BROADENING OF FERROMAGNETIC RESONANCE LINE BY DISLOCATIONS

By J. Morkowski

Ferromagnetics Laboratory, Institute of Physics of the Polish Academy of Sciences, Poznań*

(Received July 16, 1968)

The relaxation time for uniform magnons scattered on dislocations is calculated. The semiphenomenological theory of magnons is adopted. It is assumed that magnons are coupled to the strain field of a dislocation by magnetostrictive effects, and the magnetoelastic energy is taken for the magnon interaction Hamiltonian. Only two-magnon relaxation processes are considered. The calculated relaxation time is used to estimate the influence of dislocations on the ferromagnetic resonance line-width. Results of numerical calculations of the line-width are presented.

1. Introduction

After two decades of studies of ferromagnetic resonance there are still problems concerning the width of the resonance line which remain to be clarified. Numerous efforts have been undertaken in past years to understand processes determining the line-width of ferromagnetic resonance (*cf.* [1, 2]) and the first real successes in comparing theoretical predictions with experimental data came only with the studies of the ferromagnetic resonance in the yttrium iron garnet.

Until quite recently there was a particularly wide gap between the theoretical estimates and observed resonance line-widths in the classical ferromagnetic metals, such as iron or nickel. Studies by Rodbell [3, 4, 5] and others [6] of the ferromagnetic resonance in whiskers or thin platelets of iron and nickel revealed that the resonance line may be surprisingly narrow if the crystal is highly perfect. The resonance line-width in bulk monocrystals of silicon-iron [7] amounted to 400 oersted, whereas some selected iron-whisker crystals had line-widths as low as 30 oersted (at room temperature). The latter value can be satisfactorily explained by the Ament-Rado conductivity-exchange mechanism of damping [8], with perhaps a small contribution of intrinsic relaxation. In nickel, the line-width found in bulk monocrystals reaches several hundred oersted [9] while selected nickel-whiskers or thin platelets have line-widths often less than 150 oersted [4, 5].

The whiskers and platelets are apparently highly perfect crystals, therefore, the experimental data suggest that the ferromagnetic resonance line-width is closely related to the

* Address: Zakład Ferromagnetyków, Instytut Fizyki PAN, Poznań, ul. Fredry 10, Polska.

structural perfection of the crystal [5]. Imperfections which inevitably are present in bulk crystals cause extra broadening of the resonance line either by inducing local fluctuations of the resonance frequency or by promoting relaxation due to scattering of magnons on local defects of the crystal lattice.

Dislocations are an important class of defects always present in crystals. They are particularly important for various properties of metals. In the present paper we try to estimate the broadening of the ferromagnetic resonance line caused by dislocations. The results of the paper may be useful for an eventual explanation the difference between the line-widths in whiskers and in bulk crystals, provided the density of dislocations in the samples is known.

We shall calculate the relaxation time of the uniform resonance mode due to two-magnon scattering processes on dislocations. We assume that magnons are coupled to dislocations by the magnetoelastic energy. The phenomenological approach to magnons is used. The results can be applied to any cubic ferromagnet, numerical estimates are given for nickel. Preliminary results of the calculations have been published recently [10].

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

2. *Model assumptions*

In the present paper the results of calculations of the relaxation time for magnons scattered on dislocations are reported. Only lowest order, *i. e.* two-magnon processes are considered. The results are to be applied in estimating the effect of dislocations on the ferromagnetic resonance line-width; therefore, we are interested only in the relaxation of long-wavelength magnons, which are excited in a ferromagnetic sample by microwave field.

For long-wavelength magnons it is admissible, and most convenient for the present purpose, to use the semiphenomenological theory of magnons [11] (see also [2]). In this approach the ferromagnetic crystal is considered to be a continuous medium and the operator $\boldsymbol{M}(\boldsymbol{r})$ of the local magnetic moment density at a point $\boldsymbol{r}$ is defined. The local magnetization $\boldsymbol{M}(\boldsymbol{r})$ is expanded into plane waves, whose amplitudes are determined by magnon creation and annihilation operators (consult [2] for details). In the lowest order approximation, correct up to the terms quadratic in the magnon operators, the components of the locol magnetization can be expressed as follows (see Appendix for more accurate treatment)

$$M_{+}(\boldsymbol{r}) = M_x(\boldsymbol{r}) + iM_y(\boldsymbol{r})$$
$$\cong (4\mu_B M/V)^{1/2} \sum_{\boldsymbol{k}} a_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \ldots, \tag{1a}$$

$$M_z(\boldsymbol{r}) = M - (2\mu_B/V) \sum_{\boldsymbol{k}\boldsymbol{k}'} a^*_{\boldsymbol{k}'} a_{\boldsymbol{k}'+\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}. \tag{1b}$$

Here $a^*_{\boldsymbol{k}}$ and $a_{\boldsymbol{k}}$ are the operators of creation and annihilation of a magnon having the wave vector $\boldsymbol{k}$ (this interpretation is valid only in the limit of very high magnetic field, as in general only a linear combination of $a^*_{\boldsymbol{k}}$ and $a_{-\boldsymbol{k}}$ represents the true magnon creation operator, as is explained below). They satisfy boson commutation rules. M denotes the saturation magnetization at a given temperature. (The theory is, in principle, applicable to low temperatures. However, two-magnon relaxation is relatively insensitive to temperature and it is likely that the results presented here may retain their validity even for temperatures not very far from the Curie point.) V is the volume of the ferromagnetic crystal and μ_B is the Bohr magneton.

The total energy of the ferromagnetic medium, including the exchange energy, the Zeeman energy in an external magnetic field and the energy of magnetic dipolar interaction, can be expressed in terms of magnon operators. In the lowest order approximation, corresponding to the theory of non-interacting magnons, we have [12]

$$\mathcal{H}_0 = \sum_k \left(A_k a_k^* a_k + \frac{1}{2} B_k a_k a_{-k} + \frac{1}{2} B_k^* a_k^* a_{-k}^* \right). \tag{2}$$

The coefficients A_k, B_k are quoted in the Appendix.

The bilinear Hamiltonian (2) can be diagonalized by the Holstein-Primakoff [12] transformation (see Appendix for details)

$$a_k = l_k c_k - m_k c_{-k}^*, \tag{3}$$

where $l_k = l_{-k}$, $m_k = m_{-k}$, $l_k^2 - |m_k|^2 = 1$.

In terms of the new operators, c_k^* and c_k, the Hamiltonian (2) apart from an unimportant constant, takes the form,

$$\mathcal{H}_0 = \sum_k \varepsilon_k c_k^* c_k. \tag{4}$$

The energy ε_k of a magnon having the wave vector $\boldsymbol{k}$ is given by ([12, 2])

$$\varepsilon_k = 2\mu_B\{(H+\alpha k^2)(H+\alpha k^2+4\pi M \sin^2\theta_k)\}^{1/2}, \tag{5}$$

where $H = H_0 - 4\pi N_z M$. The applied magnetic field H_0 is directed along the z-axis of the coördinate system. α is the magnon dispersion coefficient, proportional to the strength of the exchange interaction. θ_k is the angle between the magnetic field direction and the wave vector $\boldsymbol{k}$. The energy of uniform magnons $\boldsymbol{k} = 0$ is given by the expression [2]

$$\varepsilon_0 = 2\mu_B\{(H+4\pi N_x M)(H+4\pi N_y M)\}^{1/2}. \tag{5'}$$

N_x, N_y and N_z denote the demagnetizing factors for the x, y and z directions, respectively.

