LABORATORY EQUIPMENT AND TECHNIQUES

INFLUENCE OF DOMAIN WIDTH ON THE THICKNESS AND ENERGY OF SYMMETRIC BLOCH WALLS IN FACE-CENTRED CUBIC FERROMAGNETIC LATTICES

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The influence of the domain width on the thickness and energy of symmetric Bloch walls in the *fcc* ferromagnetic lattice is numerically examined, by utilizing the formulae derived for those quantities in an earlier paper (Wachniewski, Ziętek 1967) where the variational principles for the walls were solved under periodic boundary conditions. For each type of wall, the critical domain width is determined below which the departure of the examined quantities from their asymptotic values (as used in practice) becomes significant. As an example, specific results for Ni are provided.

1. *Introduction*

The variational principles derived in [1] for the various symmetric[1] Bloch walls that may occur in cubic ferromagnetic crystals were solved in [3] and [4–6] for the case of the *bcc* and *fcc* lattice, respectively, under periodic boundary conditions. The solutions thus obtained are expressed in terms of Jacobi's elliptic functions, the modulus k depending on material constants and the domain width Δ. The formulae for the thickness δ and the energy σ (per cm^2) of the Bloch walls derived in [3–6] from the periodic solutions involve, generally, the modulus k, the incomplete $F(\varphi, k)$ and complete $\boldsymbol{K} = \boldsymbol{K}(k)$ elliptic integrals of the first

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[1] See Footnote 1 in [2] for the definition of a symmetric Bloch wall.

kind, and the complete elliptic integrals $\boldsymbol{E} = \boldsymbol{E}(k)$ and $\boldsymbol{\Pi}(n, k)$ of the second and third kind, respectively[2]. Since φ and n are in that case functions of k, the thickness and the energy depend — apart from material constants — on the domain width Δ, *i.e.*, $\delta = \delta(\Delta)$ and $\sigma = \sigma(\Delta)$.

With the exception of the (71°[110]180°) Bloch wall[3], for all the symmetric walls considered in [3–5] the relationship between k and Δ is unique and such that

$$0 \leqslant \Delta(k) < \Delta(k') \leqslant \infty, \qquad (1)$$

$$0 \leqslant k(\Delta) < k(\Delta') \leqslant 1$$

for

$$0 \leqslant k < k' \leqslant 1, \quad 0 \leqslant \Delta < \Delta' \leqslant \infty. \qquad (2)$$

Hence, the limiting processes $k \to 0$, $k \to 1$ and $\Delta \to 0$, $\Delta \to \infty$ are respectively equivalent.

As regards the (71°[110]180°) wall, it was shown in [6] that, depending on the value of the first integration constant, there are three different periodic solutions of the variational principle which lead to different relationships between k and Δ, each corresponding to a specific interval for Δ. As shown in Appendix I, Eqs (I.16), (I.23) and (I.25), this splits the whole interval $\langle 0, \infty \rangle$ for Δ in three intervals: $\langle 0, \Delta_1 \rangle$, $\langle \Delta_1, \Delta_2 \rangle$ and $\langle \Delta_2, \infty \rangle$ in which different functions $\Delta(k)$ must be used, with corresponding intervals for k. With k° as defined by Eq. (17) in [6], *i.e.*,

$$k^\circ = \sqrt{(11-4\sqrt{7})/22} \approx 0.1, \qquad (3)$$

we have for this wall instead of (1), (2) the set of inequalities:

$$0 \leqslant \Delta(k) < \Delta(k') \leqslant \Delta_1,$$

$$0 \leqslant k(\boldsymbol{k}) < k(\Delta') \leqslant k^0 \qquad (4)$$

for

$$0 \leqslant k < k' \leqslant k^0, \quad 0 \leqslant \Delta < \Delta' \leqslant \Delta_1 \qquad (5)$$

with $\Delta(k)$ defined by choosing the positive sign in Eq. (18) in [6];

$$\Delta_1 \leqslant \Delta(k) < \Delta(k') \leqslant \Delta_2,$$

$$k^0 \geqslant k(\Delta) > k(\Delta') \geqslant 0 \qquad (6)$$

for

$$k^0 \geqslant k > k' \geqslant 0, \quad \Delta_1 \leqslant \Delta < \Delta' \leqslant \Delta_2 \qquad (7)$$

with $\Delta(k)$ defined by choosing the negative sign in Eq. (18) in [6]; and

$$\Delta_2 \leqslant \Delta(k) < \Delta(k') \leqslant \infty,$$

$$0 \leqslant k(\Delta) < k(\Delta') \leqslant 1 \qquad (8)$$

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[2] A systematic error has been made in calculating the energies σ in [3, 4]; correct formulae for the *bcc* case are given in Appendix III in [2], and for the *fcc* case in Appendix I of the present paper.

[3] In denoting the Bloch walls we use the notation introduced in [1] and refined in [7] (see also [2]).

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for

$$0 \leqslant k < k' \leqslant 1, \qquad \Delta_2 \leqslant \Delta < \Delta' \leqslant \infty \tag{9}$$

with $\Delta(k)$ defined by Eq. (8) in [6]. Hence, in each of the intervals (5), (7), (9) the relationship between Δ and k is unique and the limiting processes $\Delta \to 0$, $\Delta \to \Delta_1$, $\Delta \to \Delta_2$, $\Delta \to \infty$ are equivalent to $k \to 0$, $k \to k^\circ$, $k \to 0$, $k \to 1$ in the respective intervals. Of course, the functions $\delta(\Delta)$ and $\sigma(\Delta)$ differ accordingly in each of the intervals; however, one can show that there is no discontinuity at the points $\Delta = \Delta_1$ and $\Delta = \Delta_2$ (see Section 2b and Appendix I). Thus, with the notation

$$\delta_1 \equiv \delta(\Delta_1), \qquad \delta_2 \equiv \delta(\Delta_2), \qquad \delta_\infty \equiv \delta(\infty);$$
$$\sigma_1 \equiv \sigma(\Delta_1), \qquad \sigma_2 \equiv \sigma(\Delta_2), \qquad \sigma_\infty \equiv \sigma(\infty) \tag{10}$$

and because of $\delta(0) = 0$, $\sigma(0) = \infty$ (see Section 2b) one has for $\delta(\Delta)$ and $\sigma(\Delta)$ the intervals

$$\langle 0, \delta_1 \rangle, \qquad \langle \delta_1, \delta_2 \rangle, \qquad \langle \delta_2, \delta_\infty \rangle; \tag{11}$$

$$\langle \infty, \sigma_1 \rangle, \qquad \langle \sigma_1, \sigma_2 \rangle, \qquad \langle \sigma_2, \sigma_\infty \rangle \tag{12}$$

that correspond respectively to the intervals (5), (7), (9) for k and Δ.

