

ABSORPTION OF INFRARED RADIATION IN *n*-TYPE GERMANIUM UNDER HIGH [111] COMPRESSION IN A WEAK MAGNETIC FIELD

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The absorption of polarized electromagnetic waves at helium temperatures in the wavelength region from 5×10^{-2} cm to 2 cm, in *n*-type Ge subjected to high uniaxial [111] compression and in a magnetic field from 8 kgauss to 12 kgauss, is investigated theoretically on the basis of a model of direct absorption process model. A formula is derived for the magnetic field — dependent correction to the absorption coefficient calculated by Blinowski and the numerical results are presented for waves polarized with the electric vector parallel and perpendicular to the [111] direction of the crystal with magnetic field parallel to this direction. Concentrations of donors up to 6×10^{15} cm⁻³ and compensations up to 0.2 are considered. The range of validity of the results is discussed.

1. Introduction

The present paper is a continuation of the previous work [2–5] dealing with the absorption of the electromagnetic radiation in *n*-Ge and *n*-Si at helium temperatures. The work was based on the model of the direct absorption process proposed by Tanaka and Fan [1]. According to this model, the absorption of electromagnetic radiation is due to photon-induced transitions between the electronic states in the singly ionized two-donor pairs in the presence of an electrostatic field of ionized acceptors. Because the donor wave functions in Si and Ge are quite complicated, in the papers mentioned above several approximations were introduced. In papers [2], [3], [5] rather exact wave functions were applied, but in view of difficulties encountered in the integrations, the angularly averaged values of the overlap integrals and resonance energies were used in the formula for the transition rate. Lately Milward and Neuringer [6] measured the absorption coefficient of *n*-Si and their results confirm the theory derived in papers [2], [3]. In the paper [4] and in our work such approximations were not used, because donor wave functions in the case of *n*-Ge subjected to the high uniaxial [111] compression are relatively simple and the integration over directions can be performed in the final formulae.

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The purpose of the present work is to study the direct absorption by hopping in uniaxially stressed n -Ge in the presence of an external weak magnetic field at temperatures near to 0°K . In papers [2], [3] the electron-photon interaction was assumed to be weak. Only in such a case the whole formalism used in these papers was applicable. For this reason only pairs with $\Delta E > 2E_{LR}$ were taken into account. (E_{LR} is the relaxation energy of the lattice deformed by the interaction with an electron on an isolated donor, ΔE is the energy difference between the electronic states in the singly ionized two-donor pair). This led to the limitation of the wavelengths of radiation for which the theory given in papers [2], [3] was valid, because $\hbar\omega = \Delta E$, where ω is the angular frequency of the incident radiation. For this reason the calculations of the absorption coefficients in the papers [2–5], were confined to higher frequencies for which $\hbar\omega \gg E_{LR}$ with the electron-phonon interaction assumed weak. Here we also confine ourselves to these higher frequencies; we do it however for other reasons which will be given later. The limitation $\hbar\omega \gg E_{LR}$ stems from the adiabatic treatment of the electron-phonon interaction [7–8]. Takeyama [9] showed that the adiabatic treatment is not justified for pairs which are important in the microwave experiment (“microwave pairs”). Since it was shown in paper [3] that the effect of the electron-phonon interaction on the magnitude of the absorption coefficient is small in n -Ge, we can assume that at $T \rightarrow 0^\circ\text{K}$ the photon-induced hopping is the only possible mechanism of absorption; such mechanisms as lattice vibrations, absorption by the neutral impurity centres and free carries can be considered absent in these temperature and frequency regions.

2. Donor states in n -Ge

According to Nguyen Van Huong [5] the ground singlet state wave function of an electron on the shallow donor state in a nondeformed Ge crystal in the presence of an external weak magnetic field can be written as follows

$$u(\mathbf{r}, \mathbf{H}) = \frac{1}{2} \sum_{p=1}^4 F_p(\mathbf{r}', \mathbf{H}) \Phi_p(\mathbf{r}'). \quad (1)$$

