HIGHER HARMONICS GENERATED BY FREE CARRIERS IN SEMICONDUCTORS

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The third harmonic in the electric current density generated by free carriers in semiconductors is derived and numerical calculations are performed. The effects discussed in this paper are associated with the energy dependence of the relaxation time for a parabolic band and with the nonparabolicity of the band. It is shown that a large enhancement of nonlinear effects due to the energy dependence of the relaxation time at high magnetic fields can be observed.

1. *Introduction*

The generation of higher harmonics by free carriers in semiconductors have been the subject of a number of recent papers. In an earlier paper by Lax *et al.* [2] the nonparabolicity of the energy band was found to be an important source of nonlinear excitation of electrons in solids. This has been observed experimentally by Patel *et al.* [3] and explained theoretically by Wollf and Pearson [4].

Theoretical calculations [2], [4] and [5] of the higher harmonics generation in solids have taken into account only the effect of nonparabolicity of the band, with the relaxation time assumed to be a constant phenomenological parameter [4] or even neglected entirely [5]. All these theoretical calculations have been carried out starting from the simple classical equation of motion.

In the paper published by Kołodziejczak [1] it was shown that there are two most important sources for the higher harmonics generation by free carriers in semiconductors. The first one is the energy dependence of the relaxation time, and the second is the nonparabolicity of the band. This theory starts from the nonlinear Boltzmann equation and allows

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us to calculate the magneto-conductivity tensor in the presence of a strong electric a.c. field. It was shown that the effect of free carriers on the generation of higher harmonics is associated with nonlinearities in the Boltzmann equation.

In this paper the third harmonic in the current density, associated with the energy depenedence of the relaxation time and with the nonparabolicity of the band in the presence of a strong magnetic field, is derived and numerical claculations are performed.

The final results can be easily simplified for spherical energy surfaces. The results obtained in this paper have been applied in an analysis of the third harmonic generated by free carriers.

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2. Magneto-conductivity tensor of the third harmonic

In an earlier paper [1] a theory describing the nonlinear excitation of electrons in semiconductors was proposed. This theory starts from the nonlinear Boltzmann equation and gives the electron current density in the presence of a strong a.c. electric field. For this electric field, $\boldsymbol{E} = \boldsymbol{E}^{(1)}e^{i\omega t}$ the following expression for the electric current density corresponding to the third harmonic was obtained

$$J_{\varrho}^{(3)} = \sigma_{\varrho\mu}^{(3)} E_{\mu}^{(1)} e^{3i\omega t} \tag{1}$$

where $\sigma_{\varrho\mu}^{(3)}$ is the magneto-conductivity tensor. The third harmonic described by the tensor $\sigma_{\varrho\mu}^{(3)}$ is associated with two independent reasons for nonlinear excitation of electrons: 1) nonparabolicity of the band 2) energy dependence of the relaxation time.

It was shown [6] that the tensor $\sigma_{\varrho\mu}^{(3)}$ is given by

$$\sigma_{\varrho\mu}^{(3)} = -i\,\frac{q^2 N E_{\alpha}^{(1)} E_{\beta}^{(1)}}{3\omega}\,\frac{\left\langle G_{\varrho\mu}^{(3)} \dfrac{d}{d\varepsilon}\,\tilde{F}\{f_0' G_{\alpha\beta}^{(1)}\}\right\rangle}{\langle f_0' \rangle} \tag{2}$$

where f_0 is the distribution function in the absence of an external field, N is the carrier concentration, ω is frequency of the strong electric a.c. field. The tensor $G_{\varrho\mu}^{(n+1)}$ in equation (2) is given by the formula

$$G_{\varrho\mu}^{(n+1)} = \frac{1}{\Delta_{n+1}}\left\{q m_{\varrho\mu}^{-1}\tau\Omega_{n+1} - \frac{q^2\tau^2}{c}\,\varepsilon_{\alpha\beta\gamma}H_{\alpha}m_{\varrho\gamma}^{-1}m_{\beta\mu}^{-1} + \frac{q^3\tau^3\,|m^{-1}|}{c^2}\,\frac{H_{\varrho}H_{\mu}}{\Omega_{n+1}}\right\} \tag{3}$$

where

$$\Delta_{n+1} = \Omega_{n+1}^2 + \omega_c^2\tau^2 \tag{4}$$

$$\Omega_{n+1} = [1+(n+1)i\omega\tau] \tag{5}$$

ω_c is the cyclotron frequency, $|m^{-1}|$ is the determinant of the momentum effective mass tensor $m_{\varrho\mu}^{-1}$ defined by:

$$m_{\varrho\mu}^{-1} = m_{\varrho\mu}^{-1}(0)\left(\frac{d\gamma}{d\varepsilon}\right)^{-1}. \tag{6}$$