Strictly speaking, we are not allowed to call the operators a_k^*, a_k the magnon operators. In fact, a_k^*, a_k describe deviations of the magnetization from its equilibrium value, but they are not normal modes of the system, as is seen from (2). On the contrary, the operators c_k^*, c_k diagonalize the Hamiltonian of the system and they represent magnons (*cf.* [1] or [2] for detailed discussion). However, if the terms of dipolar origin in the energy are relatively unimportant, the difference between a_k and c_k is insignificant and we can assume that the Fourier components of the local magnetization, Eq. (1), define approximate normal modes. This is the case if $H \gg 2\pi M$; then $l_k \cong 1$ and $m_k \cong 0$ (see Appendix), and hence $c_k \cong a_k$.

First, the case $H \gg 2\pi M$ will be discussed and the theory will be formulated in terms of the approximate normal mode operators, a_k^* and a_k. Generalization of the theory for arbitrary values of $2\pi M/H$ is discussed in the Appendix. However, even for $2\pi M/H \ll 1$ we will retain the exact expression for the magnon energy (5). The results of this approach[1] appear to be exact up to terms of the order of $(2\pi M/H)^2$.

[1] A similar procedure was used by Sparks *et al.* [13] and results were found to differ only by a few per cent from the exact calculations [14], taking the diagonalizing transformation (3) into account.

We assume that the ferromagnetic medium is deformed by a single dislocation described by the strain tensor e_{ij} $(i, j = x, y, z)$. The basic assumption made here is that the macroscopic magnetoelastic energy provides the coupling of magnons to the deformation field of the dislocation (Kittel and Abrahams [15] proposed similar idea for the first time in a different problem, namely for magnon-phonon scattering). We assume cubic symmetry of the ferromagnet and take the energy of coupling of magnons to the deformation field of the dislocation in the form [16]

$$\mathcal{H}_{me} = \int d\boldsymbol{r}\{(B_1/M^2)\,[M_x^2 e_{xx} + M_y^2 e_{yy} + M_z^2 e_{zz}] + $$
$$+ (B_2/M^2)\,[(M_x M_y)e_{xy} + (M_y M_z)e_{yz} + (M_z M_x)e_{zx}]\}. \tag{6}$$

B_1 and B_2 are the phenomenological magnetoelastic constants. $(M_i M_j)$ denotes the symmetrized product of the magnetization vector components, $\frac{1}{2}(M_i M_j + M_j M_i)$. Obviously, M_i as well as e_{ij} vary with the position $\boldsymbol{r}$ and the integration extends over the volume V of the sample.

3. *Two-magnon relaxation time*

The classical expression for the magnetoelastic energy (6) is now quantized by replacing the components of the magnetization vector by operators according to the approximate relations (1). Apart from an unimportant constant we obtain thus the lowest order expansion of the form

$$\mathcal{H}_{me} = \sum_{k} \bar{D}_k a_k + \sum_{kk'} (W_{k-k'} a_k^* a_{k'} + V_{k-k'} a_k a_{k'}) + \text{c.c.} \tag{7}$$

Terms linear in the magnon operators can be eliminated by a canonical transformation to a new set of oscillator operators, namely $a_k = a_k' + \gamma_k$, $a_k^* = a_k'^* + \gamma_k^*$ [17]. The complex numbers γ_k are determined by the requirement of vanishing the terms linear in the new operators $a_k'^*$, a_k' in the total Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{me}$. Terms for $\boldsymbol{k} = \boldsymbol{k}'$ in (7) represent corrections to the magnon energy, which are negligible in our case and will be ignored. The terms $\Sigma W_{k-k'} a_k^* a_{k'}$ describe processes of scattering of a magnon $\boldsymbol{k}'$ into a magnon $\boldsymbol{k}$ on the deformation field (two-magnon processes). The remaining part of (7) represents higher order processes and will not be considered here.

In the present paper we intend to calculate the relaxation time of the uniform magnons $\boldsymbol{k} = 0$ scattered into the degenerate spectrum of $\boldsymbol{k} \neq 0$ magnons on a dislocation. The terms in (7) responsible for these transitions are

$$\mathcal{H}_i = \sum_{k} W_k a_k^* a_0 + \text{c.c.}, \tag{8}$$

where

$$W_k = (2\mu_B B_1/MV) \int d\boldsymbol{r} (e_{xx} + e_{yy} - 2e_{zz}) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}. \tag{9}$$

In the first approximation there is no contribution to W_k from the second magnetoelastic constant B_2. Corrections to W_k in a more accurate treatment are discussed in the Appendix.

The system of magnons in our model is thus described by the total Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_i$, Eqs (4) and (8). By the standard perturbation theory [2] we obtain the relaxation time of the uniform magnons

$$1/\tau = (2\pi/\hbar) \sum_{k} |W_k|^2 \delta(\varepsilon_0 - \varepsilon_k). \tag{10}$$

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

In order to proceed further on we have to calculate the scattering matrix elements W_k. This will be done in the subsequent sections for the case of a screw and an edge dislocation.

4. Scattering on a screw dislocation

Now we calculate the scattering matrix element W_k (9) for a single screw dislocation oriented at an angle ϑ relative to the magnetic field direction. The Burgers vector $\boldsymbol{b}$ of the screw dislocation, in the coordinate system (x, y, z) with the z-axis parallel to the applied magnetic field, is $\boldsymbol{b} = (0, b \sin \vartheta, b \cos \vartheta)$. It is convenient to introduce a new Cartesian coordinate system, (x_1, x_2, x_3), with its x_3-axis parallel to the dislocation line or to the Burgers vector $\boldsymbol{b}$, and $x_1 = x$. The components of the strain tensor e_{ij} are calculated under the assumption that the ferromagnet can be treated as an elastic continuum. This assumption introduces some limitations which, fortunately, are not very essential for final results. First, we have to exclude from the considerations the region of high deformation along the dislocation line, to which the linear elasticity theory does not apply. Let r_0 be the shortest distance from the dislocation line for which the formulae of the elasticity theory are still valid. Secondly, we have to adopt a finite range r_1 of the deformation field assuming that the components of the strain tensor vanish at points distant by more than r_1 from the dislocation line.

The only non-vanishing components of the strain tensor are [18]

$$e_{13} = -\frac{b}{4\pi}\frac{x_2}{x_1^2+x_2^2}, \qquad e_{23} = \frac{b}{4\pi}\frac{x_1}{x_1^2+x_2^2}$$

for $r_0 \leqslant (x_1^2+x_2^2)^{1/2} \leqslant r_1$, and $e_{ij} = 0$ otherwise. Thus, in the (x_1, x_2, x_3)-system we have

$$e_{xx}+e_{yy}-2e_{zz} = \frac{3b}{4\pi}\frac{x_1}{x_1^2+x_2^2}\sin 2\vartheta. \tag{11}$$

The domain of integration in (9) is confined to the hollow cylinder $(r_0 \leqslant (x_1^2+x_2^2)^{1/2} \leqslant r_1$, $|x_3| \leqslant L)$, $2L$ being the length of the dislocation, and the result follows

$$W_k = i(6\mu_B B_1 b/MV)\sin 2\vartheta \frac{k_1}{k_1^2+k_2^2}\frac{\sin k_3 L}{k_3}\{J_0(r_0\sqrt{k_1^2+k_2^2}) - J_0(r_1\sqrt{k_1^2+k_2^2})\}, \tag{12}$$

where

$$k_1 = k_x,\ k_2 = k_y \cos\vartheta - k_z \sin\vartheta,\ k_3 = k_y \sin\vartheta + k_z \cos\vartheta, \tag{13}$$

and J_0 is the Bessel function.

