphi+\varphi_0-\alpha)+Q_{23}\sin(\varphi+\varphi_0-\alpha)]\right)\Phi_{\pi/\chi}(\alpha-\varphi_0)-$$

$$-\frac{1}{2\chi}\int\limits_{B_1+B_2} A(R_\alpha) r\sin(\varphi-\alpha)\{\sin\varphi_0[Q_{11}\cos(\varphi+\varphi_0-\alpha)+Q_{12}\sin(\varphi+\varphi_0-\alpha)]-$$

$$-\cos\varphi_0[Q_{12}\cos(\varphi+\varphi_0-\alpha)+Q_{22}\sin(\varphi+\varphi_0-\alpha)]\}\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$= -\frac{\cos\varphi_0}{2\chi}\frac{\partial}{\partial\varrho}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}[Q_{11}\cos(\varphi+\varphi_0-\alpha)+Q_{12}\sin(\varphi+\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha-$$

$$-\frac{\sin\varphi_0}{2\chi}\frac{\partial}{\partial\varrho}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}[Q_{12}\cos(\varphi+\varphi_0-\alpha)+Q_{22}\sin(\varphi+\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha-$$

$$-\frac{1}{2\chi}\frac{\partial}{\partial z_0}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}(Q_{13}\cos(\varphi+\varphi_0-\alpha)+Q_{23}\sin(\varphi+\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha+$$

$$+\frac{\sin\varphi_0}{2\chi\varrho}\int\limits_{B_1+B_2}[Q_{11}\cos(\varphi+\varphi_0-\alpha)+Q_{12}\sin(\varphi+\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha-\varphi_0)\frac{\partial}{\partial\alpha}\left(\frac{e^{-ikR_\alpha}}{R_\alpha}\right)d\alpha-$$

$$-\frac{\cos\varphi_0}{2\chi\varrho}\int\limits_{B_1+B_2}[Q_{12}\cos(\varphi+\varphi_0-\alpha)+Q_{22}\sin(\varphi+\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha-\varphi_0)\frac{\partial}{\partial\alpha}\left(\frac{e^{-ikR_\alpha}}{R_\alpha}\right)$$

Now we integrate by parts the last two integrals in this formula and make use of the fact that the expressions

$$[Q_{a1}\cos(\varphi+\varphi_0-\alpha)=Q_{a2}\sin(\varphi+\varphi_0-\alpha)]\ \Phi_{\pi/\chi}(\alpha-\varphi_0)\ e^{ikR_\alpha}/R_\alpha$$

vanish at the ends of the paths B_1 and B_2 lying at infinity, respectively.

Taking into account the relation:

$$\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}\frac{\partial}{\partial\alpha}\{[Q_{a1}\cos(\varphi+\varphi_0-\alpha)+Q_{a2}\sin(\varphi+\varphi_0-\alpha)\Phi_{\pi/\chi}(\alpha-\varphi_0)\}d\alpha$$

$$=-\frac{\partial}{\partial\varphi_0}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}[Q_{a1}\cos(\varphi+\varphi_0-\alpha)+Q_{a2}\sin(\varphi+\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

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we obtain the following expressions for Z'_r and (in a similar way) for the remaining components:

$$Z'_r = -\frac{1}{2\chi}\frac{\partial}{\partial x_{0l}}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}\left[Q_{l1}\cos(\varphi+\varphi_0-\alpha)+Q_{l2}\sin(\varphi+\varphi_0-\alpha)\right]\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$Z'_\varphi = -\frac{1}{2\chi}\frac{\partial}{\partial x_{0l}}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}\left[Q_{l2}\cos(\varphi+\varphi_0-\alpha)-Q_{l1}\sin(\varphi+\varphi_0-\alpha)\right]\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$Z'_z = -\frac{1}{2\chi}Q_{l3}\frac{\partial}{\partial x_{0l}}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha \tag{1.5}$$

The integration paths B_1 and B_2 can be continuously transformed into two paths C_1 and C_2 and circle around the poles $\varphi_0+2m\chi$; if they are within the region $\varphi-\pi \leqslant \varphi_0 + 2m\chi \leqslant \varphi+\pi$ (Fig. 3). Integration over C_1 and C_2 gives this part of the vector $\boldsymbol{Z}'$ (1.5) from which the diffraction wave is obtained; this part will be denoted by $\boldsymbol{Z}'^d$. Integration

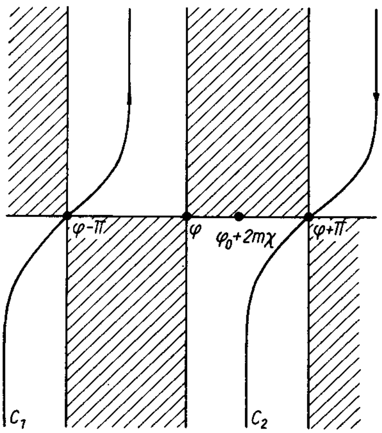

Fig. 3. The integration paths B_1 and B_2 can be continuously transformed into two paths C_1 and C_2 and circle around the poles

over the circles around the poles gives its other part $\boldsymbol{Z}'^g$ from which we obtain a field such as should be expected according to the laws of geometrical optics from the sources $L_m(\varrho, \varphi_0 + 2m\chi, z_0)$ (the only source in physical space is L_0). The vector $\boldsymbol{Z}'^g$ is given by the following formulae:

$$Z'^g_z = -\frac{\partial}{\partial x_{0l}}\sum_{m=-\infty}^{+\infty} v_m(\varphi)\frac{e^{-ikR_m}}{R_m}\left[Q_{l1}\cos(\varphi-2m\chi)+Q_{l2}\sin(\varphi-2m\chi)\right] \tag{1.6}$$