The quantity $m_{\varrho\mu}^{-1}(0)$ is interpreted as a reciprocal effective mass tensor for the energy corresponding to the bottom of the energy band. The function $\gamma(\varepsilon)$ describes the energy band, according to the formula

$$\gamma(\varepsilon) = \frac{\hbar^2}{2m_0} \mathcal{A}_{\alpha\beta} k_\alpha k_\beta \tag{7}$$

where $\mathcal{A}_{\alpha\beta}$ is some tensor with constant components and m_0 is the free electron mass. Equation (7) describes the nonparabolic band of ellipsoidal symmetry. For $\gamma(\varepsilon) = \varepsilon$ the band becomes parabolic, and for $\mathcal{A}_{\alpha\beta} = a\delta_{\alpha\beta}$ the energy surfaces become spheres. If the energy surfaces have spherical symmetry we obtain

$$m_{\varrho\mu}^{-1}(0) = \frac{1}{m_0^*} \delta_{\varrho\mu}. \tag{8}$$

The momentum relaxation time is a function of the energy

$$\tau(\varepsilon) = \tau_r \varepsilon^{\frac{r}{2}} \left(\frac{d\gamma}{d\varepsilon}\right)^{-1}, \qquad \tau_r = \text{const.} \tag{9}$$

The index r describes the type of the scattering mechanism.

The expression in the bracket of Eq. (2)

$$I = \left\langle G_{\varrho\mu}^{(3)} \frac{d}{d\varepsilon} \tilde{F}\{f_0' G_{\varrho\beta}^{(1)}\} \right\rangle \tag{10}$$

can be rewritten in the form [6]

$$I = \left\{ \gamma^{5/2} \left(\frac{d\gamma}{d\varepsilon}\right)^{-1} \left[\frac{d}{d\varepsilon} (f_0' G_{\alpha\beta}^{(1)}) G_{\varrho\mu}^{(3)} - f_0' G_{\alpha\beta}^{(1)} \frac{d}{d\varepsilon} (G_{\varrho\mu}^{(3)}) \right] \right\}_0^\infty + \tag{11}$$
$$+ \int_0^\infty f_0' G_{\alpha\beta}^{(1)} \frac{d}{d\varepsilon} \left[\gamma^{5/2} \left(\frac{d\gamma}{d\varepsilon}\right)^{-1} \frac{d\varepsilon}{d} (G_{\varrho\mu}^{(3)}) \right] d\varepsilon$$

where the prime denotes derivation over energy, and $\gamma(\varepsilon)$ is a function of energy as described by Eq. (7). The symbol $\langle A \rangle$ is defined as

$$\langle A \rangle = \int_0^\infty \gamma^{3/2} A d\varepsilon. \tag{12}$$

Now we are able to calculate the electric current density corresponding to the harmonic at a high magnetic field associated with the energy dependende of the relaxation time and nonparabolicity of the energy bands.

3. *Effect of the energy dependence of the relaxation time*

For the parabolic bands $\gamma = \varepsilon$ the components of the momentum effective mass tensor are independent of energy and from (6) we have $m_{\varrho\mu}^{-1} = m_{\varrho\mu}^{-1}(0)$. The momentum relaxation time in this case is expressed in the following form:

$$\tau(\varepsilon) = \tau_r \varepsilon^{r/2}. \tag{13}$$

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By neglecting the effect of nonparabolicity of the band, the pure effect associated with the energy dependence of the relaxation time can be calculated. In the case of a parabolic band and for strong degeneracy of the electron gas, the relaxation time corresponding to the energy of the Fermi level is equal to $\tau(\zeta) = \tau_r \zeta^{r/2}$ according to Eq. (13) and

$$f_0' = -\delta(\varepsilon-\zeta). \tag{14}$$

According to equation (12) we have

$$\langle f_0' \rangle = -\zeta^{3/2}. \tag{15}$$

The equation (2) can be simplifed since the relation holds

$$E_\alpha^{(1)} E_\beta^{(1)} G_{\alpha\beta}^{(1)} = E_\alpha^{(1)} E_\beta^{(1)} \operatorname{sym} G_{\alpha\beta}^{(1)} \tag{16}$$

where sym $G_{\alpha\beta}^{(1)}$ denotes the symmetric part of the tensor $G_{\alpha\beta}^{(1)}$. According to (3) we have

$$\operatorname{sym} G_{\alpha\beta}^{(1)} = \frac{1}{\Delta_1}\left\{q m_{\alpha\beta}^{-1}(0)\tau\Omega_1 + \frac{q^3\tau^3|m^{-1}|}{c^2\Omega_1} H_\alpha H_\beta\right\}. \tag{17}$$