The parametr r_0 determining the region of high deformation is of the order of magnitude of a few lattice constants. For long-wavelength magnons $r_0(k_1^2+k_2^2)^{1/2}$ is thus small as compared with unity, and with negligible error we can put $r_0 = 0$ in (12) and in all subsequent calculations.

In order to calculate the relaxation time the summation in (10) is replaced by integration. The most convenient integration variables are k_1, k_2, k_3. The length of the dislocation is of the order of magnitude of the linear dimensions of the ferromagnetic crystal, thus for non-zero k_3, k_3L is a large number and $(\sin k_3L)^2/k_3^2$ which appears in $|W_k|^2$ behaves like the δ-function. Consequently, the integration over k_3 in (10) is performed using the formula

$$\int_{-\pi/a}^{\pi/a} dk_3 \left(\frac{\sin k_3 L}{k_3}\right)^2 f(k_3) = \pi L f(0). \tag{14}$$

It is convenient to introduce polar coordinates $k_1 = \varkappa \cos \varphi$, $k_2 = \varkappa \sin \varphi$. Elementary calculations give for $k_3 = 0$

$$\delta(\varepsilon_0 - \varepsilon_k) = \left(\frac{\partial \varepsilon_k}{\partial \varkappa}\right)^{-1} \delta(\varkappa - \varkappa_0), \tag{15a}$$

where

$$\varkappa_0 = (2\pi M/\alpha)^{1/2}\{[(\varepsilon_0/4\pi\mu_B M)^2 + (1 - \sin^2\vartheta \sin^2\varphi)^2]^{1/2} - \\ -(H/2\pi M) - 1 + \sin^2\vartheta\sin^2\varphi\}^{1/2} \tag{15b}$$

for

$$\sin^2\vartheta \sin^2\varphi > N_z - (4\pi N_x N_y M/H). \tag{15c}$$

Outside the region determined by (15c) we have $\delta(\varepsilon_0 - \varepsilon_k) = 0$.

After simple calculations we find the following expression for the relaxation time of uniform magnons due to the single screw dislocation

$$1/\tau_s = (9/\pi^2)(2\mu_B/\hbar)(b^2L/V)(B_1^2/M^3)\sin^2\vartheta\cos^2\vartheta \cdot F_s(\vartheta) \tag{16}$$

with the definition

$$F_s(\vartheta) = \int_{\gamma}^{\pi/2} d\varphi \left(\frac{\cos\varphi}{u}\right)^2 [1 - J_0(\lambda u)]^2 [1 + \chi^2(1 - \sin^2\vartheta\sin^2\varphi)^2]^{-1/2}, \tag{17}$$

where

$$u = \left\{\sin^2\vartheta\sin^2\varphi - N_z + \frac{\chi(1-\sin^2\vartheta\sin^2\varphi)^2}{1 + [1+\chi^2(1-\sin^2\vartheta\sin^2\varphi)^2]^{1/2}}\right\}^{1/2}, \tag{18a}$$

$$\chi = 2\pi M\{(H + 4\pi N_x M)(H + 4\pi N_y M)\}^{-1/2}, \tag{18b}$$

$$\lambda = r_1(2\pi M/\alpha)^{1/2}, \tag{18c}$$

and

$$\gamma = \arcsin G^{1/2},$$

with

$$G = [N_z - (4\pi N_x N_y M/H)]/\sin^2\vartheta \tag{18d}$$

if $0 \leqslant G \leqslant 1$. For $G < 0$ we take $\gamma = 0$, whereas for $G > 1$, $\gamma = \pi/2$ and the integral (17) vanishes.

In the preliminary report of this work [10] an approximate analytical expression for $1/\tau_s$ has been given for a special case (Eq. (3) of [10]). The formula in mention was based on rather crude approximations. In order to obtain more accurate results numerical work is needed.

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

5. Scattering on an edge dislocation

Calculations for the case of an edge dislocation are completely analogous to the ones for the screw dislocation described in the preceding Section, so there is no need for discussing details.

As previously, we introduce the coordinate system (x_1, x_2, x_3) with the x_3-axis parallel to the dislocation line. Let ϑ denote the angle between the magnetic field direction (z-axis) and the dislocation line. In order to simplify calculations we assume the Burgers vector parallel to the x_1-axis, $\boldsymbol{b} = (b, 0, 0)$. The components of the strain tensor e_{ij} can be found from the elasticity theory [18] and

$$e_{xx}+e_{yy}-2e_{zz} = -\frac{b}{2\pi(1-\nu)}\frac{x_2}{x_1^2+x_2^2}\left(1-2\nu+3\nu\sin^2\vartheta+\frac{3}{2}\frac{x_1^2-x_2^2}{x_1^2+x_2^2}\sin^2\vartheta\right), \qquad (19)$$

where ν denotes the Poisson ratio.

The scattering matrix element is now

$$W_k = -i[4\mu_B B_1 b/MV(1-\nu)]\frac{k_2}{k_1^2+k_2^2}\frac{\sin k_3L}{k_3}\left\{\left(1-2\nu+3\nu\sin^2\vartheta-3\frac{k_1^2}{k_1^2+k_2^2}\sin^2\vartheta\right)\times\right.$$

$$\left.\times[1-J_0(r_1\sqrt{k_1^2+k_2^2})]+\frac{3}{2}\sin^2\vartheta\left(4\frac{k_1^2}{k_1^2+k_2^2}-1\right)J_2(r_1\sqrt{k_1^2+k_2^2})\right\}. \qquad (20)$$

The notation is the same as in Section 4, in particular k_i are given by (13) and J_n denote the Bessel functions. The approximation $r_0 = 0$ was used in (20).

The relaxation time due to the edge dislocation at the angle ϑ to the magnetic field, resulting from (20) is finally given by

$$1/\tau_e = [1/\pi(1-\nu)]^2(2\mu_B/\hbar)\,(b^2L/V)\,(B_1^2/M^3)F_e(\vartheta), \qquad (21)$$

where

$$F_e(\vartheta) = \int_\gamma^{\pi/2} d\varphi\left(\frac{\sin\varphi}{u}\right)^2\left\{(1-2\nu+3\nu\sin^2\vartheta-3\sin^2\vartheta\cos^2\varphi)[1-J_0(\lambda u)]+\right.$$

$$\left.+\frac{3}{2}\sin^2\vartheta(4\cos^2\varphi-1)J_2(\lambda u)\right\}^2[1+\chi^2(1-\sin^2\vartheta\sin^2\varphi)^2]^{-1/2} \qquad (22)$$

with the notation (18).

For $\vartheta = 0$ it is easy to calculate $1/\tau_e$ in an elementary way, the result for a special case is given in [10]. For the other extreme case $\sin^2\vartheta \gg 2\pi M/H$ an approximate analytical expression for $1/\tau_e$ was given in [10][2].

---

[2] We take the opportunity to point out on an error in [10], namely the minus sign at the expression $3\nu\sin^2\vartheta$ in the formula for $F(\xi, \sin^2\vartheta)$ should be replaced by the plus sign. Consequently, the numerical value of the factor A for edge dislocations is reduced to the value $A = 0.2$.

6. *Ferromagnetic resonance line-width*

The results of the last two sections will be applied now in estimating the influence of dislocations on the ferromagnetic resonance line-width. The arguments which are presented below indicate that dislocations of sufficiently high density may play an important role in broadening the resonance line. In order to simplify the discussion we assume that the eddy current damping and scattering of magnons on dislocations produce approximately additive contributions to the resonance line-width.

Let us assume that a microwave field of frequency ω_0 excites magnons of energy $\varepsilon_0 = \hbar\omega_0$, whose wavelengths, roughly equal to the skin-depth, are large enough to make negligible the exchange contributions to the magnon energy. This assumption means that we have to calculate the damping of nearly uniform magnons of energy ε_0. Let $1/\tau$ be the inverse relaxation time of the uniform magnons, determined by a single dislocation. The corresponding contribution to the magnon line-width is $(\hbar/2\mu_B\tau)$ (see *e.g.* [1]).