$\Phi_p(\mathbf{r}')$ is the Bloch function corresponding to the p -th minimum in the lowest conduction band and $F_p(\mathbf{r}', \mathbf{H})$ is the envelope function satisfying the effective mass equation. This function can be approximately represented, up to terms quadratic in the magnetic field H , by the functional form

$$\begin{aligned} F_p(\mathbf{r}, \mathbf{H}) = & (\pi a^2 b)^{-1/2} \exp(-\zeta'_p) \left\{ 1 + \frac{11e^2 a^2 b^2}{24c^2 \hbar^2} \left(H_{x_p}^2 + H_{y_p}^2 + \frac{a^2}{b^2} H_{z_p}^2 \right) - \right. \\ & - \frac{e^2}{24c^2 \hbar^2} \left\{ H_{x_p}^2 \left[\left(\zeta'_p + \frac{3}{2} \right) (a^2 z_p'^2 + b^2 y_p'^2) + a^2 b^2 \zeta_p'^2 \right] + H_{y_p}^2 \left[\left(\zeta_p + \frac{3}{2} \right) \times \right. \right. \\ & \times (a^2 z_p'^2 + b^2 x_p'^2) + a^2 b^2 \zeta_p'^2 \left. \right] + a^2 H_{z_p}^2 \left[\left(\zeta'_p + \frac{3}{2} \right) (x_p'^2 + y_p'^2) + a^2 \zeta_p'^2 \right] - \\ & \left. \left. - 2 \left[\left(\zeta'_p + \frac{3}{2} \right) (a^2 H_{x_p}' H_{z_p}' x_p' z_p' + a^2 H_{y_p}' H_{z_p}' y_p' z_p' + b^2 H_{x_p}' H_{y_p}' x_p' y_p') \right] \right\} \right\}, \quad (1a) \end{aligned}$$

where ζ'_p is defined by

$$\zeta'_p = \left(\frac{x_p'^2 + y_p'^2}{a^2} + \frac{z_p'^2}{b^2} \right)^{1/2}, \quad p = 1, 2, 3, 4.$$

z'_p axis is chosen along the p -th valley axis.

The constants a and b are the transverse and longitudinal orbit radii of electron, respectively, and their values are found by using two relations given by Miller and Abrahams [10]:

$$m_l/m_t = a^2/b^2, \quad (2)$$

$$a = \hbar(2m_t E_{\text{obs}} E)^{-1/2}, \quad (3)$$

where m_l and m_t are the longitudinal and transverse effective masses, respectively, and E_{obs} is the observed ionization energy for the donor ground state.

According to Fritzsche [11] in the absence of external fields the ground singlet state wave function of an electron on the shallow donor in Ge subjected to high uniaxial [111] compression ($\geq 10^9$ dyne cm^{-2}) becomes relatively simple because only the states close to the [111] minimum in the conduction band contribute to this wave function. In the presence of a weak magnetic field we assume that the contributions to the wave function of the ground singlet state due to other minima are also negligible, if the uniaxial [111] compression reaches 10^9 dyne cm^{-2} . Thus we assume that this function can be approximately represented — in the high stress limit and in the presence of a weak magnetic field — by the functional form

$$u(\mathbf{r}, \mathbf{H}) = F_1(\mathbf{r}, \mathbf{H}) \Phi_1(\mathbf{r}). \quad (4)$$

\mathbf{r} is the position of a shallow electron in a deformed Ge crystal. The envelope function $F_1(\mathbf{r}, \mathbf{H})$ is given by the expression (1a) for $p = 1$ and $\mathbf{r}' = \mathbf{r}$. The coordinates x_1 , y_1 and z_1 are taken in the coordinate system with the z axis parallel to the [111] direction in a deformed Ge crystal. According to [11] in high uniaxial [111] stress limit E_{obs} for various donor impurities tends to about $E_0 = 9.2 \times 10^{-3}$ eV, therefore we assume that the radii a and b for all donor impurities can be approximately calculated from the formulae

$$\begin{aligned} a &= \hbar(2m_t E_0)^{-1/2}, \\ b &= \hbar(2m_l E_0)^{-1/2}. \end{aligned} \quad (5)$$

Using the values of the effective masses

$$m_l = (1.60 \pm 0.008) m_0$$

and

$$m_t = (0.0813 \pm 0.002) m_0$$

given by Fletcher, Yager and Merritt [12] we obtain

$$\begin{aligned} a &= 71.5 \text{ \AA}, \\ b &= 16.1 \text{ \AA}. \end{aligned} \quad (6)$$