The problem can be simplified if we assume transverse polarization of the incident radiation

$$\boldsymbol{E}^{(1)} \perp \boldsymbol{H}. \tag{18}$$

For this case we have $E_\alpha^{(1)} H_\alpha = 0$. The integral (11) can be simplified and in result we obtain the following expression for the magneto-conductivity tensor of the third harmonic

$$\sigma_{\varrho\mu}^{(3)} = -i\,\frac{q^4 N E_\alpha^{(1)} E_\beta^{(1)} m_{\alpha\beta}^{-1}(0) r\tau^2}{6\omega\zeta}\,\frac{\Omega_1}{\Delta_1}\,\frac{1}{\Delta_3^2}\times$$

$$\times\left\{m_{\varrho\mu}^{-1}(0)\left[\left(\frac{3+r}{2}\right)[\Omega_3^2-\omega_c^2\tau^2]+r[3i\omega\tau\Omega_3-\omega_c^2\tau^2]-\frac{2r[3i\omega\tau\Omega_3+\omega_c^2\tau^2][\Omega_3^2-\omega_c^2\tau^2]}{\Delta_3}\right]-\right.$$

$$\left.-\frac{2q\tau}{c}\left[\left(r+\frac{3}{2}\right)\Omega_3+\frac{3r}{2}i\omega\tau-\frac{2r\Omega_3[3i\omega\tau\Omega_3+\omega_c^2\tau^2]}{\Delta_3}\right]\times\varepsilon_{\alpha\beta\gamma}H_\alpha m_{\varrho\gamma}^{-1}(0) m_{\beta\mu}^{-1}(0)\right\}. \tag{19}$$

Making use of Eqs (1) and (19) we obtain for a high magnetic field

$$J_\varrho^{(3)} = -i\,\frac{q^4 N E_\alpha^{(1)} E_\beta^{(1)} m_{\alpha\beta}^{-1}(0) r\tau^2}{6\omega\zeta}\,E_\mu^{(1)} e^{3i\omega\tau}\times$$

$$\times\frac{[1+i\omega\tau][1+(\omega_c^2-\omega^2)\tau^2-2i\omega\tau][1+(\omega_c^2-9\omega^2)\tau^2-6i\omega\tau]^2}{[1+(\omega_c^2-\omega^2)^2\tau^4+2(\omega_c^2+\omega^2)\tau^2][1+(\omega_c^2-9\omega^2)^2\tau^4+2(\omega_c^2+9\omega^2)\tau^2]^2}\times$$

$$\times\left\{m_{\varrho\mu}^{-1}(0)\left[\left(\frac{3+r}{2}\right)-\frac{3+2r}{2}[\omega_c^2+9\omega^2]\tau^2+(3+2r)3i\omega\tau-\right.\right.$$

$$-\left.\frac{2r[3i\omega\tau+(\omega_c^2-9\omega)^2\tau^2][\Omega_3^2-\omega_c^2\tau^2]}{\Delta_3}\right]-\frac{2q\tau}{c}\left[\left(\frac{3+2r}{2}\right)+\left(\frac{3+3r}{2}\right)3i\omega\tau-\right.$$

$$\left.\left.-\frac{2r\Omega_3[3i\omega\tau+(\omega_c^2-9\omega^2)\tau^2]}{\Delta_3}\right]\varepsilon_{\alpha\beta\gamma}H_\alpha m_{\varrho\gamma}^{-1}(0)\, m_{\beta\mu}^{-1}(0)\right\} \tag{20}$$