$$Z'^g_\varphi = -\frac{\partial}{\partial x_{0l}}\sum_{m=-\infty}^{+\infty} v_m(\varphi)\frac{e^{-ikR_m}}{R_m}\left[Q_{l2}\cos(\varphi-2m\chi)-Q_{l1}\sin(\varphi-2m\chi)\right]$$

$$Z'_z = -Q_{l3}\frac{\partial}{\partial x_{0l}}\sum_{m=-\infty}^{+\infty} v_m(\varphi)\frac{e^{-ikR_m}}{R_m}$$

where:

$$R_m = [r^2+\varrho^2+(z-z_0)^2-2r\varrho \cos(\varphi-\varphi_0-2m\chi)]^{\frac{1}{2}}$$

$$v_m(\varphi) = \begin{cases} 0 \text{ for } \varphi < \varphi_0+2m\chi-\pi \quad \text{or } \varphi > \varphi_0+2m\chi+\pi \\ 1 \text{ for } \varphi_0+2m\chi-\pi \leqslant \varphi \leqslant \varphi_0+2m\chi+\pi \end{cases}$$

Each Cartesian coordinate of the Hertz vector $\boldsymbol{Z}'$ (1.5) satisfies the oscillatio nequation. It can be easily seen from (1.5) that the integrands of the integrals corresponding to these coordinates depend on the coordinates of the observation point only trough the factor e^{-ikR_α}/R_α. Besides, the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ obtained from Eq. (1.5) satisfy Maxwell's equations. This solution of the Maxwell equations is in general a regular solution everywhere in infinite-sheet Riemann space. This regularity is ensured by the factor exp $(-ikR_\alpha)$ and by putting the integration poths B_1 and B_2 in those parts of the plane α where R_α has a negative imaginary part. Singularities may occur only in the case when the branch points α_0^+ and α_0^- will lie on the real axis at $\alpha = \varphi$, and in addition, there will be a pole $\varphi_0+2m\chi$ at the same point. This may happen according to Eq. (1.3) only in such a case when $r = \varrho$, $z = z_0$ and $\varphi = \varphi_0+2m\chi$, *i. e.*, when the observation point P coincides with the point where there is a source L_m.

The solution of our diffraction problem must satisfy the following conditions:

1. the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ shold be in general regular solutions of Maxwell equations in infinite-sheet Riemann space,
2. if the observation point P approaches to any of the radiation sources L_m, singularities corresponding to a quadrupole source should appear,
3. the field components E_r, E_z, and H_φ must vanish on the perfectly conducting surfaces of the wedge $\varphi = 0$ and $\varphi = \chi$ (boundary condition),
4. Meixner's edge condition,
5. Sommerfeld's conditions of finiteness and radiation emission.

In particular the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ obtained from the Hertz vector do not satisfy the boundary condition. To obtain a solution of Maxwell's equations which has all necessary properties we introduce another Hertz vector $\boldsymbol{Z}''$ formed from (1.5) by substracting from $\boldsymbol{Z}'$ those expressions which represent the field of the quadrupole with the moment Q'_{ab} situated at the point $L'(x'_{0l})$. The point L' is the reflection of L in the semi-plane $\varphi = 0$ and the tensor Q'_{ab} is defined in terms of Q_{ab} as follows:

$$\begin{aligned} Q'_{2b} &= -Q_{2b} && \text{for } b \neq 2 \\ Q_{ab} &= Q_{ab} && \text{for } a \neq 2 \text{ and } b \neq 2 \\ & && \text{or } a = 2 \text{ and } b = 2 \end{aligned} \tag{1.7}$$

The vector $\boldsymbol{Z}''$ is given by the following relations:

$$Z''_r = -\frac{1}{2\chi}\frac{\partial}{\partial x_{0l}} \int\limits_{B_1+B_2} \frac{e^{-ikR_\alpha}}{R_\alpha} [Q_{l1}\cos(\varphi+\varphi_0-\alpha)+Q_{l2}\sin(\varphi+\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha +$$

$$+\frac{1}{2\chi}\frac{\partial}{\partial x'_{0l}} \int\limits_{B_1+B_2} \frac{e^{-ikR_\alpha}}{R_\alpha} [Q'_{l1}\cos(\varphi-\varphi_0-\alpha)+Q'_{l2}\sin(\varphi-\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha+\varphi_0)d\alpha$$

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$$Z''_\varphi = -\frac{1}{2\chi}\frac{\partial}{\partial x_{0l}}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}[Q_{l2}\cos(\varphi+\varphi_0-\alpha)-Q_{l1}\sin(\varphi+\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha+$$

$$+\frac{1}{2\chi}\frac{\partial}{\partial x'_{0l}}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}[Q'_{l2}\cos(\varphi-\varphi_0-\alpha)-Q'_{l1}\sin(\varphi-\varphi_0-\alpha)]\Phi_{\pi/\chi}(\alpha+\varphi_0)d\alpha$$

$$Z''_z = -\frac{1}{2\chi}Q_{l3}\frac{\partial}{\partial x_{0l}}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha+\frac{1}{2\chi}Q'_{l3}\frac{\partial}{\partial x'_{0l}}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}\Phi_{\pi/\chi}(\alpha+\varphi_0)d\alpha \tag{1.8}$$

From Eqs (1.8) we can obtain in a standard manner the fields $\boldsymbol{E}$ and $\boldsymbol{H}$:

$$\boldsymbol{E} = \mathrm{grad}_P\,\mathrm{div}_P\,\boldsymbol{Z}''+k^2\boldsymbol{Z}''$$

$$\boldsymbol{H} = ik\,\mathrm{curl}_P\,\boldsymbol{Z}'' \tag{1.9}$$

We have already proved that the fields presented by means of the vector $\boldsymbol{Z}'$ (1.5) satisfy the conditions (1) and (2). The proof that the fields (1.9) satisfy these conditions is identical. In the next three sections we shall show that these fields (1.9) satisfy the remaining conditions *i. e.* (3)—(5). This will be the proof that (1.9) is the solution of our diffraction problem.