3. *Electronic states in the two-center Hamiltonian*

In the localized pair model the direct absorption of the electromagnetic radiation is due to transitions of electrons between the eigenstates of the Hamiltonian

$$H = T(\mathbf{r}, \mathbf{H}) + V - e^2/\kappa r_a - e^2/\kappa r_b. \quad (7)$$

$T(\mathbf{r}, \mathbf{H})$ is the Hamiltonian of the electron in the pure but stressed crystal in a weak magnetic field, V is the potential mainly due to the negatively charged acceptor nearest to the given pair of donors a and b , κ is the static dielectric constant, \mathbf{r}_a and \mathbf{r}_b are

$$\mathbf{r}_a = \mathbf{r} - \mathbf{R}_a, \quad \mathbf{r}_b = \mathbf{r} - \mathbf{R}_b,$$

where \mathbf{R}_a and \mathbf{R}_b are the positions of donor ions.

Following Miller and Abrahams, we obtain the two lowest states of the Hamiltonian H using the variational procedure

$$\psi_a(\mathbf{r}, \mathbf{H}) = C_a^+ u(\mathbf{r}_a, \mathbf{H}) + C_b^+ u(\mathbf{r}_b, \mathbf{H}), \quad (8)$$

$$\psi_b(\mathbf{r}, \mathbf{H}) = C_a^- u(\mathbf{r}_a, \mathbf{H}) + C_b^- u(\mathbf{r}_b, \mathbf{H}), \quad (9)$$

the coefficients C_a^\pm and C_b^\pm are given by the formulae

$$\begin{aligned} C_a^\pm &= z^\pm [1 + (z^\pm)^2 + 2Sz^\pm]^{-1/2}, \\ C_b^\pm &= [1 + (z^\pm)^2 + 2Sz^\pm]^{-1/2}, \end{aligned} \quad (10)$$

where

$$z^\pm = (\Delta/2W) [1 \pm (1 + 4WS/\Delta + 4W^2/\Delta^2)^{1/2}].$$

We denote

$$W = W_R - \frac{1}{2} S\Delta + Z,$$

$$W_R = L - SJ,$$

$$L = -\langle u(\mathbf{r}_a, \mathbf{H}) | e^2/\kappa r_a | u(\mathbf{r}_b, \mathbf{H}) \rangle,$$

$$J = -\langle u(\mathbf{r}_a, \mathbf{H}) | e^2/\kappa r_a | u(\mathbf{r}_a, \mathbf{H}) \rangle,$$

$$S = \langle u(\mathbf{r}_a, \mathbf{H}) | u(\mathbf{r}_b, \mathbf{H}) \rangle,$$

$$Z = \langle u(\mathbf{r}_a, \mathbf{H}) | V | u(\mathbf{r}_b, \mathbf{H}) \rangle - \frac{1}{2} S [\langle u(\mathbf{r}_a, \mathbf{H}) | V | u(\mathbf{r}_a, \mathbf{H}) \rangle + \langle u(\mathbf{r}_b, \mathbf{H}) | V | u(\mathbf{r}_b, \mathbf{H}) \rangle],$$

$$\Delta = \langle u(\mathbf{r}_a, \mathbf{H}) | V | u(\mathbf{r}_a, \mathbf{H}) \rangle - \langle u(\mathbf{r}_b, \mathbf{H}) | V | u(\mathbf{r}_b, \mathbf{H}) \rangle. \quad (10a)$$

$u(\mathbf{r}_a, \mathbf{H})$ and $u(\mathbf{r}_b, \mathbf{H})$ are the ground state wave functions of the electron on the isolated donors a and b , respectively, in a Ge crystal under high [111] compression in a weak magnetic field. Mikoshiba and Gonda [13] showed that the magnetic field introduces a phase difference between the wave functions of neighbouring donors ("phase effect"). We fix the origin of

the vector potential at the donor nucleus a ; then the ground wave functions $u(\mathbf{r}_a, \mathbf{H})$ and $u(\mathbf{r}_b, \mathbf{H})$ are given by the expressions:

$$u(\mathbf{r}_a, \mathbf{H}) = F_1(\mathbf{r}_a, \mathbf{H}) \Phi_1(\mathbf{r}_a),$$

$$u(\mathbf{r}_b, \mathbf{H}) = F_1(\mathbf{r}_b, \mathbf{H}) \Phi_1(\mathbf{r}_b) \exp \left[-\frac{ie}{2c\hbar} (\mathbf{H} \times \mathbf{R}) \mathbf{r}_a \right]. \quad (10b)$$

W_R is the so-called resonance energy.