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where

$$\omega_c \equiv \omega_c(0) \tag{21}$$

$\omega_c(0)$ is the cyclotron frequency for an energy corresponding to the bottom of the energy band. Magnetic resonance can be observed for $\omega_c = \omega$ and $\omega_c = 3\omega$. The nonlinear current given by equation (20) arises from the energy dependence of the relaxation time for parabolic bands. The results can be easily simplified for spherical energy surfaces. In the case of the spherical energy surfaces the tensor $\sigma^{(3)}_{\varrho\mu}$ is proportional to $[E^{(1)}]^2$, because $m^{-1}_{\alpha\beta}(0) = \frac{1}{m_0^*}\delta_{\alpha\beta}$ and $m^{-1}_{\varrho\mu}(0) = \frac{1}{m_0^*}\delta_{\varrho\mu}$. If the magnetic vector $\boldsymbol{H}$ is assumed to be parallel to the z axis of the coordinate system, $\boldsymbol{H}(0, 0, H)$, the magneto-conductivity tensor of the third harmonic is given by

$$\hat{\sigma}(^3) = \begin{bmatrix} \sigma^{(3)}_{11} & \sigma^{(3)}_{12} & 0 \\ \sigma^{(3)}_{21} & \sigma^{(3)}_{22} & 0 \\ 0 & 0 & \sigma^{(3)}_{33} \end{bmatrix} \tag{22}$$

where

$$\sigma^{(3)}_{11} = \sigma^{(3)}_{22} = \sigma^{(3)}_{33} \tag{23}$$

and

$$\sigma^{(3)}_{12} = -\sigma^{(3)}_{21}. \tag{24}$$

The components of the tensor $\sigma^{(3)}_{\varrho\mu}$ for the spherical energy surfaces and for $\boldsymbol{H}(0, 0, H)$ are described by

$$\sigma^{(3)}_{\varrho\mu} = -i\,\frac{q^4N(E^{(1)})^2r\tau^2}{6\zeta\omega(m_0^*)^2}\,\frac{\Omega_1}{\Delta_1(\Delta_3)^2}\times$$
$$\times\left\{\delta_{\varrho\mu}\left[\left(\frac{3+r}{2}\right)[\Omega_3^2-\omega_c^2\tau^2]+r[3i\omega\tau\Omega_3-\omega_c^2\tau^2]-\frac{2r[3i\omega\tau\Omega_3+\omega_c^2\tau^2][\Omega_3^2-\omega_c^2\tau^2]}{\Delta_3}\right]-\right.$$
$$\left.-2\,\frac{q\tau}{m_0^*c}\,\varepsilon_{\alpha\mu\varrho}H_\alpha\left[\left(\frac{2}{3+2r}\right)\Omega_3+\frac{3ri\omega\tau}{2}-\frac{2r\Omega_3[3i\omega\tau\Omega_3+\omega_c^2\tau^2]}{\Delta_3}\right]\right\}. \tag{25}$$

Now we shall analyse the real and imaginary part of the equation (25). After some algebraic calculations we can find the expressions for the diagonal and nondiagonal components of the tensor $\sigma^{(3)}_{\varrho\mu}$ given by Eq. (25).

The theoretical results of the equation describing the real part of Eq. (25) for the diagonal components are plotted in Fig. 1 and Fig. 2, in the form of the ratio of Re $[\sigma^{(3)}_{11}]$ to Re $[\sigma^{(3)}_{11}(0)]$, as a function of the magnetic field through the values of $x = \omega_c/\omega$, with $y = \omega\tau$ as a parameter. Here, Re $[\sigma^{(3)}_{11}(0)]$ is the real part of the diagonal components of the tensor $\sigma^{(3)}_{\varrho\mu}$ at zero magnetic field $\boldsymbol{H} = 0$. The magnetic resonance of the third harmonic associated with the energy dependence of the relaxation time $\tau = \tau(\varepsilon)$ depends on the scattering mechanism through the index r.

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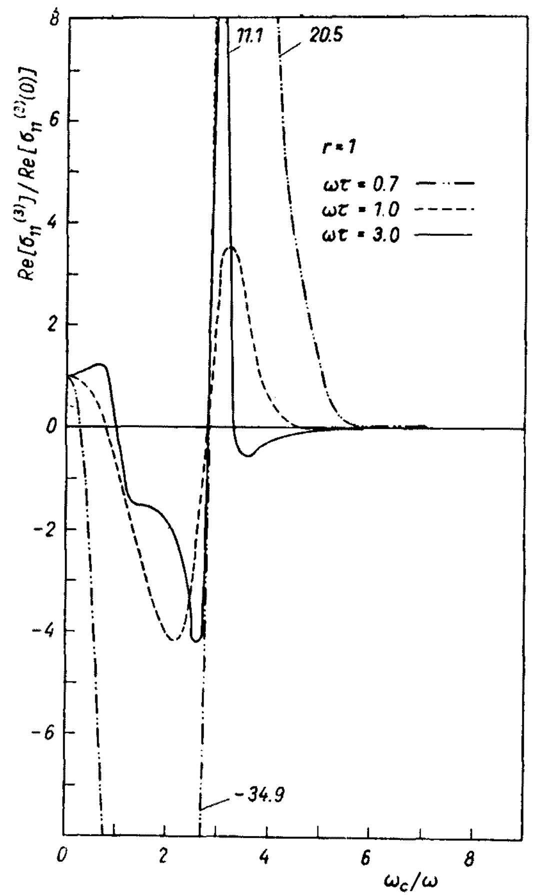