Assume a distribution of dislocations in a ferromagnetic sample, and let n be the density of dislocations, defined as the number of dislocations crossing at right angle a unit area taken at random in the ferromagnet [18]. Assume also that each dislocation scatters magnons independently. This is a resonable assumption for sufficiently low density of dislocations; some aspects of this approximation will be discussed in another paper.

The dislocation line cannot end inside the crystal, but extends from boundary to boundary, so the dislocation length $2L$ is comparable with the sample dimensions. Thus the area of the sample cross section is roughly $V/2L$ and consequently the total number of dislocations (having the length $2L$) is of the order of magnitude $(V/2L)n$; each of them contributes $(\hbar/2\mu_B\tau)$ to the line-width ΔH. Therefore, the ferromagnetic resonance line-width due to dislocations, up to the order of magnitude, amounts to[3]

$$\Delta H = (\hbar/2\mu_B\tau)\ (V/2L)n. \tag{23}$$

In general, the dislocation distribution may be anisotropic. In this case the line-width (23) depends on the relative orientation of the magnetic field and preferred direction of the system of dislocations.

The final expression for the dislocation controlled line-width of the ferromagnetic resonance is

$$\Delta H_s = (9/2\pi^2)\ (b^2B_1^2/M^3)n \sin^2\vartheta \cos^2\vartheta\, F_s(\vartheta) \tag{24}$$

for screw dislocations, and

$$\Delta H_e = [1/2\pi^2(1-\nu)^2]\ (b^2B_1^2/M^3)nF_e(\vartheta) \tag{25}$$

---

[3] Perhaps a somewhat more accurate expression can be derived with the use of the notion of the total length of dislocation line in the unit volume of the sample, equal to $2n$ (*cf.* [19], p. 44). The contribution to ΔH from unit length of a dislocation is $(\hbar/2\mu_B\tau)/2L$ and the total length of dislocations in the sample of volume V is $2nV$, thus

$$\Delta H = (\hbar/2\mu_B\tau)(V/L)n \tag{23'}$$

is twice as large as in (23).

for edge dislocations. In the above expressions b plays the role of an average value of the Burgers vector. Numerical values of b are of the order of magnitude of the lattice constant, and we replace b by the lattice constant a in the following numerical estimates. Another parameter of the theory, unfortunately not a very well defined one, is the range of the deformation field of dislocation, r_1. According to estimates [18], r_1 is typically of the order of magnitude of one micron. Numerical values of ΔH appear not changing rapidly with changes of r_1.

For an isotropic distribution of dislocations the resonance line-width due to screw or edge dislocations is the average value $\overline{\Delta H} = (2/\pi) \int_0^{\pi/2} d\vartheta \Delta H$ of the right hand sides of (24) or (25), respectively.

7. *Numerical results*

The theory described above will be illustrated now by some numerical examples. Of the ferromagnetic metals, nickel seems to be the most appropriate material for an experimental check of the theoretical predictions. In nickel B_1 is larger and M smaller than the corresponding quantities in iron, thus the factor B_1^2/M^3 is by two orders of magnitude larger than in iron. The effect of dislocations on the line-width is thus much stronger in nickel than in iron.

According to [16] for nickel at room temperature $B_1 = 6.2 \times 10^7$ ergs/cm^3. The value $M = 485$ gauss was taken for the magnetization at room temperature [16]. The exchange parameter α in (5) can be estimated from precise measurements of the temperature dependence of the spontaneous magnetization in nickel [20] with the result $(2\mu_B/k_B)\alpha = 4.66 \times 10^{-13}$ (1°K cm^2) (here k_B denotes the Boltzmann constant) or $\alpha = 3.47 \times 10^{-9}$ gauss cm^2. The lattice constant for nickel was taken to be $a = 3.52$ Å and the Poisson ratio $\nu = 0.36$.

In order to secure the condition $\chi \ll 1$ and still have the numerical results useful for comparison with eventual measurements of the dislocation induced broadening of the resonance, we calculate ΔH for the resonance frequency 34.9 kMc/sec, *i.e.* for the 8 mm microwave band. This corresponds to an energy of magnons excited by the microwave field equal to $\varepsilon_0 = 2\pi\hbar \times 34.9 \times 10^9$ and thus $\chi = 0.245$.

Application of the theory presented in this paper to nickel encounters a difficulty which is briefly outlined now and will be elaborated in the Appendix. The calculations in the preceding sections are based on the expansion (1), which gives the better approximation, the larger is the effective magnetic moment per atom. In nickel the effective magnetic moment is only 0.6 μ_B per atom and an expansion more accurate than (1) is required. As is demonstrated in the Appendix, the higher order terms in the expansion for $\boldsymbol{M}(\boldsymbol{r})$ can be approximately accounted for by multiplying the lowest-order results, (24) and (25), by the correction term $[1-(N\mu_B/MV)]^2$, which is nearly unity for large $MV/N\mu_B$. For nickel the correction factor is equal to $[1-(1/0.6)]^2 = 4/9$. In all results presented in Figs 1 to 11, the correction for higher-order terms was taken into account by multiplying the values of ΔH calculated from (24) or (25) by the factor 4/9.

Taking the above mentioned values of parameters for nickel we obtain $\Delta H_s = 0.85(10^{-8}n) \sin^2\vartheta \cos^2\vartheta F_s(\vartheta)$ as the contribution to the ferromagnetic resonance line-

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

width from screw dislocations, and $\Delta H_e = 0.23(10^{-8}\, n)F_e(\vartheta)$ as the corresponding quantity for edge dislocations (n is measured in units of cm^{-2}, ΔH in oersted). The functions $F(\vartheta)$ were calculated by numerical integration from (17) or (22), respectively, for the parameters: $\chi = 0.245$ and $\lambda = 12$, (24), 60, 240. The parameter λ, proportional to the range r_1 of the

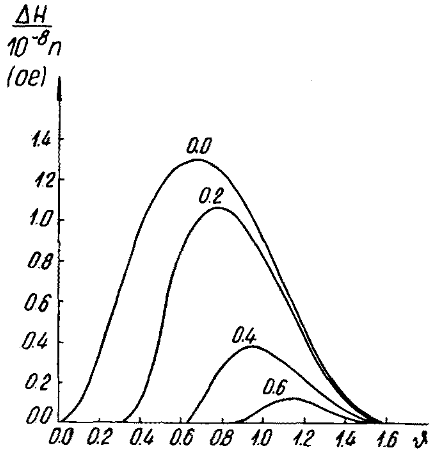

Fig. 1. Ferromagnetic resonance line-width ΔH caused by screw dislocations of density n, oriented at an angle ϑ relative to the magnetic field. The curves ΔH *vs.* ϑ were calculated for nickel, for $\chi = 0.245$, $\lambda = 12$, and for the demagnetizing factor N_z equal to 0.0, 0.2, 0.4 and 0.6, respectively. ΔH is measured in oersted times $(n \times 10^{-8}\, \mathrm{cm}^2)$

deformation field of dislocation, equals $(9.36 \times 10^5\, \mathrm{cm}^{-1})\, r_1$ for nickel. Thus the corresponding values of r_1 for chosen λ's are 0.128, (0.256), 0.64, and 2.56 micron. Typical values of r_1 are estimated to be of the order of magnitude of 1 micron [18], hence, we believe that the

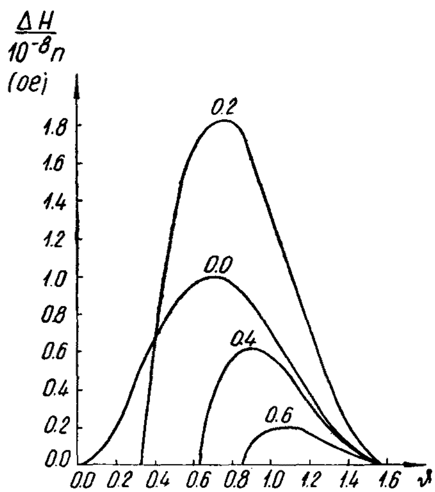

Fig. 2. Ferromagnetic resonance line-width ΔH due to screw dislocations, calculated for $\lambda = 24$. All other parameters are the same as in Fig. 1

chosen range of λ values is relevant to nickel. This statement is corroborated by the observation that in most cases variation of F with r_1 is rather slow (see Figs 5 and 10).