Similarly as in [2] for the *bcc* case, one can show that for all the symmetric Bloch walls in the *fcc* lattice the functions $\delta(\Delta)$ and $\sigma(\Delta)$ derived in [4–6] — with the corrections for the latter given in Appendix I — have the following properties:

$$1 \geqslant \frac{\delta(\Delta)}{\Delta} > \frac{\Delta'}{\delta(\Delta')} \geqslant 0, \tag{13}$$

$$0 \leqslant \delta(\Delta) < \delta(\Delta') \leqslant \delta_\infty < \infty, \tag{14}$$

$$\infty \geqslant \sigma(\Delta) > \sigma(\Delta') \geqslant \sigma_\infty > 0 \tag{15}$$

for $0 \leqslant \Delta < \Delta' \leqslant \infty$. Here, $\delta_\infty = \delta(\infty)$ and $\sigma_\infty = \sigma(\infty)$ are the so-called assymptotic values of the Bloch wall thickness and energy as obtained from non-periodic solutions of the variational principles [8, 9]. These values are commonly used in practice (see, *e.g.*, [10—12]) and, since $\delta \to 0$ and $\sigma \to \infty$ as $\Delta \to 0$, the question arises how large must be the domain width Δ, for a given material and a particular Bloch wall, in order that the asymptotic values δ_∞ and σ_∞ be fair approximations of the exact ones $\delta(\Delta)$ and $\sigma(\Delta)$. The rough estimation carried through in [13] led to the inequality

$$0 < \sigma(\Delta) - \sigma_\infty < \mathrm{const}/\Delta, \tag{16}$$

and in [7] the formula

$$\Delta_c = 10\, \delta_\infty \tag{17}$$

for the critical domain width Δ_c was derived below which $\delta/\Delta > 0.1$. In [2], the results of a systematic numerical analysis were given showing the dependence of δ/δ_∞, σ/σ_∞ and δ/Δ on Δ in the region where the departure of δ and σ from their asymptotic values becomes significant, for the symmetric Bloch walls in the *bcc* lattice. The purpose of the present paper is

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in carrying through an analogous numerical analysis for the symmetric Bloch walls in the *fcc* lattice.

Of the six walls considered in [4—6], *i.e.*, the (71°[100]90°), (71°[110]180°), (180°[112]180°), (109°[111]120°), (71°[110]71°) and (109°[110]109°) Bloch wall, only the first four are examined in the present paper, as for the last two walls more accurate tables of elliptic integrals would have to be used than those currently available [14].

No models of the walls are given here, as they can be found in [4, 5].

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2. *Numerical calculations*

The microscopic material constants A, K, a, S used in [1, 4] and denoting respectively the (negative) exchange integral, the microscopic anisotropy constant (which is negative for the *fcc* lattice), the lattice constant, and the maximum spin eigenvalue, can for the *fcc* lattice be replaced by the conventional macroscopic (positive) exchange constant A^{fcc} and the first phenomenological anisotropy constant K_1^{fcc} (which is negative for the *fcc* lattice) as follows:

$$A^{fcc} = -4a^{-1} AS^2, \qquad K_1^{fcc} = 4a^{-3} KS^2(S-\tfrac{1}{2})^2 \tag{18}$$

(see [7, 9]). Upon introducing the quantities

$$\delta_0 = \sqrt{-A^{fcc}/K_1^{fcc}}, \qquad \sigma_0 = \sqrt{-A^{fcc}K_1^{fcc}} \tag{19}$$

depending on the material and having respectively the dimensions of length and energy per surface area the formulae derived in [4–6] (with the corrections given in Appendix I of the present paper) for the Bloch wall energy σ and thickness δ, as well as the relations between the domain width Δ and the modulus k can for each Bloch wall be written in the form

$$\Delta = \delta_0 D(k), \qquad \delta = \delta_0\, d(k), \qquad \sigma = \sigma_0 f(k), \tag{20}$$

where D, d and f are dimensionless functions of the modulus k only which characterize the type of wall regardless of the material. Accordingly, the asymptotic values defined by Eqs (14), (15) follow from Eq. (20) for $k \to 1$, *i.e.*,

$$\delta_\infty = \delta_0 \lim_{k\to 1} d(k) \equiv \delta_0 d(1), \tag{21}$$

$$\sigma_\infty = \sigma_0 \lim_{k\to 1} f(k) = \sigma_0 f(1),$$

and, like in [2] for the *bcc* lattice, the functions to be numerically examined in the present paper are the dimensionless ratios

$$\frac{\delta}{\delta_\infty} = \frac{d(k)}{d(1)}, \quad \frac{\sigma}{\sigma_\infty} = \frac{f(k)}{f(1)}, \quad \frac{\delta}{\Delta} = \frac{d(k)}{D(k)} \equiv R(k), \tag{22}$$

of which the first two describe the departure of δ and σ from their asymptotic values with decreasing domain width Δ, while the last one illustrates the gradual transition of the well-defined domain structure for large Δ (with domains much wider than the narrow, sharply pronounced Bloch walls separating them) to a spiral-like magnetic structure for small Δ.

The procedure applied in carrying out the numerical calculations is much the same as in [2] and therefore requires no further explanation. As in [2], in the diagrams presented here the ratios (22) are plotted as functions of the domain width Δ which, in order that our results be applicable to any material, is measured in δ_0 as unit length. The latter quantity — as well as σ_0 — is to be specified for each material. For instance, according to [11], p. 293, one has[4] for Ni at room temperature:

$$A^{fcc} = 3.4\times10^{-7}\text{erg/cm}, \quad K_1^{fcc} = -0.5\times10^5\text{erg/cm}^3,$$
$$\delta_0 = 260\ \text{Å}, \quad \sigma_0 = 0.13\ \text{erg/cm}^2. \tag{23}$$

Each diagram is supplemented by a table containing a representative set of numerical data for the ratios (22), so that each curve can be reconstructed for practical purposes. Moreover, specific results for Ni are provided to illustrate the dependence of δ and σ on Δ for a typical four-axial ferromagnet with *fcc* lattice.

a) The (71°[100]90°) Bloch wall

With the coefficients u_0, u_4 from Table I in [4],

$$u_0 = -\frac{2}{3}\,a^2\,AS^2, \quad u_4 = -\frac{1}{18}\,KS^2\left(S-\frac{1}{2}\right)^2, \tag{24}$$

we have from Eqs (18), (19) and Eq. (I.7) in Appendix I the relation

$$\Delta = \sqrt{6}k\boldsymbol{K}\delta_0 \equiv \delta_0 D(k) \tag{25}$$

between the domain width Δ and the modulus k, and from Eq. (13) in [4] and Eq. (I.8) in Appendix I the formulae

$$\delta = \pi k\sqrt{3/2}\,\delta_0 \equiv \delta_0 d(k), \tag{26}$$

$$\sigma = \sigma_0(\sqrt{6}/9k)\{2\boldsymbol{E}-(1-k^2)\boldsymbol{K}\} \equiv \sigma_0 f(k) \tag{27}$$

for the Bloch wall thickness and energy.