We assume that both donor ions have the same position in the elementary cells of the crystal. Following Miller and Abrahams, we assume that

$$J \approx -e^2/\kappa R. \quad (10c)$$

In the a magnetic field we expand the phase factor as

$$\exp \left[-\frac{ie}{2c\hbar} (\mathbf{H} \times \mathbf{R}) \mathbf{r}_a \right] \approx 1 - \frac{ie}{2c\hbar} (\mathbf{H} \times \mathbf{R}) \mathbf{r}_a - \frac{e^2}{8c^2\hbar^2} [(\mathbf{H} \times \mathbf{R}) \mathbf{r}_a]^2 +$$

$$+ \frac{ie^3}{48c^3\hbar^3} [(\mathbf{H} \times \mathbf{R}) \mathbf{r}_a]^3. \quad (10d)$$

Because of symmetry the terms proportional to H and H^3 vanish in the calculation of S and L (and, therefore, W_R). We also assume that in the region of the two donors a and b the field of the acceptor is homogeneous enough to enable us to neglect Z in (10a) and to obtain for Δ the following expression

$$\Delta = (e^2/\kappa) (1/R_a - 1/R_b). \quad (11)$$

We assume here that the potential V is the Coulomb potential of the ionized acceptor. Therefore we neglect the dipole potentials arising from the other ionized acceptor — ionized donor pairs present in the crystal. Because of the above assumptions the values of L , S , Z , J and Δ are real.

Without loss of generality we assume

$$\Delta \geq 0.$$

If $\Delta > 0$ the function (9) has a larger amplitude near the ion b and (8) near the ion a . The higher energy corresponds to the state (8). The energy difference between the two states is

$$\Delta E = (1 - S^2)^{-1} (\Delta^2 + 4W^2 + 4WS\Delta)^{1/2}.$$

Now we assume that $|S| \ll 1$. This assumption leads to

$$\Delta E \approx (\Delta^2 + 4W_R^2)^{1/2}. \quad (12)$$

We shall investigate now the photon-induced transitions between the states (9) and (8).

4. Transition rate

The formula for the transition rate W_{ab} given in the paper [2], applied to the states (8) and (9), neglecting the electron-phonon interaction, yields

$$W_{ab} = \frac{4\pi^2 e^2}{c\hbar^2 \kappa^{1/2}} W_R^2 (\Delta^2 + 4W_R^2)^{-1} (\mathbf{R} \cdot \mathbf{s})^2 \delta[\omega - (\Delta^2 + 4W_R^2)^{1/2}/\hbar]. \quad (13)$$

\mathbf{s} is the unit vector parallel to the electric vector of the radiation. In the derivation of this formula it was assumed that the temperatures are very low, *i. e.*, $kT \ll \hbar\omega$, and that the intensity of the radiation is low enough to avoid any serious change in the electron state occupation. The assumption $|S| \ll 1$ was also used. W_R^2 with an exactness to the term proportional to H^2 is

$$W_R^2(\mathbf{H}, R, \theta, \varphi) = W_R^2(\mathbf{H} = 0, R, \theta) + 2W_R(\mathbf{H} = 0, R, \theta) \delta W_R(\mathbf{H}, R, \theta, \varphi), \quad (14)$$

where

$$W_R(\mathbf{H} = 0, R, \theta) = -\frac{2e^2 R}{3\kappa a^2} (1 + \eta \cos^2 \theta)^{1/2} \exp \left[-\frac{R}{a} (1 + \eta \cos^2 \theta)^{1/2} \right],$$

$$\delta W_R(\mathbf{H}, R, \theta, \varphi) = \frac{e^4 R^4 H^2 (1 + \eta \cos^2 \theta)}{240\kappa a^3 c^2 \hbar^2} f(\theta, \varphi, \theta_H, \varphi_H) \exp \left[-\frac{R}{a} (1 + \eta \cos^2 \theta)^{1/2} \right],$$

$$f(\theta, \varphi, \theta_H, \varphi_H) = \frac{11}{3} (1 + \eta \cos^2 \theta) \{ b^2 \sin^2 \theta_H \sin^2 (\varphi_H - \varphi) +$$

$$+ a^2 [\sin \theta_H \cos \theta \cos (\varphi_H - \varphi) - \sin \theta \cos \theta_H]^2 \} +$$

$$+ 3a^2 \{ 1 - [\sin \theta \sin \theta_H \cos (\varphi_H - \varphi) + \cos \theta \cos \theta_H]^2 \}. \quad (14a)$$

θ, φ and θ_H, φ_H are the polar angles of the vectors \mathbf{R} and \mathbf{H} , respectively, taken in the coordinate system x_1, y_1, z_1 .