The portion of the magnon spectrum ε_k which is degenerate with the uniform mode ε_0 depends on the shape of the sample through the demagnetizing factors. In consequence ΔH is a function of the demagnetizing factor N_z.

Figure 1 presents results of numerical calculations of ΔH for screw dislocations in nickel, for $\lambda = 12$. ΔH is plotted *versus* the angle ϑ between the dislocation line and the magnetic

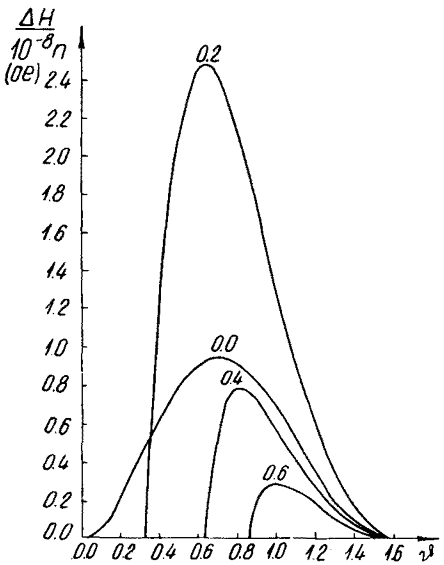

Fig. 3. Ferromagnetic resonance line-width ΔH for screw dislocations. $\lambda = 60$, and remaining parameters as in Fig. 1

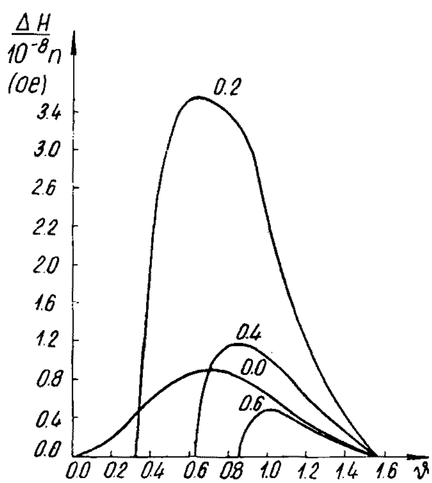

Fig. 4. Ferromagnetic resonance line-width ΔH for screw dislocations. $\lambda = 240$, and remaining parameters as in Fig. 1

field direction; the different plots correspond to different values of the demagnetizing factor N_z. The dislocation density n is measured in units of cm^{-2} and, the line-width ΔH in oersted. Similar data, but calculated for $\lambda = 24$, 60 and 240 are plotted in Figs 2, 3, and 4, respectively.

The factor $\sin^2 \vartheta \cos^2 \vartheta$ highly reduces ΔH due to screw dislocations for ϑ close to 0 and $\pi/2$. The energy conservation condition $\varepsilon_k = \varepsilon_0$ in (10) prohibits scattering processes for ϑ smaller than a critical value, ϑ_0, dependent on N_z. Thus the curves of ΔH *versus* ϑ start from $\Delta H = 0$ at $\vartheta = \vartheta_0$, with ϑ_0 increasing with N_z. Obviously, the same critical angle appears for the corresponding plots of ΔH for edge dislocations.

All these results are directly useful for highly anisotropic distributions of dislocations, with only one direction represented. In other cases of known angular distributions of dislocations, suitable average values of ΔH have to be calculated from plots shown in Figs 1 to 4. For the isotropic distribution, if n is independent of ϑ, the average values $\overline{\Delta H}$ can be obtained by integration of these curves. The averages, $\overline{\Delta H}$, calculated in this way for screw dislocations in nickel are plotted in Fig. 5 as functions of λ, for several values of N_z. An

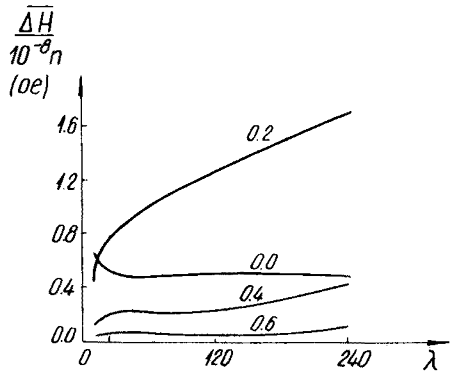

Fig. 5. Average line-width $\overline{\Delta H}$ due to isotropic distribution of screw dislocations *versus* λ, calculated for nickel, $\chi = 0.245$, $N_z = 0.0$, 0.2, 0.4 and 0.6, respectively

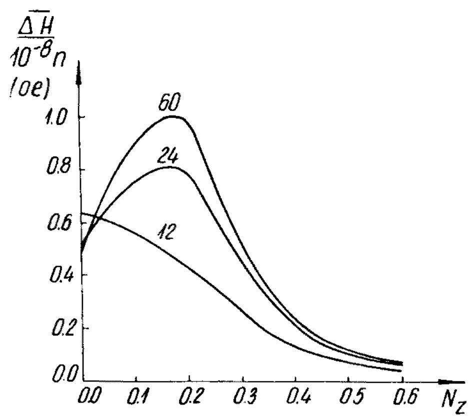

Fig. 6. Average line-width $\overline{\Delta H}$ for screw dislocations in nickel *versus* demagnetizing factor N_z for $\chi = 0.245$, $\lambda = 12$, 24 and 60, respectively

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

interesting thing is that $\overline{\Delta H}$ does not change very rapidly with λ or r_1. The relative insensitivity of the theory to the particular value of the parameter r_1 is fortunate, because the precise value of r_1 is not known.

Figure 6 presents ΔH plotted against the demagnetizing factor N_z for several values of the parameter λ. The effect of dislocations on the line-width appears strongly dependent on the sample shape through N_z, as should be expected for two-magnon processes.