§2. Proof of the fulfilment of the boundary condition

The boundary condition, requiring that the tangential component of $\boldsymbol{E}$ and the normal component of $\boldsymbol{H}$ vanish on both surfaces of the wedge, can be reduced according to (1.9) to the condition:

$$Z''_r = 0, \quad Z''_z = 0, \quad \frac{\partial z''_\varphi}{\partial\varphi} = 0$$

$$\text{for } \varphi = 0 \text{ and } \varphi = \chi \tag{2.1}$$

To prove the validity of (2.1) let us write the Hertz vector $\boldsymbol{Z}''$ in a slightly different form. It the integration variable in (1.8) is changed by substituting $\xi = \alpha-\varphi$ we obtain for the component Z''_r:

$$Z''_r = -\frac{1}{2\chi}\frac{\partial}{\partial x_{0l}}\int\limits_{B'_1+B'_2}\frac{e^{-ikR_\zeta}}{R_\zeta}[Q_{l1}\cos(\varphi_0-\zeta)+Q_{l2}\sin(\varphi_0-\zeta)]\frac{d\zeta}{1-e^{i\pi(\varphi_0-\varphi-\zeta)/\chi}}+$$

$$+\frac{1}{2\chi}\frac{\partial}{\partial x'_{0l}}\int\limits_{B'_1+B'_2}\frac{e^{-ikR_\zeta}}{R_\zeta}(Q'_{l1}\cos(\varphi_0+\zeta)-Q'_{l2}\sin(\varphi_0+\zeta)]\frac{d\zeta}{1-e^{i\pi(\varphi_0+\varphi+\zeta)/\chi}} \tag{2.2}$$

where $R_\zeta = [r^2+\varrho^2+(z-z_0)^2-2r\varrho\cos\zeta]^{\frac{1}{2}}$ and the paths B'_1 and B'_2 are the paths B_1 and B_2 transformed into the ξ plane (see Fig. 4). Let us now note that when changing the integration variable ζ into $-\zeta'$ the path B'_1 goes over into B'_2 and conversely. We perform this

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change in the second integral in the expression (2.2) and, in addition, make use of the relations:

$$Q'_{l1}\frac{\partial}{\partial x'_{0l}} = Q_{l1}\frac{\partial}{\partial x_{0l}}, \qquad Q'_{l2}\frac{\partial}{\partial x'_{0l}} = -Q_{l2}\frac{\partial}{\partial x_{0l}}$$

which result from (1.7) and the fact that $x'_{01} = x_{01}$, $x'_{02} = -x_{02}$, $x'_{03} = x_{03}$.

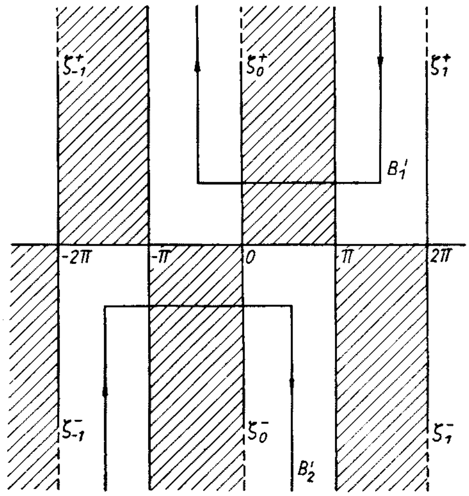

Fig. 4. ζ — plane, ζ_0^+, ζ_0^- are the branch points of R_ζ

We then obtain for Z''_r:

$$Z''_r = -\frac{1}{2\chi}\frac{\partial}{\partial x_{0l}}\int\limits_{B'_1+B'_2}\frac{e^{-ikR_\zeta}}{R_\zeta}\left[Q_{l1}\cos(\varphi_0-\zeta)+Q_{l2}\sin(\varphi_0-\zeta)\right]\frac{e^{i\pi/\chi\,\varphi}-\cos\frac{\pi}{\chi}(\varphi_0-\zeta)}{\cos\frac{\pi}{\chi}\varphi-\cos\frac{\pi}{\chi}(\varphi_0-\zeta)}\,d\zeta$$

After making use of the equality:

$$e^{i\pi/\chi\varphi}-\cos\frac{\pi}{\chi}(\varphi_0-\zeta) = \cos\frac{\pi}{\chi}\varphi-\cos\frac{\pi}{\chi}(\varphi_0-\zeta)+i\sin\frac{\pi}{\chi}\varphi$$

we have:

$$Z''_r = -\frac{1}{2\chi}\frac{\partial}{\partial x_{0l}}\int\limits_{B'_1+B'_2}\frac{e^{-ikR_\zeta}}{R_\zeta}\left[Q_{l1}\cos(\varphi_0-\zeta)+Q_{l2}\sin(\varphi_0-\zeta)\right]d\zeta-$$

$$-\frac{i\sin\frac{\pi}{\chi}\varphi}{2\chi}\frac{\partial}{\partial x_{0l}}\int\limits_{B_1+B'_2}\frac{e^{-ikR_\zeta}}{R_\zeta}\left[Q_{l1}\cos(\varphi_0-\zeta)+Q_{l2}\sin(\varphi_0-\zeta)\right]\frac{d\zeta}{\cos\frac{\pi}{\chi}\varphi-\cos\frac{\pi}{\chi}(\varphi_0-\zeta)}$$