We denote

$$\eta = a^2/b^2 - 1.$$

Because of the anisotropy of the lowest conduction band ($a^2/b^2 \gg 1$) and in view of the assumption $2R/a \gg 1$, the exponential factor $\exp [(-2R/a)(1 + \eta \cos^2 \theta)^{1/2}]$ in the expression (13) rapidly vanishes as $\eta \cos^2 \theta$ increases. Therefore, the main contribution to the integral of W_{ab} over the angle θ (we shall perform this integration later computing the absorption coefficient) come from the pairs for which $\eta \cos^2 \theta \approx 0$. For this reason in computing W_R we have used in some places — following Miller and Abrahams — the approximation $R(1 + \eta \cos^2 \theta)^{1/2}/R = 1$.

The formula (13) can be rewritten in an approximate form:

$$W_{ab} = \frac{4\pi^2 e^2}{c\hbar^2 \kappa^{1/2}} \left\{ \frac{W_R^2(\mathbf{H} = 0, R, \theta)}{\Delta^2 + 4W_R^2(\mathbf{H} = 0, R, \theta)} + \frac{2W_R(\mathbf{H} = 0, R, \theta) \delta W_R(\mathbf{H}, R, \theta, \varphi) \Delta^2}{[\Delta^2 + 4W_R^2(\mathbf{H} = 0, R, \theta)]^2} \right\} +$$

$$\times (\mathbf{R} \cdot \mathbf{s})^2 \delta \left\{ \omega - \frac{[\Delta^2 + 4W_R^2(\mathbf{H} = 0, R, \theta) + 8W_R(\mathbf{H} = 0, R, \theta) \delta W_R(\mathbf{H}, R, \theta, \varphi)]^{1/2}}{\hbar} \right\}.$$

Because $W_{\mathbf{R}}(\mathbf{H} = 0) < 0$ and $\delta W_{\mathbf{R}}(\mathbf{H}) > 0$, the magnetic field decreases both the transition rate and the resonance frequency of the radiation absorbed by the direct absorption process. Therefore we can expect that the presence of the magnetic field causes the decrease of the absorption coefficient. The decrease of the transition rate is proportional to $\delta W_{\mathbf{R}}(\mathbf{H})$.

Considering the dependence of W_{ab} on the orientation of the vector \mathbf{H} , let us consider for a moment the donor pairs lying on the directions perpendicular and nearly perpendicular to the [111] direction ($\eta \cos^2 \theta \approx 0$). Then $\delta W_{\mathbf{R}}(\mathbf{H})$ is proportional to the following function of the orientations of the vectors \mathbf{H} and \mathbf{R} :

$$\frac{11}{3} \cos^2 \theta_H + 3 - 3 \sin^2 \theta_H \cos^2 (\varphi_H - \varphi). \quad (14b)$$

Here we have neglected the term proportional to $b^2/a^2 \ll 1$ because it is small as compared with the remaining terms. The function (14b) has a maximum at $\theta_H = 0$. Therefore, assuming that the essential contribution to the absorption is given by the pairs for which $\eta \cos^2 \theta \approx 0$, we may expect that the magnetic field diminishes the absorption coefficient most strongly for \mathbf{H} parallel to the [111] direction. If $\theta_H = \pi/2$ and the difference $\varphi_H - \varphi$ is equal to 0 or π the value of (14b) is zero. Thus the magnetic field has no influence on the contributions to the absorption arising from the donor pairs lying in the direction parallel to the vector \mathbf{H} .

5. Absorption coefficient

We shall investigate the case of small compensation, *i. e.*, when the ratio of the number of acceptors to donors present in the crystal is smaller than 0.2. At such compensation there are at least a few donors in the neighborhood of each acceptor. We shall treat the vicinity of each acceptor independently. Close to 0°K all acceptors are ionized and in every such vicinity only the donor nearest to the compensating acceptor is ionized, all other donors are neutral.