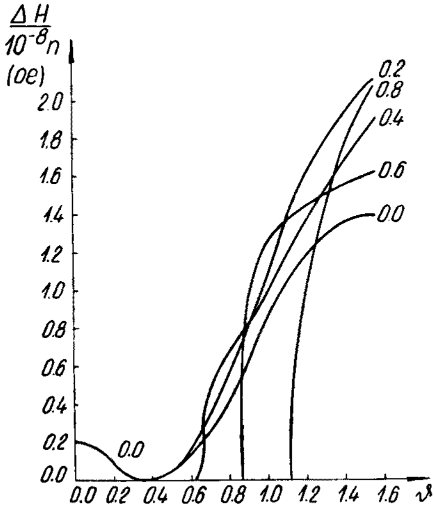

Fig. 7. Ferromagnetic resonance line-width ΔH caused by edge dislocations of density n. Details are the same as in Fig. 1 (in particular, $\lambda = 12$)

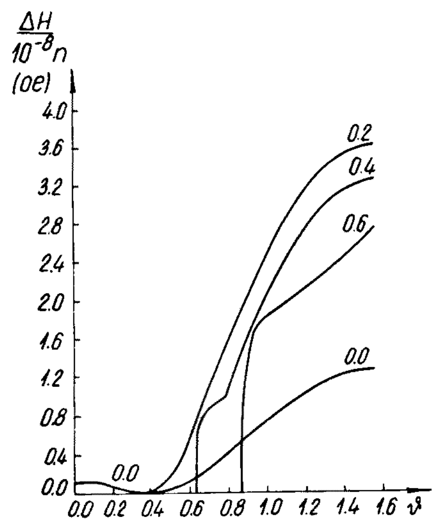

Fig. 8. Line-width for edge dislocations. Parameters as for Fig. 3 ($\lambda = 60$)

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

Figures 7, 8 and 9 depict the line-widths of the ferromagnetic resonance due to edge dislocations in nickel, calculated for $\lambda = 12$, 60, and 240, respectively. Different curves of ΔH *versus* ϑ correspond to different values of N_z.

The λ-dependence of the line-width $\overline{\Delta H}$ averaged over all directions is shown in Fig. 10; each curve corresponds to a different value of N_z. In Fig. 11 $\overline{\Delta H}$ is plotted against the demagnetizing factor N_z for several values of λ.

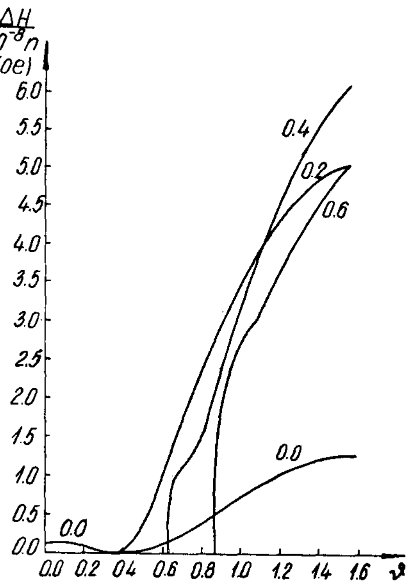

Fig. 9. Line-width for edge dislocations. Parameters as for Fig. 4 ($\lambda = 240$)

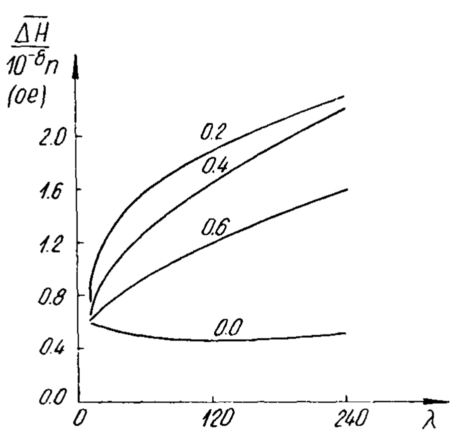

Fig. 10

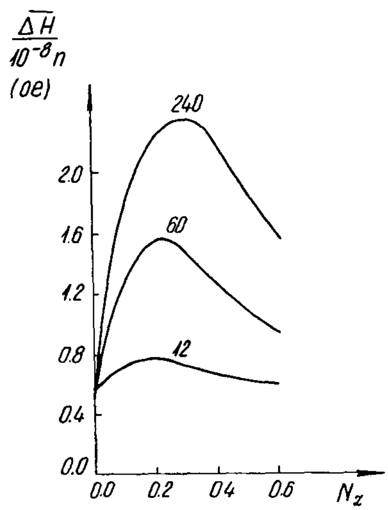

Fig. 11

Fig. 10. Average line-width for edge dislocations *versus* λ for nickel; $\chi = 0.245$ and specified values of N_z

Fig. 11. Average line-width for edge dislocations plotted against N_z for nickel; $\chi = 0.245$ and specified values of λ

8. *Conclusions*

The ferromagnetic resonance line-width due to dislocations calculated for nickel is typically of the order of magnitude $\Delta H \cong (10^{-8}n)$ oersted. It is believed that the basic assumptions of the calculations are justified[4] for dislocation densities even as high as $n \cong 10^{10}$ cm^{-2}. Thus the broadening of a resonance line by dislocations is easily within the reach of experimental possibilities. The simplest way to check the predictions of the theory experimentally would be to investigate the increase of the ferromagnetic resonance line-width with increasing ratio of plastic deformation of a monocrystalline sample.

The temperature dependence of ΔH is essentially determined by the factor B_1^2/M^3. The magnetoelastic constant B_1 is proportional to the magnetostriction constant λ_{100} [16], $B_1 = -(3/2)(c_{11}-c_{12})\,\lambda_{100}$, where c_{ij} are the elastic moduli for a cubic crystal. The elastic moduli change rather slowly with temperature, consequently, the temperature dependence of ΔH is approximately governed by the factor (λ_{100}^2/M^3). From experimental data for λ_{100} and M it follows that ΔH due to dislocations is fairly constant at low temperatures, but decreases with temperatures approaching the Curie point.

The problem of the influence of dislocations on the ferromagnetic resonance line-width was discussed in the paper [21], which appeared after the draft of the present paper was completed. Perhaps the following comment will be useful. In the present paper we do not use relations between the density of dislocations and the parameter r_1 determining the range of the dislocation field (for parallel equidistant dislocations the relation $n \sim r_1^{-2}$ can be anticipated). Thus ΔH is apparently proportional to the dislocation density n, but the functions F_s and F_e, Eqs (24) and (25), and hence $\Delta H/n$, depend on r_1. In the paper [21] the relation between n and r_1 is explicitly taken into account, resulting in a different dependence of the line-width on the dislocation density in different regions of values of parameters.

APPENDIX

Two aspects of improving the approximations made in the paper will be discussed now. First, the transformation (3) will be fully taken into account. This makes it possible to relax the condition $2\pi M/H \ll 1$ and to extend the results for an arbitrary magnetic field strength. Next, we shall calculate some corrections from the higher order terms in the expansion (1a) and corrections associated with them from the linear terms in the magnetoelastic coupling Hamiltonian (7).

We start from the generalized expansion of $M_{\pm}(\boldsymbol{r})$, correct up to the terms of third order with respect to the magnon operators (see [2], Eq. (30.17))

$$M_{+}(\boldsymbol{r}) = (4\mu_B M/V)^{1/2}\{\sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} a_{\boldsymbol{k}} - $$

$$-(\mu_B/2MV)\sum_{\boldsymbol{k}_1\boldsymbol{k}_2\boldsymbol{k}_3} e^{i(-\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{r}} a^*_{\boldsymbol{k}_1} a_{\boldsymbol{k}_2} a_{\boldsymbol{k}_3} + \ldots\}. \tag{A.1}$$

The expansion (1b) for $M_z(\boldsymbol{r})$ is exaet.

[4] The most subtle point is the assumption of additivity of contributions from single dislocations to the total line-width ΔH for a high dislocation density. Work is now in progress on the influence of dislocation dipoles on the resonance line-width, and should clarify some aspects of the problem.