The limit values for $k\to0$, $k\to1$ of the functions (22) as defined by Eqs (25)–(27) are

$$d(0) = D(0) = R(1) = 0, \quad R(0) = 1, \quad D(1) = f(0) = \infty,$$
$$d(1) = \pi\sqrt{3/2} \approx 3.847, \quad f(1) = 2\sqrt{6}/9 \approx 0.544, \tag{28}$$

which proves Eqs (13)–(15) and, due to Eq. (21) leads to

$$\delta_\infty = 3.85\ \delta_0, \quad \sigma_\infty = 0.54\ \sigma_0. \tag{29}$$

For Ni we obtain from Eq. (23)

$$\delta_\infty = 1000\ \text{Å}, \quad \sigma_\infty = 0.07\ \text{erg/cm}^2. \tag{30}$$

[4] It is to be noted that the accuracy in determining experimentally the material constants —particularly A^{fcc} — is still very poor, which results in remarkable differences between the values quoted by different authors. For example, according to [12], p. 53, we have for Ni at room temperature: $A^{fcc} = 8.6\times10^{-7}$ erg/cm, $K_1^{fcc} = -0.42\times10^5$ erg/cm³, which leads to $\delta_0 = 450$ Å, nearly twice the value from Eq. (23).

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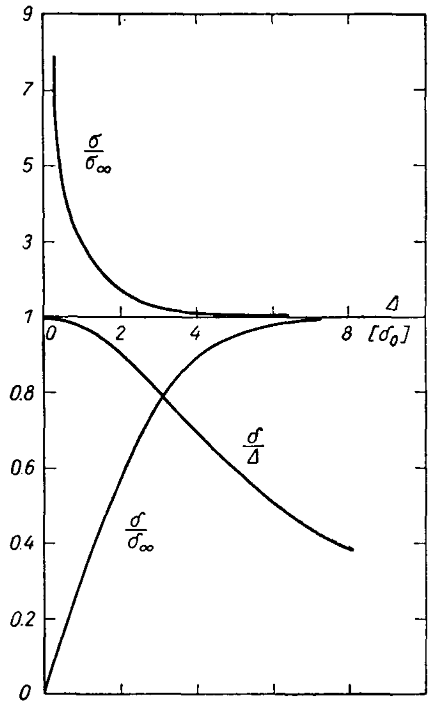

Fig. 1. Curves of the ratios (22) as defined by Eq. (31) for the (71°[100]90°) and (109°[111]120°) Bloch walls (cp. Table I)

TABLE I

Representative set of numerical data for Δ/δ_0 and the ratios (22) as defined by Eqs (25), (31) for the (71°[100]90°) and (109°[111]120°) Bloch walls (cp. Fig. 1), with results specified for Ni in the last four columns

k^2	Δ/δ_0	δ/δ_∞	δ/Δ	σ/σ_∞	Ni			
					Δ [Å]	δ [Å]	σ [erg/cm^2]	
							(71°|90°)	(109°|120°)
0	0	0	1	∞	0	0	∞	∞
0.01	0.39	0.100	0.996	7.87	101	100	0.558	1.320
0.1	1.25	0.316	0.972	2.55	326	318	0.180	0.427
0.3	2.31	0.548	0.914	1.54	600	550	0.109	0.259
0.5	3.22	0.707	0.845	1.26	837	708	0.089	0.211
0.7	4.27	0.837	0.755	1.11	1110	838	0.079	0.186
0.9	6.01	0.949	0.608	1.03	1560	950	0.073	0.173
0.99	9.04	0.995	0.424	1.00	2350	996	0.071	0.168
1	∞	1	0	1	∞	1000	0.071	0.168

According to Eqs (25)–(28) the ratios (22) take the form

$$\delta/\delta_\infty = k, \qquad \sigma/\sigma_\infty = 1.852 f(k), \qquad \delta/\Delta = \pi/2\boldsymbol{K} \tag{31}$$

with $f(k)$ from Eq. (27). These ratios are involved functions of Δ, as k is an involved function of Δ defined by Eq. (25). The dependence of the ratios (31) on Δ (in units of δ_0) is shown in Fig. 1. A representative set of numerical data is given in Table I, along with specific results for Ni. The first two columns show the dependence of Δ on k (or vice versa), according to Eq. (25).

b) The (71°[110]180°) Bloch wall

From Table I in [4] we have

$$u_0 = -\frac{1}{3}a^2 AS^2, \quad u_2 = -\frac{11}{72} KS^2\left(S-\frac{1}{2}\right)^2, \tag{32}$$

and from Eq. (30) in [4] and Eqs (18), (19), (I.10) and (I.13)

$$\Delta = 8\sqrt{3}\,\delta_0\boldsymbol{K}/\sqrt{11\omega-3} \equiv \delta_0 D(k), \tag{33}$$

$$\delta = 4\pi\delta_0\sqrt{3\omega/11}/(\omega+1) \equiv \delta_0 d(k), \tag{34}$$

$$\sigma = \sigma_0\frac{1}{9}\sqrt{3(11\omega-3)}\Big\{\boldsymbol{E} + \frac{11(\omega-1)}{2\omega(11\omega-3)}\left[2(\omega+1)\,\boldsymbol{\Pi}(-\omega^{-2}, k)-(3\omega+1)\,\boldsymbol{K}\right]\Big\} \equiv \sigma_0 f(k) \tag{35}$$

where $\omega = \omega(k)$ is defined by

$$k^2 = (11-3\omega)/\omega(11\omega-3) \tag{36}$$

from Eq. (26) in [4].

