MAGNETIZATION AND THE SPECTRUM OF ELEMENTARY EXCITATIONS OF THE UNIAXIAL ANISOTROPIC FERROMAGNET

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The expressions determining the magnetization and the spectrum of elementary excitations are derived for the uniaxial anisotropic ferromagnet. Their dependence on the temperature and the external magnetic field is discussed. The calculations were made using double-time temperature-dependent Green-functions.

In the previous paper [1] an equation for the relative magnetization was obtained and the spectrum of elementary excitations was calculated in the case of uniaxial anisotropic ferromagnet. Double-times Green-functions were applied and a special procedure was introduced for the decoupling of the higher order Green-functions. The free energy was minimized in order to get the direction of the magnetization. It was shown in the paper [2], that for the determination of the direction of the magnetization an equation can be obtained if some requirements are imposed for the analytic properties of the Green-functions.

The method of double-time anticommutative Green-functions was applied in the work [3] to investigate a system, described by the Hamiltonian of the Ising-model with perpendicular external magnetic field.

In this paper the method of the work [3] will be applied for the investigation of the properties of uniaxial anisotropic ferromagnet in the framework of Heisenberg's model.

It is worth-while to note, that using this method it is possible to obtain a correlation function of the type $\langle b_f(t)b_g(t')\rangle$ vanishing if $t = t'$ and $f = g$. Furthermore expressions will be deduced for the spectrum of elementary excitations and for the relative magnetization at various field orientations with respect to the anisotropy axis of the crystal.

1. We consider uniaxial anisotropic ferromagnet in the framework of Heisenberg's model with spin $s = \frac{1}{2}$. The axis of the anisotropy is chosen as the Z-axis of the coordinate system. For the sake of sipmlicity we consider the case, when the external magnetic field H

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lies in the (x, z) plane. In the simplest case of the anisotropy, the Hamiltonian is given by

$$\mathcal{H} = -\frac{1}{2}\sum_{f_1f_2} I(f_1,f_2)\,\mathbf{S}_{f_1}\mathbf{S}_{f_2} - \frac{1}{2}\sum_{f_1f_2} R(f_1,f_2)\, S^z_{f_1}S^z_{f_2} - \mu H^x \sum_f S^x_i - \mu H_z \sum_f S^z_f, \tag{1.1}$$

where the isotropic and anisotropic part of the exchange integral are denoted by $I(f_1, f_2) = I(|f_1-f_2|) > 0$ and $R(f_1,f_2) = R(|f_1-f_2|)$ respectively. $\boldsymbol{S}_f$ means the spin operator of the electron, situated at the point f of the lattice, and μ stands for the Bohr-magneton.

We introduce the Pauli operators b_f, b_f^+ by means of the transformation

$$S^z_f = \frac{1}{2}\{(1-2n_f)\cos\vartheta-(b_f^++b_f)\sin\vartheta\}$$

$$S^x_f = \frac{1}{2}\{(1-2n_f)\sin\vartheta+(b_f^++b_f)\cos\vartheta\}$$

$$S^y_f = \frac{i}{2}(b_f^+-b_f), \tag{1.2}$$

where $n_f = b_f^+b_f$. The parameter ϑ of the transformation will be determined later on.

The transformed Hamiltonian takes the form:

$$\mathcal{H} = E_0 - \frac{1}{2}\sum_f \tilde{\varepsilon}_1(b_f^++b_f) - \frac{1}{4}\sum_{f_1,f_2}\tilde{W}(f_1,f_2)\, b_{f_1}^+b_{f_2} -$$

$$-\frac{1}{8}\sum_{f_1,f_2} W(f_1,f_2)\,(b_{f_1}^+b_{f_2}^++b_{f_1}b_{f_2}) + \sum_f \tilde{\varepsilon}_2 n_f -$$

$$-\frac{1}{2}\sum_{f_1,f_2} V(f_1,f_2)\,(b_{f_1}^++b_{f_1})n_{f_2} - \frac{1}{2}\sum_{f_1,f_2} U(f_1,f_2)n_{f_1}n_{f_2}, \tag{1.3}$$

