INTERBAND FARADAY EFFECT AND ELLIPTICITY IN SILICON IN WEAK MAGNETIC FIELDS

By H. Stramska

Institute of Physics, Warsaw University, Warsaw*

and J. Kołodziejczak

Institute of Physics, Polish Academy of Sciences, Warsaw**

(Received November 5, 1968)

The results of measurements of the Faraday rotation and ellipticity in silicon are presented. These effects are associated with indirect interband transitions.

Comparison of the experimental results with theory gives rather qualitative than quantitative agreement.

The earliest measurements of the interband Faraday rotation in silicon associated with indirect transitions were reported in Refs [1], [2], [3].

Our previous paper [3] also presented the experimental results on interband ellipticity in Si at room temperature and in weak magnetic fields up to 25 kGs.

In the present paper the semiclassical theory by Kołodziejczak, Lax and Nishina (KLN) [4] and the quantum theory by Halpern, Lax and Nishina (HLN) [5] are used for interpreting the experimental results. We shall extend the semiclassical [4] and quantum [5] theories and then obtain corresponding expressions for the Faraday rotation and ellipticity associated with indirect transitions, including the damping effect. We shall compare the numerical calculations with the experimental results obtained for silicon.

The theoretical formula which corresponds to the nonoscillatory Faraday rotation associated with the indirect transitions for energies below the energy gap was obtained in Ref. [4] in the limit of small (γH), where (γH) is the splitting of the n-th state to the magnetic field H. The phenomenological factor γ is given by

$$\gamma = \frac{1}{2}\mu_B \frac{1}{\hbar}(g_v + g_c) = \frac{\mu_B}{2\hbar} g^* \tag{1}$$

where μ_B is the Bohr magneton and g^* is an effective g-factor.

---

* Address: Instytut Fizyki Uniwersytetu Warszawskiego, Warszawa, ul. Hoża 69, Polska.

** Address: Instytut Fizyki PAN, Warszawa, ul. Zielna 37, Polska.

In this treatment the expression for the Faraday rotation was obtained for the case when the damping effect was neglected by setting $\tau = \infty$, τ being the relaxation time. To obtain the line shape in the vicinity of the energy gap it is necessary to introduce a damping term with a phenomenological relaxation time τ by putting

$$\omega^2 \to \left(\omega^2 - i\,\frac{\omega}{\tau}\right). \tag{2}$$

Substituting Eq. (2) into Eq. (43a) in paper [4] and after some algebraic manipulation the Faraday rotation θ_i due to indirect transitions becomes

$$\theta_i = D(\gamma H)\left\{1 + \frac{\omega_g^2}{\omega^2}\ln\left[\frac{\omega_g^4\tau^4}{(\omega_g^2-\omega^2)^2\tau^4+\omega^2\tau^2}\right]^{1/2} + \right.$$
$$\left. + \frac{\omega_g}{\omega}\ln\left[\frac{\left[\omega_g - \sqrt{\frac{1}{2}\frac{\omega}{\tau}[\omega\tau+\sqrt{1+\omega^2\tau^2}]}\right]^2 + \frac{1}{2}\frac{\omega}{\tau}\frac{1}{\omega\tau+\sqrt{1+\omega^2\tau^2}}}{[\omega_g+\omega]^2}\right]^{1/2}\right\} \tag{3}$$

where ω is the optical frequency, ω_g is the frequency corresponding to the energy gap. The constant D is determined by the transition matrix element and various physical constants which are assumed to be invariant over the photon energy range of interest [3]. From semiclassical theory (KLN) [4] for the interband ellipticity Δ_i associated with indirect transitions we have

$$\Delta_i = D(\gamma H)\left\{- \frac{\omega_g^2}{\omega^2}\arcsin\left[\frac{\omega^2\tau^2}{(\omega_g^2-\omega^2)^2\tau^4+\omega^2\tau^2}\right]^{1/2} + \right.$$
$$\left. + \frac{\omega_g}{\omega}\arcsin\left[\frac{\frac{1}{2}\frac{\omega}{\tau}\frac{1}{\omega\tau+\sqrt{1+\omega^2\tau^2}}}{\left[\omega_g - \sqrt{\frac{1}{2}\frac{\omega}{\tau}[\omega\tau+\sqrt{1+\omega^2\tau^2}]}\right]^2 + \frac{1}{2}\frac{\omega}{\tau}\frac{1}{\omega\tau+\sqrt{1+\omega^2\tau^2}}}\right]^{1/2}\right\}. \tag{4}$$

The line shapes of the Faraday effect have been also studied in terms of the quantum mechanical formalism by Halpern *et al.* (HLN) [5].