According to [6], formulae (33)–(36) are restricted to the interval (see Appendix I)

$$\Delta_2 \leqslant \Delta \leqslant \infty \tag{37}$$

where

$$\Delta_2 = 3\pi\delta_0/\sqrt{7} \approx 3.6\,\delta_0 \tag{38}$$

from Eqs (18), (19), (32) and (I.16). For $0 \leqslant \Delta \leqslant \Delta_2$ one must use the formulae

$$\Delta = 8\delta_0\boldsymbol{K}\sqrt{3\Omega/11(\Omega^2-1)} \equiv \delta_0\tilde{D}(k), \tag{39}$$

$$\delta = 4\pi\delta_0\sqrt{3\Omega/11}/(\Omega+1) \equiv \delta_0\tilde{d}(k), \tag{40}$$

$$\sigma = \frac{\sigma_0}{3}\sqrt{11(\Omega^2-1)/3\Omega}\Big\{\boldsymbol{E} + \frac{\Omega^2}{\Omega^2-1}\boldsymbol{\Pi}((\Omega^2-1)^{-1}, k) - \frac{3\Omega+1}{2(\Omega+1)}\boldsymbol{K}\Big\} \equiv \sigma_0\tilde{f}(k) \tag{41}$$

that follow from Eqs (18), (19), (32), (I.17) and (I.20), where $\Omega = \Omega(k)$ is defined by

$$k^2 = (3\Omega-11)/11(\Omega^2-1) \tag{42}$$

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from Eq. (15) in [6]. As shown in Appendix I, Eqs (I.22)–(I.26), of the two functions $\Omega^{\pm}(k)$ following from Eq. (42) (according to whether the plus or minus sign is chosen for the square root; see Eq. (16) in [6]) one must take $\Omega^{-}(k)$ in Eqs (39)–(42) for

$$\Delta_1 \leqslant \Delta \leqslant \Delta_2 \tag{43}$$

and $\Omega^{+}(k)$ for

$$0 \leqslant \Delta \leqslant \Delta_1 \tag{44}$$

where

$$\Delta_1 = 12\,\delta_0 \boldsymbol{K}(k^\circ)/\sqrt{22\sqrt{7}} \approx 2.6\,\delta_0 \tag{45}$$

from Eqs (18), (19), (32), (I.21) and (I.26).

The intervals (44), (43) and (37) correspond respectively to the intervals (5), (7) and (9), and hence the intervals (11), (12) can be determined. By taking the limits $k \to 1$, $\omega \to 1$ and $k \to 0$, $\omega \to 11/3$ in Eqs (33)–(35) that correspond respectively to $\Delta \to \infty$ and $\Delta \to \Delta_2$ (see Eqs (I.15) and (I.16) in Appendix I) one has for the functions (22)

$$\begin{gathered} d(1) = 2\pi\sqrt{3/11} \approx 3.281, \qquad d(0) = 6\pi/7 \approx 2.697, \\ D(1) = \infty, \qquad D(0) = \Delta_2/\delta_0 = 3\pi/\sqrt{7} \approx 3.569, \\ R(1) = 0, \qquad R(0) = d(0)/D(0) = 2/\sqrt{7} \approx 0.756, \\ f(0) = \pi(77-8\sqrt{7})/126 \approx 1.392, \\ f(1) = \frac{1}{18}\,(11\pi+4\sqrt{6}-22\tan^{-1}\sqrt{8/3}) = 1.216, \end{gathered} \tag{46}$$

the last equality following from Eqs (16), (21) and from the corrected formula for σ_∞ given in [4], p. 99. According to Eqs (8)–(10), (20) and (21) we thus have

$$\begin{gathered} \delta_\infty = \delta_0 d(1) = 3.28\,\delta_0, \qquad \delta_2 = \delta_0 d(0) = 2.70\,\delta_0, \\ \sigma_\infty = \sigma_0 f(1) = 1.22\,\sigma_0, \qquad \sigma_2 = \sigma_0 f(0) = 1.39\,\sigma_0, \end{gathered} \tag{47}$$

and for Ni from Eq. (23)

$$\begin{gathered} \delta_\infty = 853\ \text{Å}, \qquad \delta_2 = 702\ \text{Å}, \\ \sigma_\infty = 0.16\ \text{erg/cm}^2, \qquad \sigma_2 = 0.19\ \text{erg/cm}^2. \end{gathered} \tag{48}$$

To determine the values of δ and σ for $\Delta = \Delta_1$ and $\Delta = 0$, we use Eqs (39)–(41) with $\Omega^{+}(k)$ and take the limits for $k \to k^0$, $\Omega^{+} \to \Omega^\circ$ and $k \to 0$, $\Omega^{+} \to \infty$, with k^0 and Ω^0 given by Eqs (I.21) and (I.26).This leads to

$$\begin{gathered} \tilde{D}(k^0) = \Delta_1/\delta_0 \approx 2.6, \qquad \tilde{D}(0) = \tilde{d}(0) = 0, \\ \tilde{d}(k^0) = 2\pi(1-2/\sqrt{7})\sqrt{(11+4\sqrt{7})/11} \approx 2.1, \\ \tilde{R}(k^0) = \tilde{d}(k^0)/\tilde{D}(k^0) \approx 0.8, \qquad \tilde{R}(0) = 1, \\ \tilde{f}(k^0) \approx 1.6, \qquad \tilde{f}(0) = \infty. \end{gathered} \tag{49}$$

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According to Eqs (10), (20) we thus obtain

$$\delta_1 = \delta_0 \tilde{d}(k^\circ) \approx 2.1\ \delta_0, \qquad \sigma_1 = \sigma_0 \tilde{f}(k^\circ) \approx 1.6\ \sigma_0 \tag{50}$$

and $\delta = 0$, $\sigma = \infty$ for $\Delta = 0$. From Eq. (23) one has for Ni

$$\delta_1 = 546\ \text{Å}, \qquad \sigma_1 = 0.21\ \text{erg/cm}^2. \tag{51}$$

Hence, by Eqs (47), (50) the intervals (11), (12) are determined and Eqs (13)–(15) proven.