Bearing in mind the assumptions given previously and choosing the origin of the coordinate system x_1, y_1, z_1 at the ionized acceptor, we obtain the same starting formula for the absorption coefficient as in [2]

$$\alpha = \hbar \omega K N_d^3 \int dv_a \exp(-4\pi N_d R_a^3/3) \int dv_b W_{ab}, \quad R_b \geq R_a.$$

N_d is the concentration of donors. Introducing the function

$$\delta[\Delta - (e^2/\kappa)(1/R_a - 1/R_b)]$$

and integrating over Δ and two other variables we obtain

$$\begin{aligned} \alpha = & - \frac{8\pi^3 \kappa^{1/2} K N_d^3}{c \hbar} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin \theta \int_{R_1(\mathbf{H}, \omega, \theta, \varphi)}^{\infty} dR \frac{R(\mathbf{R} \cdot \mathbf{s})^2 W_{\mathbf{R}}^2(\mathbf{H}, R, \theta, \varphi)}{\Delta(\mathbf{H}, \mathbf{R}, \omega)} \times \\ & \times \int_{R_{a1}[\mathbf{R}, \Delta(\mathbf{H}, \mathbf{R}, \omega)]}^{R_{a2}[\mathbf{R}, \Delta(\mathbf{H}, \mathbf{R}, \omega)]} dR_a \frac{R_a^4 \exp(-4\pi N_d R_a^3/3)}{[1 - \kappa e^{-2} R_a \Delta(\mathbf{H}, \mathbf{R}, \omega)]^3}, \end{aligned} \quad (15)$$

where $R_1(\mathbf{H}, \omega, \theta, \varphi)$ is the solution with the largest value of the equation

$$\Delta(\mathbf{H}, \mathbf{R}, \omega) = 0 \quad (15a)$$

and

$$R_{a_1}[R, \Delta(\mathbf{H}, \mathbf{R}, \omega)] = R/2 + e^2/\kappa \Delta(\mathbf{H}, \mathbf{R}, \omega) - \\ - [R^2/4 + e^4/\kappa^2 \Delta^2(\mathbf{H}, \mathbf{R}, \omega)]^{1/2}$$

$$R_{a_1}[R, \Delta(\mathbf{H}, \mathbf{R}, \omega)] = -R/2 + [R^2/4 + R e^2/\kappa \Delta(\mathbf{H}, \mathbf{R}, \omega)]^{1/2}. \quad (15b)$$

We denote

$$\Delta(\mathbf{H}, \mathbf{R}, \omega) = [\hbar^2 \omega^2 - 4W_R^2(\mathbf{H}, R, \theta, \varphi)]^{1/2}; \\ 4W_R^2(\mathbf{H}, R, \theta, \varphi) = AR^2(1 + \eta \cos^2 \theta) [1 - BR^3 H^2(1 + \eta \cos^2 \theta)^{1/2} \times \\ \times f(\theta, \varphi, \theta_H, \varphi_H)] \exp \left[-\frac{2R}{a} (1 + \eta \cos^2 \theta)^{1/2} \right], \\ A = (4e^2/3\kappa a^2)^2, \\ B = e^2/80ac^2 \hbar^2. \quad (15c)$$

In principle, we must integrate (15) over all R , but $W_R^2(\mathbf{H}, R, \theta, \varphi)$ is equal to zero at some value $R = R_{\max}$. Because the contributions from higher R 's are very small, to a good approximation we can restrict our integration to the $R < R_{\max}$, for which $W_R^2(\mathbf{H}, R, \theta, \varphi) > 0$.