Fig. 1

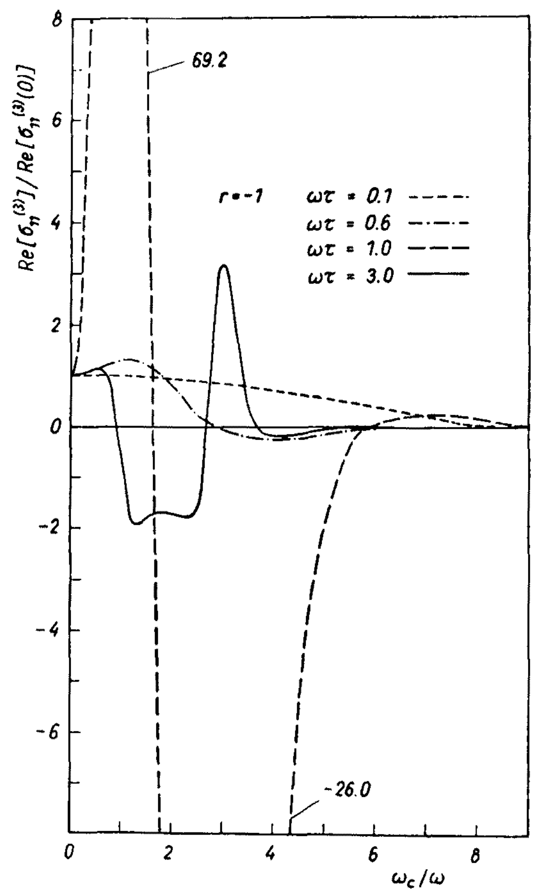

Fig. 2

Fig. 1. Theoretical curves showing the real part of Eq. (25) for the diagonal components, as a function of the magnetic field ω_c/ω, for different values of $\omega\tau$ and scattering by optical modes in polar crystals, $r = 1$

Fig. 2. Theoretical curves showing the real part of Eq. (25) for the diagonal components, as a function of the magnetic fields ω_c/ω, for different values of $\omega\tau$ and scattering by acoustic modes, $r = -1$