We shall however confine our numerical analysis to the interval (37), in which case we have for the ratios (22)

$$\delta/\delta_\infty = 0.305\ d(k), \qquad \sigma/\sigma_\infty = 0.820 f(k), \qquad \delta/\Delta = d(k)/D(k) \tag{52}$$

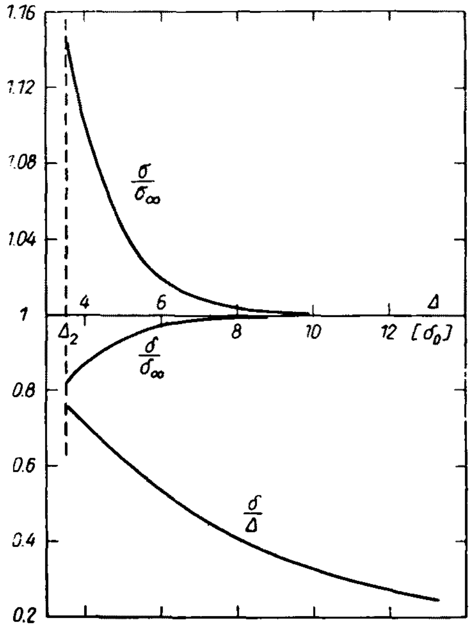

Fig. 2. Curves of the ratios (22) as defined by Eq. (52) for the (71°[110]180°) Bloch wall (cp. Table II)

from Eqs (33)–(35) and (46). Fig. 2 shows the dependence of the above ratios on Δ (in units of δ_0), and Table II contains a representative set of numerical data, along with specific results for Ni. The first three columns show the interdependence of Δ, k and ω according to Eqs (33) and (36).

c) The (180°[112]180°) Bloch wall

From Table I in [4] we have

$$u_0 = -a^2 AS^2, \qquad u_2 = -\frac{1}{24} KS^2 \left(S - \frac{1}{2}\right)^2, \tag{53}$$

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TABLE II

Rapresentative set of numerical data for Δ/δ_0 and the ratios (22) as defined by Eqs (33), (52) for the (71°[110]180°) Bolch wall (cp. Fig. 2), with results specified for Ni in the last three columns

ω^{-2}	k^2	Δ/δ_0	δ/δ_∞	δ/Δ	σ/σ_∞	Ni		
						Δ[Å]	δ[Å]	σ[erg/cm^2]
9/121	0	3.57	0.822	0.756	1.145	926	702	0.189
0.1	0.015	3.88	0.855	0.723	1.113	1010	729	0.176
0.3	0.117	5.54	0.957	0.566	1.029	1440	816	0.163
0.5	0.375	6.88	0.985	0.470	1.013	1790	841	0.160
0.7	0.611	8.56	0.998	0.383	1.002	2230	851	0.158
0.9	0.865	11.55	1.000	0.284	1.000	3000	853	0.158
1	1	∞	1	0	1	∞	853	0.158

and from Eq. (49) in [4] and Eqs (18), (19), (I.20) and (I.31)

$$\Delta = 8\sqrt{3}\,\delta_0 \boldsymbol{K}/\sqrt{7+\omega} \equiv \delta_0 D(k), \tag{54}$$

$$\delta = 4\delta_0 \sqrt{3/(7+\omega)} \left\{\boldsymbol{K} - F\left(\Gamma - \frac{\pi}{2}, k\right) + 2\gamma\sqrt{7\omega/(7\omega+1)}\right\} \equiv \delta_0\, d(k), \tag{55}$$

$$\sigma = \sigma_0 \sqrt{(7+\omega)/3} \left\{\boldsymbol{E} + \frac{\omega-1}{2\omega\,(7+\omega)}\,[2(\omega+1)\boldsymbol{\Pi}(-\omega^{-2}, k) - (3\omega+1)\boldsymbol{K}]\right\} \equiv \sigma_0 f(k), \tag{56}$$

where $\omega = \omega(k)$ is defined by

$$k^2 = (7\omega+1)/\omega(7+\omega) \tag{57}$$

from Eq. (40) in [4], and

$$\cos\gamma = \sqrt{3/7}, \qquad \gamma \approx 49^0,$$
$$\cos\Gamma = -(7+\omega)/(7\omega+1), \qquad 2\gamma \leqslant \Gamma \leqslant \pi \tag{58]}$$

from Eqs (44), (46) and (52) in [4].

As shown in [4], $\omega \to 1$ and $\Gamma \to \pi$ as $k \to 1$, and $\omega \to \infty$, $\Gamma \to 2\gamma$ as $k \to 0$. Hence, one has the limit values of the functions (22) as defined by Eqs (54)–(56):

$$d(0) = D(0) = R(1) = 0, \quad R(0) = 1, \quad D(1) = f(0) = \infty,$$
$$d(1) = \sqrt{3}\,\{\sqrt{2}\ln(\sqrt{6}+\sqrt{7}) + \sqrt{7}\sin^{-1}\sqrt{4/7}\} \approx 7.917, \tag{59}$$
$$f(1) = 2\sqrt{2/3} + (\sqrt{21}/21)\tanh^{-1}\sqrt{7/8} \approx 2.004,$$

the last two equalities following from Eq. (100) in [9]. This proves Eqs (13)–(15) and, due to Eq. (21) leads to

$$\delta_\infty = 7.92\,\delta_0, \qquad \sigma_\infty = 2.00\,\sigma_0. \tag{60}$$

For Ni we have from Eq. (23)

$$\delta_\infty = 2057\ \text{Å}, \qquad \sigma_\infty = 0.26\ \text{erg/cm}^2. \tag{61}$$

According to Eqs (54)–(56) and (59), the ratios (22) take the form

$$\delta/\delta_\infty = 0.126\, d(k), \qquad \sigma/\sigma_\infty = 0.499 f(k), \qquad \delta/\Delta = d(k)/D(k). \tag{62}$$

Their dependence on Δ is shown in Fig. 3, and Tables III and IV give representative numerical data and specific results for Ni. The reason for providing two tables for this particular wall is the same as in Section 2b in [2].

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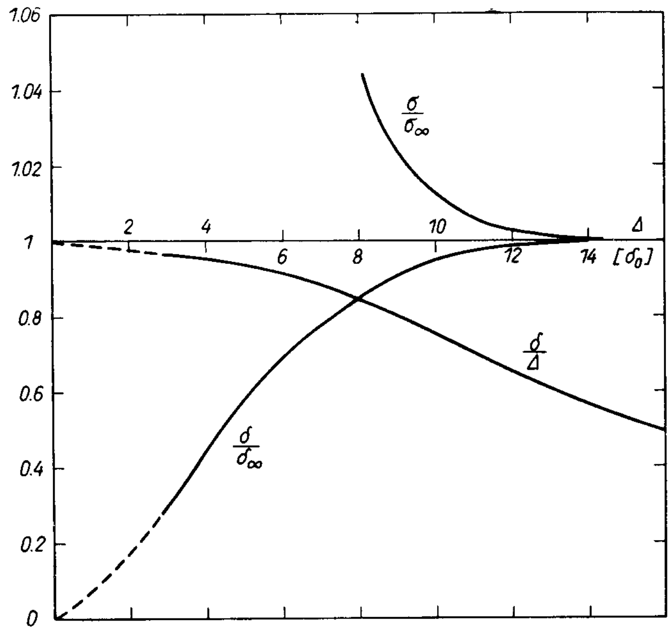