From the quantum theory by (HLN) [5] the expression describing the indirect rotation due to the background in the limit of small (γH) (assuming that the magnetic field H is small) can be obtained

$$\theta_i = \frac{D(\gamma H)}{64}\left\{-32\times\ln[\omega\tau+0.597] + \frac{4(\omega_g'-\omega)\tau}{\omega\tau}\times\right.$$
$$\left.\times\left[\ln[(\omega_g'-\omega)^2\tau^2+1] + \frac{(\omega_g'-\omega)^2\tau^2}{1+(\omega_g'-\omega)^2\tau^2}\right]\right\} \tag{5}$$

where $\omega_g' = \omega_g + \omega_{\text{ph}}$, and ω_{ph} represents the photon involved in the indirect transition. A phenomenological relaxation time τ was introduced by letting $\omega \to (\omega - i/\tau)$ where singu-

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 6

larities existed. From this theory the expression for the interband ellipticity Δ_i in low magnetic fields can be obtained

$$\Delta_i = D(\gamma H) \frac{1}{32\omega\tau} \left\{ 4(\omega_g' - \omega)\tau \operatorname{arc\,tg} [(\omega_g' - \omega)\tau] - 2 \frac{(\omega_g' - \omega)^2\tau^2}{1 + (\omega_g' - \omega)^2\tau^2} \right\}. \quad (6)$$

Equations (3) and (5) describing the Faraday rotation in the limit of small H were used in the theoretical analysis and comparison with the experimental results for silicon. All phenomenological constants could be represented by appropriate average values. The re-

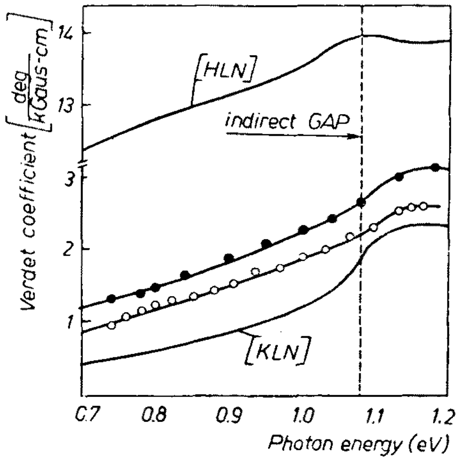

Fig. 1. Interband Faraday rotation in silicon plotted as a function of the photon energy

sults of the numerical analysis for both theories, (KLN) [4] and HLN [5], of the Faraday rotation are shown in Fig. 1. For comparison the experimental results are shown in the same figure.

It is seen from Fig. 1 that the numerical results of the Faraday rotation θ_i for the indirect transitions at energies below the energy gap apparently gives monotonically increassing positive rotation. This result is in qualitative agreement with the theory put forward by Lax and Nishina [6] that indirect transitions are resposible for the positive effects. For $\omega < \omega_g$ the expressions (3) and (5) are positive if the effective gyromagnetic constant γ describing the effective spectroscopic splitting factor $g^* = (g_v + g_c)$ is negative. From the numerical analysis and study of our experimental data (for 25 kGs and experimental value of the energy gap $\varepsilon_g = 1.08$ eV) it was found that the effective g-factor $g^* = -8$ and the relaxation time τ is approximately 10^{-13} sec at room temperature. It may be noticed that this value of the relaxation time τ is in agreement with the conclusions of Seitz [7], who found that the relaxation time for a typical conduction electron at room temperature is about 10^{-13}—10^{-14} sec.

The interband ellipticity Δ_i as a function of the photon energy in silicon for 25 kGs is shown in Fig. 2. The ellipticity (the amplitude ratio of the axes of the ellipse) is normalized to the magnetic field strength and sample thickness. Theoretical results are plotted in the

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 6

same figure. Eqs. (4) and (6) were in the numerical analysis with the above phenomenological parameters. The comparison with the experimental points indicates that the theory by Kołodziejczak, Lax and Nishina [4] is in qualitative agreement with the experimental data near the energy gap.

The numerical analysis of the expression (5) obtained on theoretical ground by (HLN) [5] for the ellipticity involving the indirect transition, gives the opposite sign of the effective

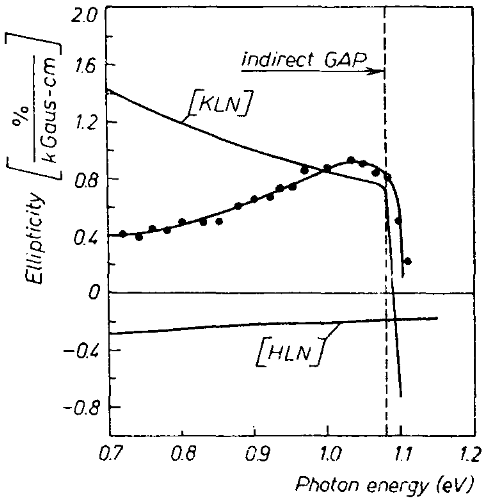

Fig. 2. Interband ellipticity in silicon plotted as a function of the photon energy

g-factor. Comparison of the experimental results with the theory for interband phenomena associated with the indirect transitions tend to support the conclusions that even the improved agreement obtained here between theory and experiment is still qualitative rather than quantitative.

REFERENCES

[1] H. Kimmel, *Z. Naturforsch.*, **12**, 1016 (1957).
[2] H. Piller and R. F. Potter, *Phys. Rev. Letters*, **9**, 203 (1962).
[3] H. Stramska, Z. Bachan, P. Byszewski and J. Kołodziejczak, *Phys. Status Solidi* **27**, K25 (1968).
[4] J. Kołodziejczak, B. Lax and Y. Nishina, *Phys. Rev.*, **128**, 2655 (1962).
[5] J. Halpern, B. Lax and Y. Nishina, *Phys. Rev.*, **134**, A 140 (1964).
[6] B. Lax and Y. Nishina, *Phys. Rev. Letters*, **6**, 464 (1961).
[7] F. Seitz, *The Modern Theory of Solids*, McGraw-Hill Book Company Inc., New York 1940.

ACTA PHYSICA POLONICA Vol. XXXV (1969) Fasc. 6