The first integral vanishes since the integrands have no more poles and the paths $B_1'+B_2'$ can be replaced by the paths $C_1'+C_2'$ (which are the paths C_1 and C_2 transformed into the ζ-plane where all integrands are identical and the integrations are performed in opposite directions). After proceeding in the same manner with the remaining components of $\boldsymbol{Z}''$ we obtain as a final result:

$$Z_r'' = -\frac{i\sin\frac{\pi}{\chi}\varphi}{2\chi}\frac{\partial}{\partial x_{0l}}\int\limits_{B_1'+B_2'}\frac{e^{-ikR_\zeta}}{R_\zeta}\left[Q_{l1}\cos(\varphi_0-\zeta)+Q_{l2}\sin(\varphi_0-\zeta)\right]\frac{d\zeta}{\cos\frac{\pi}{\chi}\varphi-\cos\frac{\pi}{\chi}(\varphi_0-\zeta)}$$

$$Z_\varphi'' = -\frac{i}{2\chi}\frac{\partial}{\partial x_{0l}}\int\limits_{B_1'+B_2'}\frac{e^{-ikR_\zeta}}{R_\zeta}\left[Q_{l2}\cos(\varphi_0-\zeta)-Q_{l1}\sin(\varphi_0-\zeta)\right]\frac{\sin\frac{\pi}{\chi}(\varphi_0-\zeta)}{\cos\frac{\pi}{\chi}\varphi-\cos\frac{\pi}{\chi}(\varphi_0-\zeta)}\,d\zeta$$

$$Z_z'' = -\frac{i\sin\frac{\pi}{\chi}\varphi}{2\chi}Q_{l3}\frac{\partial}{\partial x_{0l}}\int\limits_{B_1'+B_2'}\frac{e^{-ikR_\zeta}}{R_\zeta}\frac{d\zeta}{\cos\frac{\pi}{\chi}\varphi-\cos\frac{\pi}{\chi}(\varphi_0-\zeta)} \tag{2.3}$$

From the form of the vector Z'' (2.3) follows that it satisfies the conditions (2.1). Thus we may regard the proof to be completed.

§ 3. Field in the vicinity of the edge of the wedge

Proof of the fulfilment of Meixner's condition.

It is our aim to check whether the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ (1.9) satisfy the Meixner condition which is of fundamental importance in diffraction problems. It is necessary to satisfy this condition in order that the solution of a given diffraction problem would be unique. For this purpose we have first to check what is the behaviour of the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ near edge of the wedge, *i. e.*, to obtain approximate expressions for $\boldsymbol{E}$ and $\boldsymbol{H}$ for small r-values. To simplify the calculations let us deal with the field due to the vector $\boldsymbol{Z}'$ (1.4) which is the first term of the vector $\boldsymbol{Z}''$ (1.8), *i. e.*, the field

$$\boldsymbol{E}' = \mathrm{grad}_P\,\mathrm{div}_P\,\boldsymbol{Z}'+k^2\,\boldsymbol{Z}'$$
$$\boldsymbol{H}' = ik\,\mathrm{curl}_P\,\boldsymbol{Z}' \tag{3.1}$$

This does not affect the generality of our considerations since the behaviour of the field due to the second term on the edge is the same and the calculations are made in a similar way. Long but straight-forward calculations (which are not given here) show that the following relations hold:

$$\mathrm{div}_P\,\boldsymbol{Z}' = -\mathrm{div}_L\boldsymbol{Z}^*$$

$$[\mathrm{curl}_P\,\boldsymbol{Z}']_z = -[\mathrm{curl}_L\boldsymbol{Z}^*]_z$$

which will be used in further considerations.

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The vector $\boldsymbol{Z}^*$ is defined in the following way:

$$Z_\varrho^* = -\frac{1}{2\chi}\,[Q_{l1}\cos\varphi_0 + Q_{l2}\sin\varphi_0]\,\frac{\partial}{\partial x_{l0}}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}\,\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$Z_{\varphi_0}^* = -\frac{1}{2\chi}\,[Q_{l2}\cos\varphi_0 - Q_{l1}\sin\varphi_0]\,\frac{\partial}{\partial x_{0l}}\int\limits_{B_1+B_2}\frac{e^{-ikR_\alpha}}{R_\alpha}\,\Phi_{\pi/\chi}(\alpha-\varphi_0)d\alpha$$

$$Z_z^* = Z_z'$$

To obtain approximate expressions for the vectors $\boldsymbol{Z}'$ and $\boldsymbol{Z}^*$ which would be valid in the vicinity of the edge of the wedge let us notice that according to (1.3) the brachpoints α_0^+, α_0^- shift to infinity with $r \to 0$ parallelly to the imaginary axis of α. Together with these points one can also shift to infinity the paths B_1 and B_2. In addition the following approximate relations will be also valid

for the path B_1

$$\cos(\varphi-\alpha) \cong \frac{1}{2}\,e^{i(\varphi-\alpha)}$$

$$\cos(\varphi+\varphi_0-\alpha) \cong \frac{1}{2}\,e^{i(\varphi+\varphi_0-\alpha)}$$

$$\sin(\varphi+\varphi_0-\alpha) \cong -\frac{i}{2}\,e^{i(\varphi+\varphi_0-\alpha)} \qquad (3.3a)$$

for the path B_2

$$\cos(\varphi-\alpha) \cong \frac{1}{2}\,e^{-i(\varphi-\alpha)}$$

$$\cos(\varphi+\varphi_0-\alpha) \cong \frac{1}{2}\,e^{-i(\varphi+\varphi_0-\alpha)}$$

$$\sin(\varphi+\varphi_0-\alpha) \cong \frac{i}{2}\,e^{-i(\varphi+\varphi_0-\alpha)} \qquad (3.3b)$$