Fig. 3. Curves of the ratios (22) as defined by Eq. (62) for the (180°[112]180°) Bloch wall (cp. Tables III and IV)

d) The (109°[111]120°) Bloch wall

From Eq. (24) in [5] we have

$$u_0 = -\frac{8}{9}\, a^2 AS^2, \quad u_3 = -\left(\frac{2}{3}\right)^5 KS^2 \left(S - \frac{1}{2}\right)^2 \tag{63}$$

and, upon passing to the constants δ_0, σ_0 defined by Eqs (18), (19) we obtain from Eqs (28), (30) and (34) in [5]

$$\Delta = \sqrt{6}\, k\boldsymbol{K}\delta_0 \equiv \delta_0 D(k), \tag{64}$$

$$\delta = \pi\delta_0 k\sqrt{3/2} \equiv \delta_0 d(k), \tag{65}$$

$$\sigma = \sigma_0(2^6\sqrt{2}/3^4\sqrt{3}\, k)\{2\boldsymbol{E}-(1-k^2)\boldsymbol{K}\} \equiv \sigma_0 f(k). \tag{66}$$

TABLE III

Representative set of numerical data for Δ/δ_0, δ/δ_∞ and δ/Δ as defined by Eqs (54), (62) for the (180°[112]180°) Bloch wall (cp. Fig. 3), with results specified for Ni in the last two columns

Γ	ω	k^2	Δ/δ_0	δ/δ_∞	δ/Δ	Ni	
						Δ[Å]	δ[Å]
2γ	∞	0	0	0	1	0	0
99°	72.01	0.089	2.50	0.302	0.959	650	622
102°	14.92	0.322	5.11	0.605	0.936	1330	1243
105°	8.31	0.465	6.47	0.730	0.893	1680	1502
111°	4.40	0.634	8.16	0.857	0.831	2120	1762
130°	1.82	0.856	11.24	0.975	0.686	2920	2005
146°	1.29	0.938	13.51	0.996	0.583	3510	2048
160°	1.09	0.980	16.35	1.000	0.484	4250	2057
180°	1	1	∞	1	0	∞	2057

TABLE IV

Representative set of numerical data for Δ/δ_0 and σ/σ_∞ as defined by Eqs (54), (62) for the (180°[112]180°) Bloch wall (cp. Fig. 3), with results specified for Ni in the last two columns

ω^{-2}	k^2	Δ/δ_0	σ/σ_∞	Ni	
				Δ[Å]	σ[erg/cm^2]
0	0	0	∞	0	∞
0.05	0.630	8.13	1.044	2120	0.272
0.1	0.720	9.19	1.022	2390	0.266
0.2	0.806	10.38	1.011	2700	0.263
0.4	0.889	12.00	1.004	3120	0.262
0.6	0.938	13.54	1.001	3520	0.261
0.8	0.972	15.54	1.000	4040	0.261
1	1	∞	1	∞	0.261

When comparing the above formulae with Eqs (25)–(27) one sees that the functions $D(k)$ and $d(k)$ are identical, and that $f(k)$ differs merely by the coefficient 64/27. Therefore, the limit values are the same as in Eq. (28) except that now

$$f(1) = 2^7\sqrt{2}/3^4\sqrt{3} \approx 1.290 \tag{67}$$

which slightly changes σ_∞:

$$\sigma_\infty = 1.29\,\sigma_0; \quad \text{for Ni:} \quad \sigma_\infty = 0.17\ \text{erg/cm}^2. \tag{68}$$

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This, however, has no influence on the ratios (22) as given by Eq. (31) and plotted in Fig. 1, the only difference being in that — for the same domain width Δ — the energy of the (109°[111]120°) wall is 2.37 times larger than that of the (71°[100]90°) wall, as seen from the last two columns in Table I for Ni.

3. *Concluding remarks*

It is seen from Figs 1–3 and Tables I–IV that a significant departure of δ and σ from their asymptotic values δ_∞ and σ_∞ occurs for all the Bloch walls for comparatively small domain widths Δ, despite the fact that in this region the wall thickness and the domain width are already of the same order of magnitude. This is best seen from Tables V–VII. Table V shows that a 10% contraction of the Bloch wall thicknesses takes place in the region from 4.4 δ_0 to 9 δ_0 for the domain width, which for Ni corresponds to the interval 760 Å — 1870 Å. The increase of the wall energies in this region is even smaller, from 3%

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TABLE V

Approximate values of Δ/δ_0, δ/Δ and σ/σ_∞ for $\delta/\delta_\infty = 0.9$

Type of wall	Δ/δ_0	δ/Δ	σ/σ_∞	Ni		
				Δ[Å]	δ[Å]	σ[erg/cm²]
(71°[100]90°)	5.0	0.69	1.06	1300	900	0.075
(71°[110]180°)	4.4	0.67	1.08	1140	760	0.171
(180°[112]180°)	9.0	0.80	1.03	2340	1870	0.27
(109°[111]120°)	5.0	0.69	1.06	1300	900	0.18

TABLE VI

Approximate values of Δ/δ_0, δ/Δ and δ/δ_∞ for $\sigma/\sigma_\infty = 1.1$

Type of wall	Δ/δ_0	δ/Δ	δ/δ_∞	Ni		
				Δ[Å]	δ[Å]	σ[erg/cm²]
(71°[100]90°)	4.3	0.75	0.84	1150	850	0.078
(71°[110]180°)	4.1	0.71	0.87	1070	740	0.174
(180°[112]180°)	7.5	0.86	0.82	1950	1680	0.29
(109°[111]120°)	4.3	0.75	0.84	1150	850	0.19

TABLE VII

Approximate values of Δ/δ_0, δ/δ_∞ and σ/σ_∞ for $\delta/\Delta = 0.5$

Type of wall	Δ/δ_0	δ/δ_∞	σ/σ_∞	Ni		
				Δ[Å]	δ[Å]	σ[erg/cm²]
(71°[100]90°)	7.6	0.98	1.01	1960	980	0.072
(71°[110]180°)	6.5	0.98	1.01	1700	850	0.160
(180°[112]180°)	15.6	1.00	1.00	4120	2060	0.26
(109°[111]120°)	7.6	0.98	1.01	1960	980	0.17

to 8% of the asymptotic value, while at the same time the wall thicknesses amount already from 67% to 80% of the corresponding domain widths. As seen from Table VI, the situation is much the same for a 10% increase of the wall energies. On the other hand, Table VII shows that δ and σ differ from their asymptotic values by at most 2% for $\delta/\Delta = 0.5$, the domain width in this instance ranging from 6.5 δ_0 to 15.6 δ_0, that is for Ni from 1700 Å to 4120 Å. This means that for domain widths larger than 7.6 δ_0, 6.5 δ_0 and 15.6 δ_0 for the respective Bloch walls the asymptotic values δ_∞ and σ_∞ are, in fact, very good approximations of the exact ones δ and σ, in spite of the fact that the thickness of the wall may actually be as large as half the domain width which, in turn, lessens by at least 80% the critical domain widths given by Eq. (17).