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

With the third order terms taken into account the magnon Hamiltonian (2) has to be replaced by (*cf.* [2], Chapter A.III)

$$\mathcal{H}_m = \sum_k \left\{A_k a_k^* a_k + \left(\frac{1}{2} B_k a_k a_{-k} + \text{c.c.}\right)\right\} + \sum_{kk'} (F_k a_{k+k'}^* a_k a_{k'} + \text{c.c.}), \qquad \text{(A.2)}$$

where the coefficients for cubic ferromagnets, in the long-wavelength limit (but for magnon wavelengths smaller than the sample dimensions) are

$$A_k = 2\mu_B(H + \alpha k^2 + 2\pi M \sin^2 \theta_k),$$
$$B_k = 4\pi\mu_B M(k_x - ik_y)^2/k^2, \qquad \text{(A.3)}$$
$$F_k = -4\pi\mu_B(4\mu_B M/V)^{1/2}(k_x - ik_y)k_z/k^2.$$

For uniform magnons

$$A_0 = 2\mu_B[H + 2\pi(1-N_z)M],$$
$$B_0 = 4\pi\mu_B M(N_x - N_y),\ F_0 = 0.$$

The magnetoelastic energy, correct up to the third order terms, is (apart from a constant)

$$\mathcal{H}_{me} = \sum_k \overline{D}_k a_k + \sum_{kk'} (\overline{W}_{k-k'} a_k^* a_{k'} + \overline{V}_{k-k'} a_k a_{-k'}) + \sum_{kk'k''} D_k a_{k+k'+k''}^* a_{k'} a_{k''} + \text{c.c.} \qquad \text{(A.4)}$$

where

$$\overline{W}_k = \left(1 - \frac{N\mu_B}{MV}\right) W_k, \qquad \overline{V}_k = \left(1 - \frac{N\mu_B}{2MV}\right) V_k,$$

$$\overline{D}_k = -\frac{2}{5}\frac{MV}{\mu_B}\left[\left(1 - \frac{N\mu_B}{MV}\right)\Big/\left(1 - \frac{3}{5}\frac{N\mu_B}{MV}\right)\right] D_k. \qquad \text{(A.5)}$$

$$D_k = -\frac{5}{16}\frac{B_2}{M^3}(4\mu_B M/V)^{3/2}\left(1 - \frac{3}{5}\frac{N\mu_B}{MV}\right)\int d\boldsymbol{r}(e_{zx} - ie_{yz})e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}, \qquad \text{(A.6)}$$

$$V_k = \frac{\mu_B}{MV}\int d\boldsymbol{r}\,[B_1(e_{xx} - e_{yy}) - iB_2 e_{xy}]e^{i\boldsymbol{k}\cdot\boldsymbol{r}}. \qquad \text{(A.7)}$$

N is the number of atoms in the ferromagnetic crystal.

Summing (A.2) and (A.4) we arrive at the total Hamiltonian $\mathcal{H} = \mathcal{H}_m + \mathcal{H}_{me}$. In order to eliminate from $\mathcal{H}$ terms linear in a_k we introduce another set of oscillator operators, a_k' and $a_k'^*$, by the canonical transformation [17]

$$a_k = a_k' + \gamma_k, \qquad a_k^* = a_k'^* + \gamma_k^*. \qquad \text{(A.8)}$$

The complex numbers γ_k are determined from the condition that all coefficients standing at a_k' and $a_k'^*$ in the transformed Hamiltonian $\mathcal{H}$ vanish. These coefficients contain terms linear and quadratic in γ_k. The latter appear from the third order terms in $\mathcal{H}$, and they can be consistently neglected. The linearized equations for γ_k are

$$\sum_{k'} (\overline{W}_{k'-k}\gamma_{k'}^* + 2\overline{V}_{k'+k}\gamma_{k'}) + A_k\gamma_k^* + B_k\gamma_{-k} + \overline{D}_k = 0. \qquad \text{(A.9)}$$

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

It is evident from (A.8) that the third order terms in $\mathcal{H}$ contribute to the transformed Hamiltonian terms of the second order in a'_k, proportional to γ_k. For the discussion of two-magnon scattering the only relevant terms in the Hamiltonian $\mathcal{H}$ are those of the second order in the magnon operators a'_k. From now on we thus neglect the third order terms and, apart from an unimportant constant, $\mathcal{H}$ reads

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}' \tag{A.10}$$

with $\mathcal{H}_0$ given essentially by (2),[5]

$$\mathcal{H}_0 = \sum_k \left\{ A_k a'^*_k a'_k + \left(\frac{1}{2} B_k a'_k a'_{-k} + \text{c. c.} \right) \right\},$$

and

$$\mathcal{H}' = \sum_{kk'} \{ Q_{kk'} a'^*_{k+k'} a'_k + (R_{kk'} a'_k a'_{k'} + \text{c. c.}) \}. \tag{A.11}$$

Here,

$$Q_{kk'} = \overline{W}_k + 2 \sum_{k''} (D_{k-k''} \gamma_{k''} + D^*_{-k-k''} \gamma^*_{k''}) + (F_k + F_{k'}) \gamma_k + (F_{-k} + F_{k'}) \gamma^*_{-k} \tag{A.12}$$

for $\boldsymbol{k} \neq 0$ and $Q_{0,k'} = 0$, and

$$R_{kk'} = \overline{V}_{k+k'} + \sum_{k''} D_{k''} \gamma^*_{k+k'+k''} + \frac{1}{2} (F_k + F_{k'}) \gamma^*_{k+k'} \tag{A.13}$$

for $\boldsymbol{k} \neq -\boldsymbol{k}'$, and $R_{k,-k} = 0$.

The equations (A.9) can be solved for γ_k by iteration. In the second step, γ_k, correct up to second-order terms with respect to strains e_{ij}, is given by

$$\gamma_k \cong B^*_k \overline{D}_{-k} - A_k \overline{D}^*_k - \sum_{k'} A_k A_{k'} (\overline{W}^*_{k'-k} D^*_{-k'} + 2 \overline{V}^*_{k-k'} D_{k'}) + \ldots \tag{A.14}$$

The total Hamiltonian (A.10) is now transformed according to the transformation (3), in order to bring $\mathcal{H}_0$ into explicitly diagonal form

$$\begin{aligned} a'_k &= l_k c_k - m_k c^*_{-k}, \\ a'^*_k &= l_k c^*_k - m^*_k c_{-k}, \end{aligned} \tag{A.15}$$

where

$$l_k = \left\{ \frac{1}{2} \left(\frac{A_k}{\varepsilon_k} + 1 \right) \right\}^{1/2},$$

$$m_k = \frac{B^*_k}{|B_k|} \left\{ \frac{1}{2} \left(\frac{A_k}{\varepsilon_k} - 1 \right) \right\}^{1/2}, \tag{A.16}$$

[5] Some small corrections to A_k and B_k were ignored. Their effect on the final answer (A.19) is of higher order with respect to the deformation and is negligible within the framework of (A.18).

and $\varepsilon_k = (A_k^2 - |B_k|^2)^{1/2}$ is the magnon energy. The transformed Hamiltonian is

$$\mathcal{H} = \sum_k \varepsilon_k c_k^* c_k + \sum_{\substack{kk' \\ (k \neq k')}}' \Omega_{kk'} c_k^* c_{k'} + \ldots, \tag{A.17}$$

apart from a constant and terms proportional to $c_k c_{k'}$ or $c_k^* c_{k'}^*$ which do not contribute to two-magnon scattering processes.

Damping of the uniform magnon mode is determined by the part $\sum_{k(k\neq 0)}' (\Omega_{0k} c_0^* c_k + \text{c. c.})$ of (A.17) and the relaxation time is given by (10) with W_k replaced by Ω_{0k}.

The exact expression for Ω_{0k} is quite intricate but is simplified considerably in the first approximation with respect to strains e_{ij}. In the linear approximation with respect to strains or, formally, with respect to the magnetoelastic constants

$$\Omega_{0k} \cong l_k \overline{W}_{-k} - 2m_k^* \overline{V}_{-k}^* + 2l_k F_k(\gamma_k^* + \gamma_{-k}) - m_k^* F_k^* \gamma_{-k} + 0(e_{ij}^2), \tag{A.18}$$

where for γ_k we put the lowest order solution $\gamma_k \cong B_k^* \overline{D}_{-k} - A_k \overline{D}_k^*$, since the other terms in (A.14) are at least quadratic in the magnetoelastic constants.