If we now make the following substitutions: $\sigma = r\varrho\, e^{i(\varphi-\alpha)}$ in the case of the integral along the path B_1 and $\sigma = r\varrho\, e^{-i(\varphi-\alpha)}$ for the integral along B_2 we shall obtain with the help of (1.5) and (3.3 a and b) the following expressions for Z_r':

$$Z_r' = \frac{1}{4\chi i}\frac{\partial}{\partial x_{0l}}\left\{\frac{e^{i\varphi_0}(Q_{l1}-Q_{l2})}{r\varrho}\int\limits_U\frac{e^{-ikR_\sigma}}{R_\sigma}\,\frac{d\sigma}{1-e^{i\,\pi/\chi\,(\varphi_0-\varphi)}\sigma^{\pi/\chi}(r\varrho)^{-\pi/\chi}}\right\} -$$

$$-\frac{1}{4\chi i}\frac{\partial}{\partial x_{0l}}\left\{\frac{e^{-i\varphi_0}(Q_{l1}+Q_{l2})}{r\varrho}\int\limits_U\frac{e^{-ikR_\sigma}}{R_\sigma}\,\frac{d\sigma}{1-e^{i\,\pi/\chi\,(\varphi_0-\varphi)}\sigma^{-\pi/\chi}(r\varrho)^{\pi/\chi}}\right\} \qquad (3.4)$$

where

$$R_\sigma = [\varrho^2+(z-z_0)^2-\sigma]^{\frac{1}{2}}$$

On the complex σ plane the branch-point of the function R_σ is, as can be seen, $\sigma_0 = \varrho^2+(z-z_0)^2$. The branch-ray led from this point which corresponds to the branch rays from the points α_0^+ and α_0^- on the α plane goes to infinite along the positive real semi-axis of σ as follows from the substitutions applied. In the entire upper semi-plane σ, the imaginary part of R_α is negative, as this semi-plane corresponds to those segments of the

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plane α in which the imaginary part of R_α is negative. The integration path U which corresponds to both paths B_1 and B_2 consist of a circle around the point $\sigma = 0$ and two rays going to infinity along the positive imaginary axis of σ (Fig. 5).

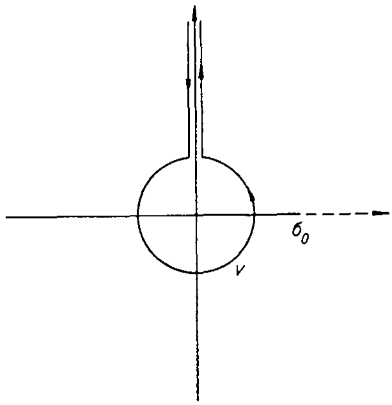

Fig. 5. σ — plane. σ_0 is the branch point of R_σ. Dashed line denotes the branch ray

For small r-values one may put in (3.4)

$$\frac{1}{1-e^{i\,\pi/\chi\,(\varphi_0-\varphi)}\sigma^{\pi/\chi}(r\varrho)^{-\pi/\chi}} \cong -\frac{e^{-i\,\pi/\chi\,(\varphi_0-\varphi)}(r\varrho)^{\pi/\chi}}{\sigma^{\pi/\chi}}$$

$$\frac{1}{1-e^{i\,\pi/\chi\,(\varphi_0-\varphi)}\sigma^{-\pi/\chi}(r\varrho)^{\pi/\chi}} \cong 1+\frac{e^{i\,\pi/\chi\,(\varphi_0-\varphi)}(r\varrho)^{\pi/\chi}}{\sigma^{\pi/\chi}}$$

We shall then have for Z_r'

$$Z_r' = -\frac{1}{4\chi i}\frac{\partial}{\partial x_{0l}}\left\{\frac{e^{-i\varphi_0}(Q_{l1}+iQ_{l2})}{r\varrho}\int\limits_U \frac{e^{-ikR_\sigma}}{R_\sigma}\,d\sigma\right\} -$$

$$-\frac{1}{2\chi i}\frac{\partial}{\partial x_{0l}}\left\{\frac{Q_{l1}\cos\left[\left(1-\frac{\pi}{\chi}\right)\varphi_0+\frac{\pi}{\chi}\varphi\right]+Q_{l2}\sin\left[\left(1-\frac{\pi}{\chi}\right)\varphi_0+\frac{\pi}{\chi}\varphi\right]}{(r\varrho)^{1-\pi/\chi}}\int\limits_U \frac{e^{-ikR_\sigma}}{R_\sigma}\frac{d\sigma}{\sigma^{\pi/\chi}}\right\}$$

The integrand of the first integral contains no branch point inside U and hence integration along rays going to infinity along the positive imaginary axis given contributions which cancel one another. There remains a closed line integral which vanishes according to the Cauchy theorem. The second integral cannot be calculated since inside U there is a branch point of the factor $\sigma^{\pi/\chi}$; it is however independent of r. Therefore using the same procedure with the components Z_φ', Z_z', Z_ϱ^*, $Z_{\varphi_0}^*$ we can write

$$Z_r' \cong A_1(\varphi, z, \varrho, \varphi_0, z_0, \chi)r^{\pi/\chi-1} \qquad Z_\varrho^* \cong A_4(\varphi, z, \varrho, \varphi_0, z_0, \chi)r^{\pi/\chi}$$

$$Z_\varphi' \cong A_2(\varphi, z, \varrho, \varphi_0, z_0, \chi)r^{\pi/\chi-1} \qquad Z_{\varphi_0}^* \cong A_5(\varphi, z, \varrho, \varphi_0, z_0, \chi)r^{\pi/\chi}$$

$$Z_z' \cong A_3(\varphi, z, \varrho, \varphi_0, z_0, \chi)r^{\pi/\chi}$$

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From Eqs (3.1) and (3.2) it follows that in the vicinity of the edge we have:

$$E_r \sim r^{\pi/\chi-1} \qquad H_r \sim r^{\pi/\chi-1}$$
$$E_\varphi \sim r^{\pi/\chi-1} \qquad H_\varphi \sim r^{\pi/\chi-1}$$
$$E_z \sim r^{\pi/\chi} \qquad H_z \sim r^{\pi/\chi}$$