Finally, we would like to point out that our curves and numerical data can be used in determining experimentally the Bloch wall thickness and energy for domain widths smaller than the critical values given in the first column in Table VII, by simply measuring the domain width which even in this region remains the only quantity easily accessible to measurements. It should however be stressed that our results are based on the formulae given in [4–6] which, as in the conventional domain theory, have been obtained without taking into account the so-called magnetostatic self-energy of the crystal (see [1]),*i.e.*, the energy of the crystal in its own demagnetizing field originating from magnetic poles on the crystal surface (see, *e,g,.* [11, 12]). Since this energy is the more significant the smaller (or thinner) the crystal, the domain width decreasing with decreasing crystal size, it may well have a non-negligible influence on the curves presented here. To examine this effect, however, one would have to solve the variational principles for the walls with the demagnetizing energy included, a problem which involves insurmountable mathematical difficulties.

APPENDIX I

A systematic error has been made in [4,6] when calculating the energies σ of the Bloch walls. Since its origin is exactly the same as in the case of the *bcc* lattice [2,3], it can be corrected in the same way as shown in detail in Appendix III in [2], by simply adding the term

$$c = N\Delta V^{-1} \{C+f(0, 0)\} \tag{I.1}$$

to the energies calculated in [4, 6] which we shall denote henceforth by σ'. Thus, the correct energies σ are

$$\sigma = \sigma' + c. \tag{I.2}$$

In Eq. (I.1), Δ is the domain width, N the number of lattice atoms, V the volume of the crystal. Since for the *fcc* lattice $N/V = 4a^{-3}$, where a is the lattice constant, Eq. (I.1) can be written as

$$c = 4a^{-3}\Delta \{C+f(0, 0)\}. \tag{I.3}$$

Further, $f(0, 0)$ is the value for $\varphi = \dot{\varphi} = 0$ of the integrand in the functional in [4], Eq. (1), which can be written in the form

$$h[\varphi] = \text{const} + 4a^{-3} \int_V f(\varphi, \dot{\varphi}) dv. \tag{I.4}$$

Hence,

$$f(0,0) = -(u_2+\varepsilon u_4) \tag{I.5}$$

with u_2, u_4 and ε given for each wall in Table I in [4]. The quantity C in Eq. (I.3) is the first integration constant of the Euler-Lagrange equation which, for a functional of the type (I.4), can be determined by applying the boundary conditions to the equation

$$\dot{\varphi}\frac{\partial f}{\partial \dot{\varphi}} - f = C \tag{I.6}$$

(see Appendix III in [2]). Thus, the constants given for the respective Bloch walls by Eqs (10), (25), (39) and (65) in [4] are simply equal to C/u_0 and hence C can readily be determined for each wall.

For the (71°[100]90°) Bloch wall we have from Eqs (10), (11) and Table I in [4]

$$C = u_4(2-k^2)/k^2, \quad \Delta = k\boldsymbol{K}\sqrt{u_0/2u_4}, \quad f(0,0) = -u_4,$$
$$c = 4a^{-3}k^{-1}\sqrt{2u_0u_4}\,(1-k^2)\boldsymbol{K}. \tag{I.7}$$

Hence, with σ' from Eq. (15) in [4] we obtain for σ as defined by Eq. (I.2)

$$\sigma = 4a^{-3}k^{-1}\sqrt{2u_0u_4}\,\{2\boldsymbol{E}-(1-k^2)\boldsymbol{K}\}. \tag{I.8}$$

For the (71°[110]180°) Bloch wall we have from Eq. (25) and Table I in [4]

$$C = u_2(22\omega^2-3\omega+22)/44\omega, \quad f(0,0) = -41u_2/44, \tag{I.9}$$

and, from Eqs (22) and (26) in [4]

$$\Delta = 2\boldsymbol{K}\sqrt{22u_0/u_2(11\omega-3)} \tag{I.10}$$

with the function $\omega = \omega(k)$ defined by

$$k^2 = (11-3\omega)/\omega(11\omega-3). \tag{I.11}$$

Hence,

$$c = 4a^{-3}\boldsymbol{K}(\omega-1)^2\omega^{-1}\sqrt{22u_0u_2/(11\omega-3)} \tag{I.12}$$

and, with σ' from Eq. (31) in [4] we obtain for σ as defined by (I.2)

$$\sigma = 8a^{-3}\sqrt{2u_0u_2\,(11\omega-3)/11}\,\Bigg\{\boldsymbol{E} + \frac{11(\omega-1)}{2\omega\,(11\omega-3)}\,[2\,(\omega+1)\boldsymbol{\Pi}(-\omega^{-2}, k) - (3\omega+1)\boldsymbol{K}]\Bigg\}. \tag{I.13}$$

However, as shown in [6] the solution for this wall given in [4] is restricted to the interval

$$41u_2/44 \leqslant C \leqslant 21u_2/11, \tag{I.14}$$

since

$$1 \leqslant \omega(k) \leqslant 11/3 \quad \text{for} \quad 1 \geqslant k \geqslant 0 \tag{I.15}$$

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as shown in [4], Eq. (29). This restricts the interval $\langle 0, \infty\rangle$ for Δ to

$$\Delta_1 \equiv \frac{\pi}{2} \sqrt{33u_0/14u_2} \leqslant \Delta \leqslant \infty \quad \text{as} \quad 0 \leqslant k \leqslant 1. \tag{I.16}$$