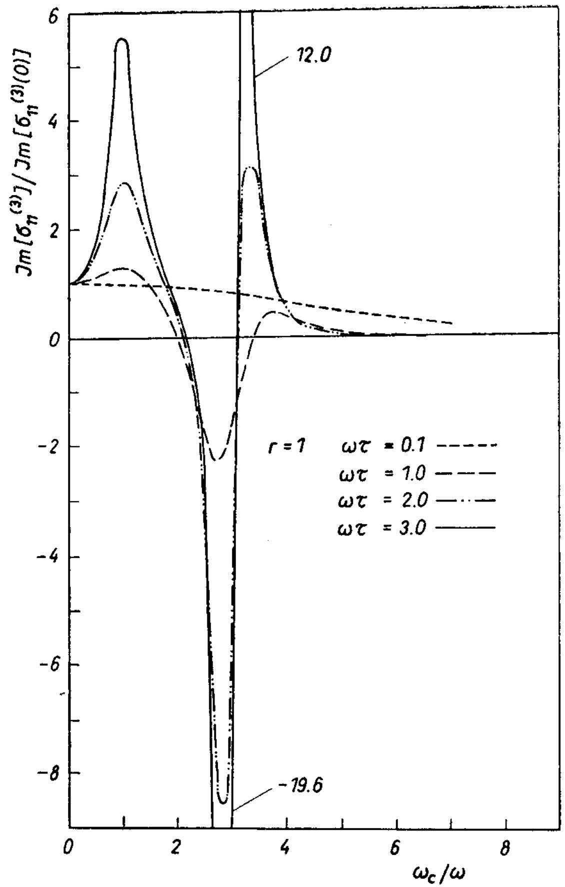

Fig. 3

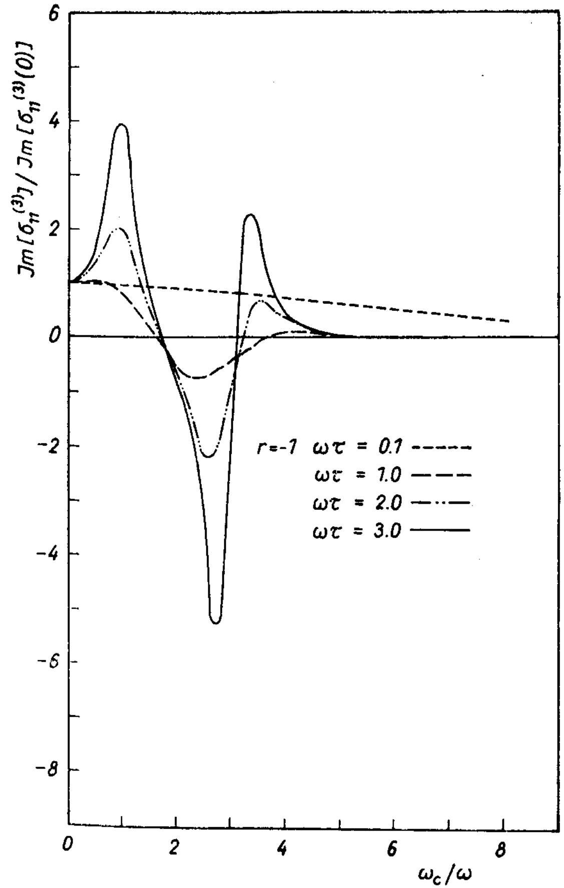

Fig. 4

Fig. 3. Theoretical curves showing the imaginary parts of Eq. (25) for the diagonal components as a fuction of the magnetic field ω_c/ω, for different values of $\omega\tau$ and scattering by optical modes in polar crystals $r = 1$

Fig. 4. Theoretical curves showing the imaginary parts of Eq. (25) for the diagonal components as a function of the magnetic field ω_c/ω, for different values of $\omega\tau$ and scattering by acoustic modes $r = -1$

In calculating the results plotted in Fig. 1 we assumed $r = 1$, *i.e.* scattering by optical modes in polar crystals. In Fig. 2 the assumption is $r = -1$, *i.e.* acoustic mode scattering and optical mode scattering in nonpolar crystals. It is seen that the resonance in quite well defined for $\omega\tau = 0.7$ in Fig. 1 and $\omega\tau = 1$ in Fig. 2.

The results plotted in Fig. 1 and Fig. 2 lead us to the conclussion that the effect of the scattering mechanism is important in this problem. The results of the above calculations supply us with information on the scattering mechanism in solids.

The numerical results of the imaginary parts of Eq. (25) are shown in Fig. 3 and Fig. 4. All results show that the large enhacement due to magnetic resonance in the third harmonic can be observed in the case when the relaxation time is energy dependent. Similar calculations can be made for the nondiagonal components of the tensor $\sigma^{(3)}_{\varrho\mu}$.

4. *Effect of nonparabolicity of the band*

For the nonparabolicity of the band of Kane's type the momentum effective mass tensor depends on energy and is given by

$$m^{-1}_{\varrho\mu} = m^{-1}_{\varrho\mu}(0)\left[1+2\,\frac{\varepsilon}{\varepsilon_g}\right]^{-1} \tag{26}$$

where ε_g is the energy gap. Let us now make the assumption that the momentum relaxation time is independent of energy, *i.e.* $\tau = \text{const}$. Neglecting the effect associated with the energy dependence of the relaxation time, we can analyse the pure effect of the non-parabolicity of the band by including the relaxation time as a phenomenological constant.

It was shown in Ref. [6] that the tensor $\sigma^{(3)}_{\varrho\mu}$ for an energy gap large enough to satisfy the inequalities $k\tau/\varepsilon_g \ll 1$ or $\zeta/\varepsilon_g \ll 1$ (ζ is the Fermi level) can be rewritten in the final form

$$\sigma^{(3)}_{\varrho\mu} = -\,i\,\frac{5q^2NE^{(1)}_\alpha E^{(1)}_\beta}{6\omega}\,G^{(1)}_{\alpha\beta}(0)\left[\frac{d}{d\varepsilon}\,G^{(3)}_{\varrho\mu}\right]_{\varepsilon=0}. \tag{27}$$