where the following notations are used:

$$U(f_1,f_2) = I(f_1,f_2)+R(f_1,f_2)\cos^2\vartheta$$

$$V(f_1,f_2) = R(f_1,f_2)\sin\vartheta\cos\vartheta$$

$$W(f_1,f_2) = R(f_1,f_2)\sin^2\vartheta$$

$$\tilde{W}(f_1,f_2) = 2I(f_1,f_2)+W(f_1,f_2)$$

$$\tilde{\varepsilon}_1 = \varepsilon_1 - \frac{1}{2}V(0) = \mu(H^x\cos\vartheta - H^z\sin\vartheta) - \frac{1}{2}R(0)\sin\vartheta\cos\vartheta$$

$$\tilde{\varepsilon}_2 = \varepsilon_2 + \frac{1}{2}U(0) = \mu(H^x\sin\vartheta+H^z\cos\vartheta) + \frac{1}{2}(I(0)+R(0)\cos^2\vartheta)$$

$$E_0 = -\frac{1}{2}\left(\varepsilon_2 + \frac{1}{4}I(0) + \frac{1}{4}R(0)\cos^2\vartheta\right)$$

$$I(0) = \sum_{f_1} I(f_1,f_2); \quad R(o) = \sum_{f_1} R(f_1,f_2). \tag{1.4}$$

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The dependence of magnetization on the temperature and on the external magnetic field will be obtained with the help of retarded double-time anticommutative Green-functions [3]:

$$G^{(1)}_{gh}(t) = \langle\langle b_g(t)|b_h(0)\rangle\rangle = \theta(t)\langle b_g(t)b_n(0)+b_h(0)b_g(t)\rangle$$

$$G^{(2)}_{gh}(t) = \langle\langle b_g^+|b_h(0)\rangle\rangle$$

$$G^{(3)}_{gh}(t) = \langle\langle n_g(t)|b_h(0)\rangle\rangle. \tag{1.5}$$

Making use of the equations of motion for the operators b_g, b_g^+ and n_g in Heisenberg's picture, we can get the following system of equations for our Green-functions:

$$i\frac{d}{dt}G^{(1)}_{gh}(t) = ia_1(g,h)\,\delta(t)-\theta(t)\,\tilde{\varepsilon}_1\bar{b}+$$

$$+\sum_f\left\{\tilde{\varepsilon}_2\Delta(f-g)-\frac{1}{4}\tilde{W}(f,g)\right\}G^{(1)}_{fh}(t)-\frac{1}{4}\sum_f W(f,g)G^{(2)}_{fh}(t)+$$

$$+\sum_f\left\{\tilde{\varepsilon}_1\Delta(f-g)-\frac{1}{2}V(f,g)\right\}G^{(3)}_{fh}(t)-$$

$$-\frac{1}{2}\sum_f V(f,g)\ll b_g(b_f^+ +b_f)|b_h\gg+\frac{1}{2}\sum_f\tilde{W}(f,g)\ll n_g b_f|b_h\gg+$$

$$+\frac{1}{2}\sum_f W(f,g)\gg b_f^+ n_g|b_h\gg-\sum_f U(f,g)\gg b_g n_f|b_h\gg+$$

$$+\sum_f V(f,g)\ll n_f n_g|b_h\gg; \tag{1.6}$$

$$i\frac{d}{dt}G^{(2)}_{gh}(t) = i\,a_2(g,h)\,\delta(t)+\theta(t)\tilde{\varepsilon}_1\,\bar{b}-$$

$$-\sum_f\left\{\tilde{\varepsilon}_2\,\Delta(f-g)-\frac{1}{4}\tilde{W}(f,g)\right\}G^{(2)}_{fh}(t)+\frac{1}{4}\sum_f W(f,g)\,G^{(1)}_{ih}(t)-$$

$$-\sum_f\left\{\tilde{\varepsilon}_1\,\Delta(f-g)-\frac{1}{2}V(f,g)\right\}G^{(3)}_{f,h}+\frac{1}{2}\sum_f V(f,g)\ll(b_f^+ +b_f)b_g^+|b_h\gg-$$