The same result is obtained by differentiating (1.4) according to (3.1) and calculating asymptotic approximate expressions for $\boldsymbol{E}'$ and $\boldsymbol{H}'$ directly from the formulae obtained in this way.

The Meixner condition requires that the energy density of the field $w = (E^2+H^2)/8\pi$ should be integrable with respect to space coordinates in the vicinity of the edge. Is it sufficient for this purpose that we had no singularity greater than $r^{-2\alpha}$, where $\alpha < 1$. In our case $w \sim r^{2\pi/\chi-2}$ which satisfies this condition.

§ 4. Proof of the fulfilment of the Sommerfeld conditions

The Sommerfeld conditions concern the behaviour of the field at infinity. In particular the finiteness condition requires:

$$\lim_{r\to\infty} r\boldsymbol{E} = \text{finite} \tag{4.1a}$$
$$\lim_{r\to\infty} r\boldsymbol{H} = \text{finite}$$

The radiation emission condition has the form

$$\lim_{r\to\infty} r(\boldsymbol{E}+\boldsymbol{r}^*\times\boldsymbol{H}) = 0 \tag{4.1b}$$
$$\lim_{r\to\infty} r(\boldsymbol{H}-\boldsymbol{r}^*\times\boldsymbol{E}) = 0$$

where r^* is the unit vector in the direction of the propagation of the wave at infinity.

Similarly as in the previous section we shall deal with the field due to the vector $\boldsymbol{Z}^r$ or more precisely only with the diffraction field which is obtained from the vector $\boldsymbol{Z}'^d$. The geometrical-optics -field which is given by $\boldsymbol{Z}'^g$ (1.6) is normal quadrupole radiation from the sources $L_m(\varrho, \varphi_0+2m\chi, z_0)$ and as such fulfils the Sommerfeld conditions. However, before proving that the diffraction field has the same property we will obtain an asymptotic expression for $\boldsymbol{Z}'^d$ by making use of the saddle-point method. For this purpose let us write the component $Z_r'^d$ with the help of the transform of the vector $\boldsymbol{Z}'$ (1.4) in the form:

$$Z_r'^d = \frac{1}{2\chi}\int_{C_1+C_2} e^{g(\alpha)}d\alpha$$

where:

$$g(\alpha) = -ikR_\alpha + \ln R_\alpha^{-2}\Phi_{\pi/\chi}(\alpha-\varphi_0)\{[r\cos(\varphi+\varphi_0-\alpha)-\varrho\cos\varphi_0][Q_{11}\cos(\varphi+\varphi_0-\alpha)+$$
$$+Q_{12}\sin(\varphi+\varphi_0-\alpha)]+[r\sin(\varphi+\varphi_0-\alpha)-\varrho\sin\varphi_0][Q_{12}\cos(\varphi+\varphi_0-\alpha)+Q_{22}\sin(\varphi+\varphi_0-\alpha)]$$
$$+(z-z_0)[Q_{13}\cos(\varphi+\varphi_0-\alpha)+Q_{23}\sin(\varphi+\varphi_0-\alpha)].$$

If we assume that k is large, the greatest contribution to the integral from the function $g(\alpha)$ will come from the term $-ikR_\alpha$. Because of this we can replace the condition $\partial g(\alpha)/\partial\alpha = 0$, from which the saddle points are determined, by the condition $\partial R(\alpha)/\partial\alpha = 0$. The saddle points will be given by the equations: $\alpha = \varphi \pm n\pi$ $(n = 0, \pm 1, \pm 2, \ldots)$. Using a similar procedure as in the scalar case of an isotropic, spherical wave (Rubinowicz 1966) we find that the steepest descent paths are given by the equations: $\alpha(ds) = \varphi \pm \pi + se^{i\pi/4}$. Putting for all functions of α except $-ikR_\alpha$ appearing in $g(\alpha)$ their values at the saddle point, we obtain for $Z_r'^d$ and in an identical way for the remaining components:

$$Z_r'^d \cong \frac{e^{i\,^3/_4\pi}\sqrt{2\pi}\,(\cos\varphi_0 + \sin\varphi_0)\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0) - \cos\dfrac{\pi^2}{\chi}\right]}\,\frac{e^{-ikR_0}}{R_0^{3/2}\sqrt{kr\varrho}}\,\{(r+\varrho)[(Q_{11}+Q_{12})\cos\varphi +$$

$$+(Q_{12}+Q_{22})\sin\varphi_0] - (z-z_0)(Q_{13}+Q_{23})\}$$

$$Z_\varphi'^d \cong \frac{e^{i\,^3/_4\pi}\sqrt{2\pi}\,(\cos\varphi_0 - \sin\varphi_0)\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0) - \cos\dfrac{\pi^2}{\chi}\right]}\,\frac{e^{-ikR_0}}{R_0^{3/2}\sqrt{kr\varrho}}\,\{(r+\varrho)[(Q_{12}-Q_{11})\cos\varphi_0 +$$