Note that $\sum_k'(\Omega_{0k} c_0^* c_k + \text{c. c.})$ with Ω_{0k} given by (A.18) represents the interaction Hamiltonian for the two-magnon processes $(\boldsymbol{k} = 0) \to (\boldsymbol{k} \neq 0)$ which, within the linear approximation with respect to strains e_{ij}, takes into account the effect of higher order terms in the magnon Hamiltonian (A.2). From these, only the third order terms contribute anything and their effect is included in (A.18) through the terms proportional to F_k.

The relaxation time is calculated from

$$1/\tau = (2\pi/\hbar) \sum_k |\Omega_{0k}|^2 \, \delta(\varepsilon_0 - \varepsilon_k)$$

essentially in the same way as in Section 4, but the calculations are now much more tedious.

The relaxation time for a screw dislocation making an angle ϑ with the direction of the applied magnetic field, *i. e.* having the Burgers vector $\boldsymbol{b} = (0, b \sin \vartheta, b \cos \vartheta)$, is given by

$$1/\tau_s = (9/\pi^2)(2\mu_B/\hbar)(b^2 L/V)(B_1^2/M^3) \left(1 - \frac{N\mu_B}{MV}\right)^2 G_s(\vartheta), \tag{A.19}$$

where

$$G_s(\vartheta) = \int_\gamma^{\pi/2} d\varphi \frac{1}{u^2} [1 - J_0(\lambda u)]^2 \frac{S(\vartheta, \varphi)}{[1 + \chi^2(1 - \sin^2 \vartheta \sin^2 \varphi)^2]^{1/2}}. \tag{A.20}$$

u, λ and γ are defined by (18a–d). The function S, for the special case of a ferromagnetic sample in the shape of an ellipsoid of revolution ($N_x = N_y$), has the form

$$S(\vartheta, \varphi) = \omega \sin^2 \vartheta \cos^2 \vartheta \cos^2 \varphi \, \{1 + (\sigma\chi/6\omega)[1 - (1 - 6\delta + \cos^2 \vartheta) \sin^2 \varphi] -$$

$$-16\chi^2 \cos^2 \vartheta \sin^2 \varphi \cos^2 \varphi + (2\delta\chi^3/\omega) \cos^2 \vartheta \sin^2 \varphi (1 - \sin^2 \vartheta \sin^2 \varphi)^2\}^2 +$$

$$+ \frac{1}{4} \omega (\delta\chi p)^2 \sin^2 \vartheta \sin^2 \varphi \cos^2 \varphi [8 - (\chi/\omega)(1 - \sin^2 \vartheta \sin^2 \varphi)^2]^2 +$$

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4

$$+(\delta^2\chi^4p^2/4\omega)\sin^2\vartheta\cos^2\vartheta\sin^4\varphi(1-\sin^2\vartheta\sin^2\varphi)^2+$$

$$+\frac{1}{4}\delta^2\chi^2\sin^2\vartheta\sin^2\varphi\left\{\sigma\left[\cos^2\vartheta-\left(1+\frac{3\delta-2}{3\delta}\cos^2\vartheta\right)\cos^2\varphi\right]-\right.$$

$$\left.-32\chi\omega\cos^4\vartheta\sin^2\varphi\cos^2\varphi+4\chi^2\cos^2\vartheta\cos 2\vartheta\sin^2\varphi\cos^2\varphi\,(1-\sin^2\vartheta\sin^2\varphi)\right\}^2+$$

$$+\frac{1}{4}\omega(\delta\chi p)^2\sin^2\vartheta\cos^2\vartheta\sin^4\varphi[8-(\chi/\omega)(1-\sin^2\vartheta\sin^2\varphi)]^2+$$

$$+(\delta^2\chi^4p^2/4\omega)\sin^2\vartheta\sin^2\varphi\cos^2\varphi(1-\sin^2\vartheta\sin^2\varphi)^2. \tag{A.21}$$

χ is given by (18b) and this reduces to $\chi = 2\pi M/(H+4\pi N_xM)$ for $N_x = N_y$. Other symbols used in (A.21) are defined as follows

$$\omega = 1+\frac{1}{2}\frac{\chi^2(1-\sin^2\vartheta\sin^2\varphi)}{1+[1\chi^2(1+-\sin^2\vartheta\sin^2\varphi)^2]^{1/2}}, \tag{A.22}$$

$$p = 1-\chi(1-\sin^2\vartheta\sin^2\varphi)\,[1-(2/\chi)\,(\omega-1)], \tag{A.23}$$

$$\sigma = \frac{1-(N\mu_B/2MV)}{1-(N\mu_B/MV)}, \qquad \delta = B_2/6B_1. \tag{A.24}$$

The result (A.21) is valid for arbitrary strength of the magnetic field. In the limit $2\pi M \ll H$ or $\chi \ll 1$, the function $S(\vartheta, \varphi)$ reduces to $\sin^2\vartheta\cos^2\vartheta\cos^2\varphi$, and the former result (16) is obtained again.

REFERENCES

[1] M. Sparks, *Ferromagnetic-relaxation Theory*, McGraw-Hill, New York 1964.
[2] F. Keffer, *Spin Wawes* in *Encyclopedia of Physics*, edited by S. Flügge (Springer Verlag), vol. XVIII/2, 1966.
[3] D. S. Rodbell, *J. Appl. Phys.*, **30,** 187 *S* (1959).
[4] D. S. Rodbell, *Phys. Rev. Letters*, **13,** 471 (1964).
[5] D. S. Rodbell, *Physics*, **1,** 279 (1965).
[6] Z. Frait, *Czech. J. Phys.*, *B***10,** 546 (1960).
[7] A. F. Kip and R. D. Arnold, *Phys. Rev.*, **75,** 1556 (1949).
[8] W. S. Ament and G. T. Rado, *Phys. Rev.*, **97,** 1558 (1959).
[9] K. H. Reich, *Phys. Rev.*, **101,** 1647 (1956); S. M. Bhagat and L. L. Hirst, *Phys. Rev.*, **151,** 401 (1966).
[10] J. Morkowski, *Physics Letters*, **26***A*, 144 (1968).
[11] C. Herring and C. Kittel, *Phys. Rev.*, **81,** 869 (1951).
[12] T. Holstein and H. Primakoff, *Phys. Rev.*, **58,** 1098 (1940).
[13] M. Sparks, R. Loudon and C. Kittel, *Phys. Rev.*, **122,** 791 (1961).
[14] E. Schlömann, *Phys. Rev.*, **121,** 1312, (1961).
[15] E. Abrahams and C. Kittel, *Phys. Rev.*, **88,** 1200 (1952).
[16] C. Kittel, *Rev. Mod. Phys.*, **21,** 541 (1949).
[17] H. B. Callen and E. Pittelli, *Phys. Rev.*, **119,** 1523 (1960).
[18] J. Friedel, *Dislocations*, Pergamon Press, Oxford 1964.
[19] F. R. N. Nabarro, *Theory of Crystal Dislocations*, Oxford University Press, 1967.
[20] B. E. Argyle, S. H. Charap and E. W. Pugh, *Phys. Rev.*, **132,** 2051 (1963).
[21] V. G. Baryakhtar, M. A. Savchenko and V. V. Tarasenko, *Zh. Eksper. Teor. Fiz.*, **54,** 1603 (1968).

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 4