$$-\frac{1}{2}\sum_f\tilde{W}(f,g)\ll b_f^+ n_g|b_h\gg-\frac{1}{2}\sum_f W(f,g)\ll b_f n_g|b_h\gg+$$

$$+\sum U(f,g)\ll n_f b_g^+|b_h\gg-\sum_f V(f,g)\gg n_f n_g|b_h\gg \tag{1.7}$$

$$i \frac{d}{dt} G^{(3)}_{gh}(t) = ia_3(g, h)\delta(t) + \frac{1}{2} \tilde{\varepsilon}_1 \{G^{(1)}_{gh}(t) - G^{(2)}_{gh}(t)\} -$$

$$-\frac{1}{4} \sum_f \tilde{W}(f, g) \ll b_g^+ b_f - b_f^+ b_g | b_h \gg - \frac{1}{4} \sum_f W(f, g) \ll b_f^+ b_g^+ - b_f b_g | b_h -$$

$$-\frac{1}{2} \sum_f V(f, g) \ll (b_g^+ - b_g) n_f | b_h \gg; \tag{1.8}$$

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where

$$a_1(g, h) = 2\langle b_g b_h \rangle,$$

$$a_2(g, h) = 2\langle b_g^+ b_h \rangle + \sigma \Delta(g-h)$$

$$a_3(g, h) = 2\langle n_g b_h \rangle + \bar{b} \Delta(g-h)$$

$$\sigma = 1 - 2\langle n_f \rangle = 1 - 2\bar{n}.$$

The expectation values $\langle b_f \rangle$, $\langle b_f^+ \rangle$, $\langle n_f \rangle$ are independent of the lattice index f, because of the translational invariance. Using the equations of motion for the operatr S_f^z and taking into account (1.2), we can easily prove that

$$\langle b_f \rangle = \langle b_f^+ \rangle = \bar{b}. \tag{1.9}$$

2. In order to get a closed system of equations for the Green-functions (1.5) we introduce the same decoupling method of the higher order Green-functions, as in the paper [3]:

$$\ll b_f b_g | b_h \gg = \bar{b}(G^{(1)}_{fg}(t) + G^{(1)}_{gh}(t))$$

$$\ll n_g b_f | b_h \gg = \bar{n} G^{(1)}_{fh}(t) + \bar{b} G^{(3)}_{gh}(t)$$

$$\ll b_f^+ b_g | b_h \gg = \bar{b}(G^{(1)}_{gh}(t) + G^{(2)}_{fh}(t))$$

$$\ll n_g n_f | b_h \gg = \bar{n}(G^{(3)}_{fh}(t) + G^{(3)}_{gh}(t)) \tag{2.1}$$

and apply similar expressions for the other higher order Green-functions.

Substituting the expressions given by (2.1) into the equations (1.6)–(1.8) and performing the Fourier transformation defined by

$$G^{(\alpha)}_{gh}(t) = \int_{-\infty}^{\infty} G^{(\alpha)}_{gh}(E)^{-iEt} dt,$$

$$G^{(\alpha)}_{gh}(E) = \frac{1}{N} \sum_\nu G^{(\alpha)}_\nu(E) e^{i(\nu, g-h)} \quad (\alpha = 1, 2, 3), \tag{2.2}$$

we get:

$$\{E - A(\nu) + B(\nu)\} G^{(1)}_\nu(E) + B(\nu) G^{(2)}_\nu(E) - \{2C(\nu) - D(\nu)\} G^{(3)}_\nu(E)$$

$$= \frac{i}{2\pi E} \{a_1(\nu) E - N\tilde{\varepsilon}_1 \bar{b} \Delta(\nu)\} \tag{2.3}$$

$$-B(\nu)G_\nu^{(1)}(E)+\{E+A(\nu)-B(\nu)\}\ G_\nu^{(2)}(E)+\{2C(\nu)-D(\nu)\}\ G_\nu^{(3)}(E)$$