$$+(Q_{22}-Q_{12})\sin\varphi_0] - (z-z_0)(Q_{23}-Q_{13})\}$$

$$Z_z'^d \cong \frac{e^{i\,^3/_4\pi}\sqrt{2\pi}\,\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0) - \cos\dfrac{\pi^2}{\chi}\right]}\,\frac{e^{-ikR_0}}{R_0^{3/2}\sqrt{kr\varrho}}\,[(r+\varrho)(Q_{13}\cos\varphi_0 + Q_{23}\sin\varphi_0) - Q_{33}(z-z_0)]$$

where

$$R_0 = [(r+\varrho)^2 + (z-z_0)^2]^{\frac{1}{2}} \tag{4.2}$$

It can be seen that the expressions (4.2) become infinite at points lying at each boundary of the shadow $\varphi = \varphi_0 + 2m\chi \pm \pi$. This results from the fact that the diffraction field is discontinuous at the boundary of the shadow and the jump which occurs here must compensate the jump of the geometrical wave. Eqs (4.2) will be valid only for points at a large distance from the boundaries of the shadow. For regions in the vicinity of these boundaries we have to look for other asymptotic approximations. For this purpose we introduce auxiliary functions (Rubinowicz 1966) by putting

$$Z_r^{**d} = \frac{\partial}{\partial k}\left(e^{ikR_m} Z_r'^d\right)$$

where

$$R_m = [r^2 + \varrho^2 + (z-z_0)^2 - 2r\varrho\cos(\varphi - \varphi_0 - 2m\chi)]^{\frac{1}{2}}$$

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Applying the saddle point method to Z_r^{**d}, integrating between $+\infty$ and k, and multiplying the expressions obtained by e^{-ikR_m} we obtain an expression for $Z_r'^d$ which will be valid both for from and near the boundary of the shadow

$$Z_r'^d = \frac{\pi e^{i\pi/4} \sin\frac{\pi^2}{\chi} \sqrt{R_0-R_m}(\cos\varphi_0+\sin\varphi_0)e^{-ikR_m}}{\chi\left[\cos\frac{\pi}{\chi}(\varphi-\varphi_0)-\cos\frac{\pi^2}{\chi}\right]\sqrt{r\varrho}\,R_0^{3/2}}\{(r+\varrho)[(Q_{11}+Q_{12})\cos\varphi_0+$$

$$+(Q_{12}+Q_{22})\sin\varphi_0]-(z-z_0)(Q_{13}+Q_{23})\}\int\limits_{+\infty}^{\sqrt{2k/\pi\,(R_0-R_m)}} e^{-i\pi v^2/2}dv$$

After introducing the diffraction angle $\psi = \varphi-(\varphi_0+2m\chi\pm\pi)$ for the m-th shadow boundary $\varphi = \varphi_0+2m\chi = \pi$ and making use of the fact that $\sqrt{R_0-R_m} = 2[r\varrho/R_0+R_m]^{\frac{1}{2}}|\sin\psi/2|$ as well as assuming that for small $\psi|\sin\psi/2| = |\psi/2|$ and $R_m = R_0$ we have:

$$Z_r'^d \cong \frac{e^{-ikR_0}}{2R_0^2}\operatorname{sgn}\psi(\cos\varphi_0+\sin\varphi_0)\{(r+\varrho)[(Q_{11}+Q_{12})\cos\varphi_0+(Q_{12}+Q_{22})\sin\varphi_0]-$$

$$-(z-z_0)(Q_{13}+Q_{23})\}$$

$$Z_r'^d \cong \frac{e^{-ikR_0}}{2R_0^2}\operatorname{sgn}\psi(\cos\varphi_0-\sin\varphi_0)\{(r+\varrho)[(Q_{12}-Q_{11})\cos\varphi_0+(Q_{22}-Q_{12})\sin\varphi_0)]-$$

$$-(z-z_0)(Q_{23}-Q_{13})\}$$

$$Z_z'^d \cong -\frac{e^{-ikR_0}}{2R_0^2}\operatorname{sgn}\psi[(r+\varrho)(Q_{13}\cos\varphi_0+Q_{23}\sin\varphi_0)-Q_{33}(z-z_0)] \tag{4.3}$$

It is our aim to investigate what is the behaviour of the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ for large r. From (3.1) as well as (4.2) and (4.3) taking into consideration (because of (4.1a) and b)) only terms proportional to r^α such that $\alpha \geqslant -1$ we obtain for the points for from the shadow boundary.

$$E_r^d \cong \frac{k^2 e^{i^3/_4\pi}\sqrt{2\pi}(\cos\varphi_0+\sin\varphi_0)\sin\frac{\pi^2}{\chi}(r+\varrho)\,e^{-ikR_0}}{2\chi\left[\cos\frac{\pi}{\chi}(\varphi-\varphi_0)-\cos\frac{\pi^2}{\chi}\right]\sqrt{kr\varrho}\,R_0^{3/2}}\left[1-\frac{(r+\varrho)^2}{R_0^2}\right][(Q_{11}+Q_{12})\cos\varphi_0+$$

$$+(Q_{12}+Q_{22})\sin\varphi_0]$$

$$E_\varphi^d \cong \frac{k^2 e^{i^3/_4\pi}\sqrt{2\pi}(\cos\varphi_0-\sin\varphi_0)\sin\frac{\pi^2}{\chi}(r+\varrho)\,e^{-ikR_0}}{2\chi\left[\cos\frac{\pi}{\chi}(\varphi-\varphi_0)-\cos\frac{\pi^2}{\chi}\right]\sqrt{kr\varrho}\,R_0^{3/2}}[(Q_{12}-Q_{11})\cos\varphi_0+(Q_{22}-Q_{12})\sin\varphi_0]$$