Using relation (3) and

$$\omega^2_c = \frac{q^2}{c^2}\,H_\alpha H_\gamma m_{\gamma\alpha}\det m^{-1}_{\mu\nu} \tag{28}$$

we obtain

$$\sigma^{(3)}_{\varrho\mu} = i\,\frac{5q^4NE^{(1)}_\alpha E^{(1)}_\beta[1+(\omega^2_c(0)+\omega^2)\tau^2+i\omega\tau[(\omega^2_c(0)-\omega^2)\tau^2-1]]}{3\omega\varepsilon_g[1+(\omega^2_c(0)-\omega^2)\tau^2+2(\omega^2_c(0)+\omega^2)\tau^2]}\times$$

$$\times\,\frac{[1+(\omega^2_c(0)-9\omega^2)^2\tau^4+2(\omega^2_c(0)-27\omega^2)\tau^2]-12i\omega\tau[1+(\omega^2_c(0)-9\omega^2)\tau^2]}{[1+(\omega^2_c(0)-9\omega^2)^2\tau^4+2(\omega^2_c(0)+9\omega^2)\tau^2]^2}\times$$

$$\times\, m^{-1}_{\alpha\beta}(0)\Big\{m^{-1}_{\varrho\mu}(0)[(1-[\omega^2_c(0)+27\omega^2]\tau^2)+3i\omega\tau[3-(\omega^2_c(0)+9\omega^2)\tau^2]]-$$

$$-\,2\,\frac{q\tau}{c}\,[1+3i\omega\tau]^2\varepsilon_{\alpha\beta\gamma}H_\alpha m^{-1}_{\varrho\gamma}(0)m^{-1}_{\beta\mu}(0)\Big\}. \tag{29}$$

It is seen that Eq. (29) does not depend on statistics and thus it is always satisfied independently of the degree of degeneracy of the electron gas.

In the case of the spherical energy surfaces and $\boldsymbol{H}(0, 0, H)$ the tensor $\sigma^{(3)}_{\varrho\mu}$ is also given by Eq. (22). The components of the tensor $\sigma^{(3)}_{\varrho\mu}$ for $\tau = \text{const}$ in this case are described by

$$\sigma^{(3)}_{\varrho\mu} = i\, \frac{5q^4N[E^{(1)}]^2\tau^2}{3(m_0^*)^2\varepsilon_g\omega}\, \frac{\Omega_1\Omega_3}{\Delta_1(\Delta_3)^2} \times \left\{\delta_{\varrho\mu}[\Omega_3^2-\omega_c^2\tau^2]-2\, \frac{q\tau}{m_0^*c}\, \Omega_3\varepsilon_{\alpha\mu\varrho}H_\alpha\right\}. \tag{30}$$

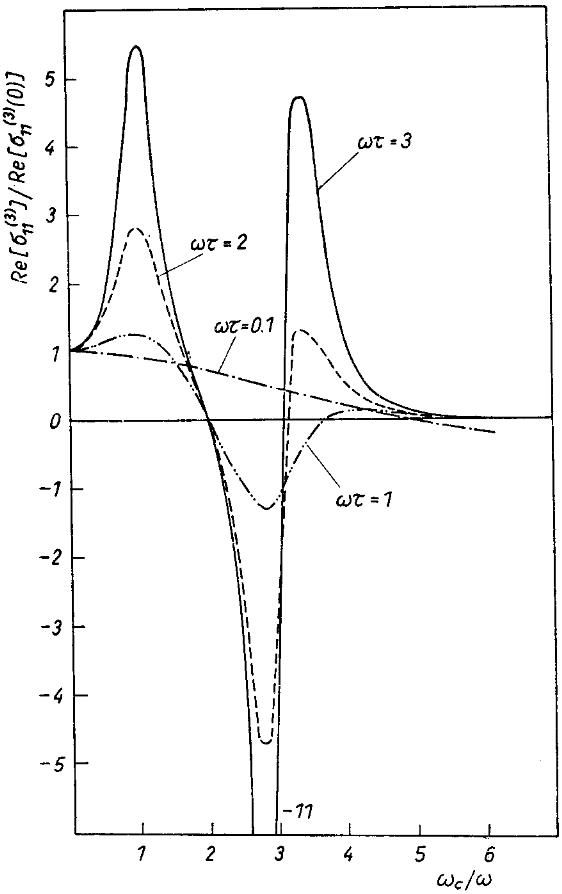

Fig. 5. Plot of the line shapes for the real part of Eq. (30) and diagonal components as a function of the magnetic field ω_c/ω, for various relaxation times in $\omega\tau$ units

In Fig. 5 we have plotted the real part of the expression (30) for the diagonal components. The numerical values of the real part (30) for nondiagonal components are demonstrated in Fig. 6, with $y = \omega\tau$ as a parameter.