$$= \frac{i}{2\pi E}\ \{a_2(\nu)E+N\tilde{\varepsilon}_1\bar{b}\Delta(\nu)\} \tag{2.4}$$

$$-\ C(\nu)\ G_\nu^{(1)}(E)+C(\nu)\ G_\nu^{(2)}(E)+E\ G_\nu^{(3)}(E) = \frac{i}{2\pi}\ a_3(\nu), \tag{2.5}$$

where:

$$A(\nu) = A + \frac{1}{2}\ I(0)\sigma\varepsilon_\nu$$

$$B(\nu) = \frac{1}{4}\ R(\nu)\sin\vartheta\ (\sigma\sin\vartheta+2\bar{b}\cos\vartheta)$$

$$C(\nu) = C + \frac{1}{2}\ I(0)\ \bar{b}\varepsilon_\nu$$

$$D(\nu) = \frac{1}{2}\ R(\nu)\cos\vartheta\ (\sigma\sin\vartheta+2\bar{b}\cos\vartheta)$$

$$A = \mu(H^x\sin\vartheta+H^z\cos\vartheta)+\frac{1}{2}\ R(0)\cos\vartheta\ (\sigma\cos\vartheta-2\bar{b}\ \mathrm{cin}\ \vartheta)$$

$$C = \frac{1}{2}\ \mu(H^x\cos\vartheta-H^z\sin\vartheta)-\frac{1}{4}\ R(0)\sin\vartheta(\sigma\cos\vartheta-2\bar{b}\sin\vartheta)$$

$$\varepsilon_\nu = 1-I(\nu)|I(0). \tag{2.6}$$

Solving the system of equations (2.3)–(2.5) and applying the standard methods of the Green-function theory (*cf.* (4)) we can get the following expressions for the spectrum of the elementary excitations

$$E(\nu) = \sqrt{A^2(\nu)+4C^2(\nu)-2[A(\nu)B(\nu)+C(\nu)D(\nu)]}. \tag{2.7}$$

For the Fourier transforms of the correlation functions defined by:

$$\Gamma_{gh}^{(1)} = \langle b_g b_h\rangle; \quad \Gamma_{gh}^{(2)} = \langle b_g^+ b_h\rangle; \quad \Gamma_{gh}^{(3)} = \langle n_g b_h\rangle \tag{2.8}$$

we get the following system of the equations:

$$\{A(\nu)-B(\nu)\}\ \Gamma_\nu^{(1)}-B(\nu)\ \Gamma_\nu^{(2)}+\{2C(\nu)-D(\nu)\}\Gamma_\nu^{(3)}$$

$$= \frac{\sigma}{2}\ B(\nu)-\frac{\bar{b}}{2}\ \{2C(\nu)-D(\nu)-N\tilde{\varepsilon}_1\Delta(\nu)\}; \tag{2.9}$$

$$B(\nu)\ \Gamma_\nu^{(1)}-\{A(\nu)-B(\nu)\}\ \Gamma_\nu^{(2)}-\{2C(\nu)-D(\nu)\}\ \Gamma_\nu^{(3)}$$

$$= \frac{\sigma}{2}\left\{A(\nu)-B(\nu)-E(\nu)\ \mathrm{cth}\ \frac{1}{2}\ \beta E(\nu)\right\}+$$

$$+\ \frac{\bar{b}}{2}\ \{2C(\nu)-D(\nu)-N\tilde{\varepsilon}_1\Delta(\nu)\}; \tag{2.10}$$

$$C(\nu)\ \{\Gamma_\nu^{(1)}-\Gamma_\nu^{(2)}\} = \frac{\sigma}{2}\ C(\nu)-\frac{\bar{b}}{2}\ E(\nu)\ \mathrm{cth}\ \frac{1}{2}\ \beta E(\nu). \tag{2.11}$$

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It is easy to see, that equations (2.9), (2.10) lead to the following relation:

$$\Gamma_\nu^{(1)} - \Gamma_\nu^{(2)} = \frac{\sigma}{2}\left\{1 - \frac{E_\nu}{A_{(\nu)}} \operatorname{cth} \frac{1}{2} \beta E(\nu)\right\}. \tag{2.12}$$