$$E_z^d \cong -\frac{k^2 e^{i^3/_4\pi}\sqrt{2\pi}\sin\frac{\pi^2}{\chi}(r+\varrho)\,e^{-ikR_0}}{2\chi\left[\cos\frac{\pi}{\chi}(\varphi-\varphi_0)-\cos\frac{\pi^2}{\chi}\right]\sqrt{kr\varrho}\,R_0^{3/2}}(Q_{13}\cos\varphi_0+Q_{23}\sin\varphi_0)$$

$$H_r^d = 0$$

$$H_\varphi^d \cong \frac{k^2 e^{i^3/_4 \pi}\sqrt{2\pi}\sin\dfrac{\pi^2}{\chi}(r+\varrho)^2 e^{-ikR_0}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]\sqrt{kr\varrho}\,R_0^{5/2}}(Q_{13}\cos\varphi_0+Q_{23}\sin\varphi_0)$$

$$H_z^d \cong \frac{k^2 e^{i^3/_4 \pi}\sqrt{2\pi}(\cos\varphi_0-\sin\varphi_0)\sin\dfrac{\pi^2}{\chi}(r+\varrho)^2 e^{-ikR_0}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]\sqrt{kr\varrho}\,R_0^{5/2}}[(Q_{12}-Q_{11})\cos\varphi_0+$$

$$+(Q_{22}-Q_{12})\sin\varphi_0] \tag{4.4}$$

and for points in the vicinity of the boundary of the shadow

$$E_r^d \cong \frac{k^2 e^{-ikR_0}}{2R_0^2}\operatorname{sgn}\psi(\cos\varphi_0+\sin\varphi_0)\left[1-\frac{(r+\varrho)^2}{R_0^2}\right][(Q_{11}+Q_{12})\cos\varphi_0+(Q_{12}+Q_{22})\sin\varphi_0]$$

$$E_\varphi^d \cong \frac{k^2 e^{-ikR_0}}{2R_0^2}\operatorname{sgn}\psi\,(\cos\varphi_0-\sin\varphi_0)(r+\varrho)[(Q_{12}-Q_{11})\cos\varphi_0+(Q_{22}-Q_{12})\sin\varphi_0]$$

$$E_z^d \cong -\frac{k^2 e^{-ikR_0}}{2R_0^2}\operatorname{sgn}\psi\,(r+\varrho)(Q_{13}\cos\varphi_0+Q_{23}\sin\varphi_0)$$

$$H_r^d = 0$$

$$H_\varphi^d \cong \frac{k^2 e^{-ikR_0}}{2R_0^3}\operatorname{sgn}\psi\,(r+\varrho)^2(Q_{13}\cos\varphi_0+Q_{23}\sin\varphi_0)$$

$$H_z^d \cong \frac{k^2 e^{-ikR_0}}{2R_0^3}\operatorname{sgn}\psi(r+\varrho)^2(\cos\varphi_0-\sin\varphi_0)[(Q_{12}-Q_{11})\cos\varphi_0+(Q_{22}-Q_{12})\sin\varphi_0] \tag{4.5}$$

We can now check the Sommerfeld conditions. From Eqs (4.4) we have for points far from the shadow boundary:

$$\lim_{r\to\infty} r\boldsymbol{E}^d = \frac{k^2 e^{i^3/_4 \pi}\sqrt{2\pi}\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\frac{e^{-ikR_0}}{\sqrt{k\varrho}}\boldsymbol{a}$$

$$\lim_{r\to\infty} r\boldsymbol{H}^d = \frac{k^2 e^{i^3/_4 \pi}\sqrt{2\pi}\sin\dfrac{\pi^2}{\chi}}{2\chi\left[\cos\dfrac{\pi}{\chi}(\varphi-\varphi_0)-\cos\dfrac{\pi^2}{\chi}\right]}\frac{e^{-ikR_0}}{\sqrt{k\varrho}}\boldsymbol{b}$$

where

$$a_r = 0,\ a_\varphi = (\cos\varphi_0-\sin\varphi_0)[(Q_{12}-Q_{11})\cos\varphi_0+(Q_{22}-Q_{12})\sin\varphi_0],\ a_z = -(Q_{13}\cos\varphi_0+$$

$$+Q_{23}\sin\varphi_0)$$

$$b_r = 0,\ b_\varphi = -a_z,\ b_z = a_\varphi$$

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and from Eqs (4.5) for points in the vicinity of the shadow:

$$\lim_{r\to\infty} r\,\boldsymbol{E}^d = \frac{k^2 e^{-ikR_0}}{2}\operatorname{sgn}\psi\,\boldsymbol{a}$$

$$\lim_{r\to\infty} r\,\boldsymbol{H}^d = \frac{k^2 e^{-ikR_0}}{2}\operatorname{sgn}\psi\,\boldsymbol{b}$$

The condition of finiteness is thus satisfied and besides it can be seen that at infinity the wave propagates in the direction defined by the vector $\boldsymbol{i}_r$. In (4.1b) we can thus put $\boldsymbol{r}^* = \boldsymbol{i}_r$, however, because of $\boldsymbol{i}_r \times \boldsymbol{b} = -\boldsymbol{a}$ and $\boldsymbol{i}_r \times \boldsymbol{a} = \boldsymbol{b}$ the radiation emission condition is satisfied as well, both in the vicinity and far from the shadow boundary. We have proved in this way that (1.9) fulfils all postulated conditions of Sect. 1 *i.e.* (1)–(5). We have thus shown that (1.9) is a unique solution of our diffraction problem.

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