For $C > 21u_2/11$, *i.e.*, for $\Delta < \Delta_2$ we have the solution (13) from [6] which changes the relations (I.10), (I.11) to

$$\Delta = 2\boldsymbol{K}\sqrt{2u_0\Omega/u_2(\Omega^2-1)}\,, \tag{I.17}$$

$$k^2 = (3\Omega-11)/11(\Omega^2-1) \tag{I.18}$$

and leads to

$$c = 4a^{-3}\boldsymbol{K}(\Omega-1)^2\Omega^{-1}\sqrt{2u_0u_2\Omega/(\Omega^2-1)}\,, \tag{I.19}$$

as the formula for C in Eq. (I.9) still holds except that ω is to be replaced by Ω. With σ' from Eq. (21) in [6] we now have for σ as defined by (I.2)

$$\sigma = 8a^{-3}\sqrt{2u_0\, u_2\, (\Omega^2-1)/\Omega}\left\{\boldsymbol{E} + \frac{\Omega^2}{\Omega^2-1}\,\boldsymbol{\Pi}((\Omega^2-1)^{-1},\, k) - \frac{3\Omega+1}{2(\Omega+1)}\,\boldsymbol{K}\right\}. \tag{I.20}$$

Eqs (I.17)–(I.20) are valid in the whole interval $\infty \geqslant C \geqslant 21u_2/11$, *i.e.*, for $0 \leqslant \Delta \leqslant \Delta_2$, provided that both the solutions for $\Omega(k)$ from Eq. (I.18) are used which we shall denote by $\Omega^+(k)$ and $\Omega^-(k)$, according to whether the plus or minus sign for the square-root is chosen (see Eq. (16) in [6]). Since in either case the modulus k is confined to the interval

$$0 \leqslant k \leqslant \sqrt{(11-4\sqrt{7})/22} \equiv k^0 \approx 0.1 \tag{I.21}$$

(see Eq. (17) in [6]), we have

$$11/3 \leqslant \Omega^-(k) \leqslant \Omega^\circ \quad \text{as} \quad 0 \leqslant k \leqslant k^\circ \tag{I.22}$$

for

$$21u_2/11 \leqslant C \leqslant 475u_2/132, \quad \Delta_2 \geqslant \Delta \geqslant \Delta_1; \tag{I.23}$$

and

$$\Omega^\circ \leqslant \Omega^+(k) \leqslant \infty \quad \text{as} \quad k^\circ \geqslant k \geqslant 0 \tag{I.24}$$

for

$$475u_2/132 \leqslant C \leqslant \infty, \quad \Delta_1 \geqslant \Delta \geqslant 0, \tag{I.25}$$

where

$$\Omega^0 = \Omega^\pm(k^0) = (11+4\sqrt{7})/3 \approx 7.2,$$

$$\Delta_1 = \boldsymbol{K}(k^0)\sqrt{3u_0/u_2\sqrt{7}}. \tag{I.26}$$

Hence, we have for the energy σ of the (71°[110]180°) Bloch wall the formula (I.13) in the interval (I.16), and the formula (I.20) with Ω^- and Ω^+ in the intervals (I.23) and (I.25), respectively.

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For the (180°[112]180°) Bloch wall we have from Eq. (39) and Table I in [4]

$$C = u_2(2\omega^2+7\omega+2)/4\omega, \quad f(0, 0) = -11u_2/4, \tag{I.27}$$

and, from Eqs (37) and (40) in [4],

$$\Delta = 2\boldsymbol{K}\sqrt{2u_0/u_2(7+\omega)} \tag{I.28}$$

with the function $\omega = \omega(k)$ defined by

$$k^2 = (1+7\omega)/\omega(7+\omega), \quad 1 \leqslant \omega \leqslant \infty. \tag{I.29}$$

Hence, according to Eq. (I.3) we have

$$c = 4a^{-3}\boldsymbol{K}(\omega-1)^2\omega^{-1}\sqrt{2u_0u_2/(7+\omega)} \tag{I.30}$$

and, with σ' from Eq. (53) in [4] we obtain for σ as defined by (I.2)

$$\sigma = 8a^{-3}\sqrt{2u_0u_2\,(7+\omega)}\left\{\boldsymbol{E} + \right.$$

$$\left. + \frac{\omega-1}{2\omega(7+\omega)}\,[2(\omega+1)\boldsymbol{\Pi}(-\omega^{-2}, k)-(3\omega+1)\,\boldsymbol{K}]\right\}. \tag{I.31}$$

For the (71°[110]71°) and (109°[110]109°) Bloch walls we have from Eq. (65) and Table I in [4]

$$C = -3u_2(2\omega^2+3\omega+2)/4\omega, \quad f(0, 0) = -11u_2/4, \tag{I.32}$$

and from Eqs (63) and (66) in [4]

$$\Delta = 4\boldsymbol{K}\sqrt{u_0\omega/6u_2(1-\omega^2)} \tag{I.33}$$

with the function $\omega = \omega(k)$ defined by

$$k^2 = (1+3\omega)/(1-\omega^2), \quad -\infty \leqslant \omega \leqslant -3. \tag{I.34}$$

Hence, according to Eq. (I.3) one has

$$c = +8a^{-3}\boldsymbol{K}(3\omega+1)(3+\omega)\sqrt{u_0u_2/6\omega(1-\omega^2)} \tag{I.35}$$

and, with the total energy σ_t' of the walls from Eq. (75) in [4] we obtain for σ_t according to Eq. (I.2)

$$\sigma_t = 8a^{-3}\sqrt{6u_0u_2\,(1-\omega^2)/\omega}\left\{\boldsymbol{E} + \right.$$

$$\left. + \frac{1}{2\,(\omega^2-1)}\,[2\omega^2\boldsymbol{\Pi}((\omega^2-1)^{-1}, k)-(9\omega^2+10\omega-3)\,\boldsymbol{K}]\right\}. \tag{I.36}$$

Although the energies σ' as given in [4, 6] and σ as corrected here have the same asymptotic values σ_∞ in the limit $\Delta \to \infty$ (see Footnote 4 in [2]), σ' decreases while σ increases with decreasing domain width Δ and, in the limit $\Delta = 0$, one has $\sigma' = 0$ while $\sigma = \infty$. This

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explains why in the example considered in Section 6 in [4] the energy of the contracted (71°[100]90°) Bloch wall turned out to be smaller than σ_∞. For the same 10% contraction of the wall considered in that example, the correct formula (I.8) leads to $\sigma = 1.06\,\sigma_\infty$ as seen from Table V, while $\sigma' = 0.8\,\sigma_\infty$, Eq. (79) in [4]. (Note that σ_0 in [4] corresponds to σ_∞ in the present notation.)

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