Now we turn to the determination of the transformation parameter ϑ. We define the transformation parameter by the requirement

$$\bar{b} = 0. \tag{2.13}$$

From the equation (2.12) immediately follows that both of $\Gamma_\nu^{(1)}$ and $\Gamma_\nu^{(2)}$ cannot be zero. Therefore the equation (2.11) leads to the following equation:

$$\mu H^x \cos \vartheta - \left(\mu H^z + \frac{1}{2} R(0) \sigma \cos \vartheta\right) \sin \vartheta = 0 \tag{2.14}$$

if we take into account the requirement $\bar{b} = 0$. Thus the parameter ϑ of the transformation is determined by the equation (2.14).

Taking into account (2.13), (2.14) and the following equations:

$$\frac{1}{N} \sum_\nu \Gamma_\nu^{(1)} = \frac{1}{N} \sum_\nu \langle b_f b_g \rangle_\nu = \langle b_f b_f \rangle \equiv 0; \tag{2.15}$$

$$\frac{1}{N} \sum_\nu \Gamma_\nu^{(2)} = \frac{1}{N} \sum_\nu \langle b_f^+ b_g \rangle_\nu = \langle b_f^+ b_f \rangle = \frac{1-\sigma}{2} \tag{2.16}$$

we can get from (2.12) a transcendental equation for the relative magnetization σ:

$$\frac{1}{\sigma} = \frac{1}{N} \sum_\nu \frac{\tilde{E}(\nu)}{\tilde{A}(\nu)} \operatorname{cth} \frac{1}{2} \beta \tilde{E}(\nu) = \frac{v}{(2\pi)^3} \int d^3\nu \frac{\tilde{E}(\nu)}{\tilde{A}(\nu)} \operatorname{cth} \frac{1}{2} \beta \tilde{E}(\nu) \tag{2.17}$$

and an expression for the spectrum of the elementary excitations:

$$\tilde{E}(\nu) = \sqrt{\tilde{A}^2(\nu) - 2\tilde{A}(\nu)\tilde{B}(\nu)}, \tag{2.18}$$

where

$$\tilde{A}(\nu) = \mu H^x \sin \vartheta + (\mu H^z + \frac{1}{2} R(0)\sigma \cos \vartheta) \cos \vartheta + \frac{1}{2} I(0)\sigma\varepsilon_\nu$$

$$\tilde{B}(\nu) = \frac{1}{4} R(\nu)\sigma \sin^2 \vartheta. \tag{2.19}$$

In the case of an isotropic ferromagnet, when $R(f, g) = 0$, we get the results of the paper [5], and the direction of the magnetization is parallel to the external magnetic field, as it can be seen from the equation (2.14). In the case, when $I(f, g) = 0$, we get the results of the paper [3].

In the following sections we shall consider two different cases, namely, the case of $R(0) > 0$ and $R(0) < 0$, for some directions of the external magnetic field.

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3. At first let us consider the case, when $R(0) > 0$.

a) When the external field is parallel to the axis of the anisotropy, that is: $H^x = 0$, $H^z = H \neq 0$, the equation (2.14) takes the form:

$$\left\{\mu H + \frac{\sigma}{2} R(0) \cos \vartheta\right\} \sin \vartheta = 0 \tag{3.1}$$

and its solution is $\sin \vartheta = 0$, $\vartheta = 0$. In this case (2.17) and (2.18) can be expressed by means of (2.19) as:

$$E(\nu) = \mu H + \frac{\sigma}{2} \{I(0) + R(0) - I(\nu)\} \tag{3.2}$$

$$\frac{1}{\sigma} = \frac{v}{(2\pi)^3} \int d^3\nu \, \mathrm{cth} \, \frac{1}{2} \beta E(\nu). \tag{3.3}$$

As we see, the magnetization is directed along the anisotropy axis.