The results plotted in Fig. 5 show the magnetic resonance for $\omega_c = \omega$ and for $2\omega < \omega_c < 4\omega$. This non-linear current given by equation (1) and (30) arises from the

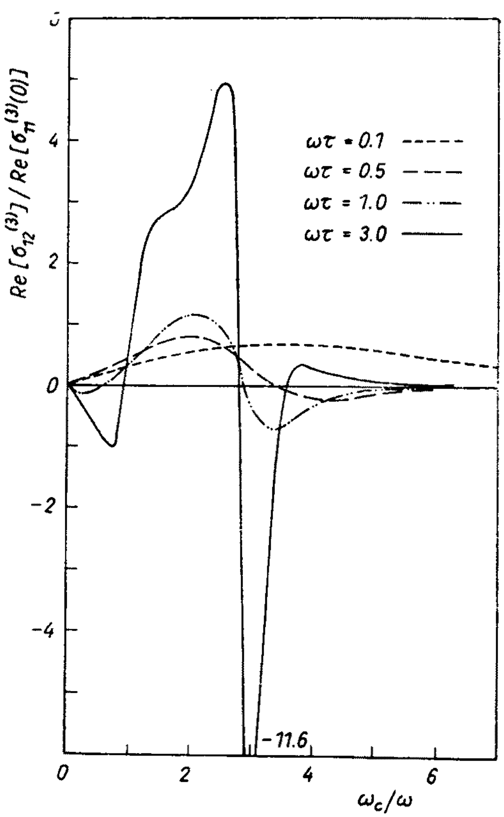

Fig. 6. Plot of the line shapes for the real part of Eq. (30) and nondiagonal components as a function of the magnetic field ω_c/ω, for various relaxation times in $\omega\tau$ units

nonparabolicity of the band, *i.e.* from the energy dependendce of the effective mass because $\tau = \text{const}$. The expression for the nondiagonal components of the tensor $\sigma^{(3)}_{\varrho\mu}$ (30) plotted in Fig. 6 has two singularities.

Similar calculations can be made for the imaginary parts of Eq. (30)

5. *Vanishing magnetic field*

The general expression for the magneto-conductivity tensor of the third harmonic associated with the energy dependence of the relaxation time is given by Eq. (19). In the case of the vanishing the magnetic field $\boldsymbol{H} = 0$ the tensor $\sigma^{(3)}_{\varrho\mu}$ can be rewritten into

$$\sigma^{(3)}_{\varrho\mu} = -i\,\frac{q^4 N E^{(1)}_\alpha E^{(1)}_\beta m^{-1}_{\alpha\beta} m^{(1)}_{\varrho\mu}(0) r\tau^2}{12\omega\zeta} \times$$

$$\times \frac{1}{\Omega_1(\Omega_3)^3}\,[(3+r)+3(3-r)i\omega\tau]. \qquad (31)$$

Assuming a high frequency ω, such that the condition $\omega\tau \gg 1$ is satisfied, we obtain

$$\sigma^{(3)}_{\varrho\mu} = \frac{q^4 N E^{(1)}_{\alpha} E^{(1)}_{\beta} m^{-1}_{\alpha\beta}(0) m^{-1}_{\varrho\mu}(0) r}{108\, \zeta^{\frac{r+2}{2}}\, \tau_r \omega^4} \left[(3-r) + i\,\frac{15-17r}{3\omega\tau}\right]. \tag{32}$$

When calculating the above expression the terms proportional to $1/\omega^6$ and $1/\omega^7$ were neglected. In the case of spherical energy surfaces the tensor $\sigma^{(3)}_{\varrho\mu}$ becomes a scalar, for the effective mass $m^{-1}_{\varrho\mu}(0)$ is described by Eq. (8). The sign of the magneto-conductivity tensor depends on the scattering mechanism. The equation (32) corresponds to Eq. (3) given in Ref. [7], but is not exactly the same.

In the case when the band is not parabolic and the magnetic field vanishes $\boldsymbol{H} = 0$, the tensor $\sigma^{(3)}_{\varrho\mu}$ (30) can be rewritten in the form

$$\sigma^{(3)}_{\varrho\mu} = \frac{1}{9}\left(\frac{q}{\omega}\right)^4 N E^{(1)}_{\alpha} E^{(1)}_{\beta} m^{-1}_{\alpha\beta}(0) m^{-1}_{\varrho\mu}(0) \frac{5}{\varepsilon_g \tau}\left[\frac{4}{3} - i\omega\tau\right]. \tag{33}$$

This equation is derived for $\omega\tau \gg 1$ and corresponds to the equation given in Ref. [7].

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