b) When the external magnetic field is perpendicular to the axis of anisotropy, that is $H^x = H \neq 0$, $H^z = 0$, the equation (2.14) can be written as:

$$\left\{\frac{\mu H}{\frac{1}{2} R(0)\sigma} - \sin \vartheta\right\} \cos \vartheta = 0, \tag{3.4}$$

which has two solutions. If

$$\frac{\mu H}{\frac{1}{2} R(0)\varrho} > 1 \tag{3.5}$$

the solution is $\cos \vartheta = 0$, $\vartheta = \frac{\pi}{2}$. Thus, in the case of strong magnetic field the magnetization is parallel to the external magnetic field, and for (2.17), (2.18) we get:

$$E(\nu) = \left[\mu H + \frac{1}{2} I(0)\sigma\varepsilon_\nu\right] \left\{1 - \frac{1}{2} \frac{\sigma R(\nu)}{\mu H + \frac{1}{2} I(0)\sigma\varepsilon_\nu}\right\}^{1/2} \tag{3.6}$$

$$\frac{1}{\sigma} = \frac{v}{(2\pi)^3} \int d^3\nu \left[1 - \frac{1}{2} \frac{\sigma R(\nu)}{\mu H + \frac{1}{2} I(0)\sigma\varepsilon_\nu}\right]^{1/2} \mathrm{cth} \, \frac{1}{2} \beta E(\nu). \tag{3.7}$$

In the case of weak magnetic fields, when

$$\frac{\mu H}{\frac{1}{2} R(0)\sigma} \leqslant 1 \tag{3.8}$$

the solution of the equation (3.4) can be written as

$$\sin \vartheta = \frac{\mu H}{1/2 \, R(0)\sigma} \tag{3.9}$$

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i.e. the direction of the magnetization differs from that of the external magnetic field. Now using Eq. (2.19), the equations (2.17) and (2.18) read as:

$$E(\nu) = \frac{\sigma}{2}\,[I(0)+R(0)-I(\nu)]\left\{1-\frac{\left(\dfrac{\mu H}{\frac{1}{2}\,R(0)\sigma}\right)^2 R(\nu)}{I(0)+R(0)-I(\nu)}\right\}^{1/2} \tag{3.10}$$

$$\frac{1}{\sigma} = \frac{v}{(2\pi)^3}\int d^3\nu\left\{1-\frac{\left(\dfrac{\mu H}{\frac{1}{2}\,R(0)\sigma}\right)^2 R(\nu)}{I(0)+R(0)-I(\nu)}\right\}^{1/2} \operatorname{cth}\frac{1}{2}\,\beta E(\nu). \tag{3.11}$$

It is evident that the expressions (3.12), (3.10) and (3.3), (3.11) take the same form if $H = 0$:

$$E(\nu) = \frac{\sigma}{2}\,\{I(0)+R(0)-I(\nu)\} \tag{3.12}$$

$$\frac{1}{\sigma} = \frac{v}{(2\pi)^3}\int d^3\nu \operatorname{cth}\frac{1}{2}\,\beta E(\nu) \tag{3.13}$$

and $\vartheta = 0$, *i.e.* without external magnetic field the magnetization is parallel to the axis of anisotropy.

4. Let us now consider the case, when $R(0) < 0$.

a) When the external field is parallel to the anisotropy axis, that is $H^x = 0$, $H^z = H \neq 0$ the equation (2.14) can be written as:

$$\left\{\frac{\mu H}{\frac{1}{2}\,|R(0)|\sigma} - \cos\vartheta\right\}\sin\vartheta = 0. \tag{4.1}$$

In the case of strong fields

$$\frac{\mu H}{\frac{1}{2}\,|R(0)|\sigma} > 1 \tag{4.2}$$

the solution is $\sin\vartheta = 0$, $\vartheta = 0$, *i.e.* the magnetization parallel to the external magnetic field. The expressions (2.17), (2.18) take the form:

$$E(\nu) = \mu H + \frac{1}{2}\,\sigma\{I(0)-|R(0)|-I(\nu)\} \tag{4.3}$$

$$\frac{1}{\sigma} = \frac{v}{(2\pi)^3}\int d^3\nu \operatorname{cth}\frac{1}{2}\,\beta E(\nu). \tag{4.4}$$

If the external field is weak

$$\frac{\mu H}{\frac{1}{2}\,|R(0)|\sigma} \leqslant 1 \tag{4.5}$$

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then the solution of (4.1) can be expressed as

$$\cos\varphi = \frac{\mu H}{\frac{1}{2}\,|R(0)|\sigma} \tag{4.6}$$

and for the expressions (2.17), (2.18) we get

$$E(\nu) = \frac{\sigma}{2}\,\{I(0)-I(\nu)\}\left\{1+\frac{|R(\nu)|\left[1-\left(\frac{\mu H}{\frac{1}{2}\,|R(0)|\sigma}\right)^2\right]}{I(0)-I(\nu)}\right\}^{1/2} \tag{4.7}$$

$$\frac{1}{\sigma} = \frac{v}{(2\pi)^3}\int d^3\nu\left\{1+\frac{|R(\nu)|\left[1-\left(\frac{\mu H}{\frac{1}{2}\,|R(0)|\sigma}\right)^2\right]}{I(0)-I(\nu)}\right\}^{1/2} \operatorname{cth}\frac{1}{2}\,\beta E(\nu). \tag{4.8}$$

Finally let us consider the case, when the external magnetic field is perpendicular to the axis of anisotropy: $H^x = H \neq 0$, $H^z = 0$. The equation (2.14) has now the feollowing form:

$$\left\{\mu H+\frac{2}{\sigma}\,|R(0)|\sin\vartheta\right\}\cos\vartheta = 0 \tag{4.9}$$

its solution is $\cos\vartheta = 0$, $\vartheta = \frac{\pi}{2}$, *i.e.* the magnetization is perpednicular to the axis of anisotropy, and by virtue of (2.19) Eqs (2.17), (2.18) can be written as:

$$E(\nu) = \left\{\mu H+\frac{1}{2}\,\sigma(I(0)-I(\nu))\right\}\left\{1+\frac{1}{2}\,\frac{\sigma|R(\nu)|}{\mu H+\frac{\sigma}{2}\,(I(0)-I(\nu))}\right\}^{1/2} \tag{4.10}$$

$$\frac{1}{\sigma} = \frac{v}{(2\pi)^3}\int d^3\nu\left\{1+\frac{1}{2}\,\frac{\sigma|R(\nu)|}{\mu H+\frac{\sigma}{2}\,(I(0)-(I\nu))}\right\}^{1/2} \operatorname{cth}\frac{1}{2}\,\beta E(\nu). \tag{4.11}$$

In this case it is obvious again, that (4.7), (4.10) and (4.8), (4.11) are identical, if $H = 0$:

$$E(\nu) = \frac{\sigma}{2}\,\{I(0)-I(\nu)\}\left\{1+\frac{|R(\nu)|}{I(0)-I(\nu)}\right\}^{1/2} \tag{4.12}$$

$$\frac{1}{\sigma} = \frac{v}{(2\pi)^3}\int d^3\nu\left\{1+\frac{|R(\nu)|}{I(0)-I(\nu)}\right\}^{1/2} \operatorname{cth}\frac{1}{2}\,\beta E(\nu) \tag{4.13}$$

and the magnetization is perpendicular to the axis of anisotropy: $\vartheta = \frac{\pi}{2}$.

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5. We summarize our results. We have obtained expressions for the spectrum of elementary excitations and an equation for the relative magnetization, which are valid for all temperature and for both of positive and negative anisotropy. We studied in details two special cases, when the direction of the external magnetic field is parallel and perpendicular to the anisotropy axis.

The expressions obtained for the spectrum of elementary excitations are the same, as in the paper [1], but our equations for the magnetization are more exact, bacause of the more consistent method used in the decoupling of the Green-functions.

Finally it is worth-while to note, that our method and the Tyablikov's method of decoupling [4], [5] leads to the same results for an isotropic ferromagnet.

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