Let us choose the y_1 axis of the coordinate system x_1, y_1, z_1 (z_1 axis is parallel to the [111] direction) to be perpendicular to the vector \mathbf{s} . In order to calculate the correction

$$\delta\alpha(H^2) \approx \left. \frac{d\alpha}{d(H^2)} \right|_{H=0}$$

we now make the following transformation from R to ϱ

$$W_R^2(\mathbf{H}, R, \theta, \varphi) = W_R^2(\varrho, \cos \theta), \quad (16)$$

where

$$4W_R^2(\varrho, \cos \theta) = A\varrho^2(1 + \eta \cos^2 \theta) \exp \left[-\frac{2\varrho}{a} (1 + \eta \cos^2 \theta)^{1/2} \right]. \quad (16a)$$

In the new variable ϱ the upper limit of the integration equals $+\infty$, and the lower limit $\varrho_1(\omega, \cos \theta)$ is now defined by the solution with the largest value of the equation

$$\Delta(\varrho, \omega, \cos \theta) = 0, \quad (17)$$

where

$$\Delta(\varrho, \omega, \cos \theta) = [\hbar^2 \omega^2 - 4W_R^2(\varrho, \cos \theta)]^{1/2}. \quad (17a)$$

The correction to the absorption coefficient due to the weak magnetic field is now

$$\delta\alpha(H^2) = \sum_{i=1}^{10} f_i(\vartheta, \theta_H, \varphi_H) I_i, \quad (18)$$

where

$$\begin{aligned}
 f_1(\vartheta, \theta_H) &= \frac{11}{3} a^2 \cos^2 \theta_H \sin^2 \vartheta, \\
 f_2(\vartheta, \theta_H, \varphi_H) &= \frac{11}{3} a^2 \left(\frac{1}{4} \sin^2 \theta_H \sin^2 \vartheta + \frac{1}{2} \sin^2 \theta_H \cos^2 \varphi_H \sin^2 \vartheta + \right. \\
 &\quad \left. + 2 \cos^2 \theta_H \cos^2 \vartheta - 4 \sin \theta_H \cos \theta_H \cos \varphi_H \sin \vartheta \cos \vartheta \right), \\
 f_3(\vartheta, \theta_H) &= -\frac{11}{3} a^2 \sin^2 \theta_H \cos^2 \vartheta, \\
 f_4(\vartheta, \theta_H, \varphi_H) &= \frac{11}{6} b^2 \sin^2 \theta_H \sin^2 \vartheta \left(\frac{1}{2} + \sin^2 \varphi_H \right), \\
 f_5(\vartheta, \theta_H) &= \frac{11}{3} b^2 \sin^2 \theta_H \cos^2 \vartheta, \\
 f_6(\vartheta, \theta_H, \varphi_H) &= -\frac{3}{2} a^2 \sin^2 \theta_H \sin^2 \vartheta \left(\frac{1}{2} + \cos^2 \varphi_H \right), \\
 f_7(\vartheta, \theta_H, \varphi_H) &= -3a^2 (\cos^2 \theta_H \sin^2 \vartheta + \sin^2 \theta_H \cos^2 \vartheta + 4 \sin \theta_H \cos \theta_H \times \\
 &\quad \times \cos \varphi_H \sin \vartheta \cos \vartheta), \tag{18a} \\
 f_8(\vartheta, \theta_H) &= -6a^2 \cos^2 \theta_H \cos^2 \vartheta, \\
 f_9(\vartheta) &= 3a^2 \sin^2 \vartheta, \\
 f_{10}(\vartheta) &= 6a^2 \cos^2 \vartheta, \\
 I_i &= -\frac{2\pi^4 \kappa^{1/2} ABKN_a^3 H^2}{c\hbar} \int_0^1 d(\cos \theta) g_i(\cos \theta) \int_{\varrho_1(\omega, \cos \theta)}^\infty d\varrho \frac{\varrho^8}{\Delta(\varrho, \omega, \cos \theta)} \times \\
 &\quad \times \frac{\exp \left[-\frac{2\varrho}{a} (1 + \eta \cos^2 \theta)^{1/2} \right]}{1 - \frac{\varrho}{a} (1 + \eta \cos^2 \theta)^{1/2}} \times \left\{ T(\varrho, \omega, \cos \theta) + \right. \\
 &\quad \left. + \left[7 + \frac{\frac{\varrho}{a} (1 + \eta \cos^2 \theta)^{1/2}}{1 - \frac{\varrho}{a} (1 + \eta \cos^2 \theta)^{1/2}} \int_{R_{a1}[\varrho, \Delta(\varrho, \omega, \cos \theta)]}^{R_{a2}[\varrho, \Delta(\varrho, \omega, \cos \theta)]} dR_a Q[R_a, \Delta(\varrho, \omega, \cos \theta)] \right] \right\}, \tag{18b} \\
 i &= 1, 2, \dots, 10.
 \end{aligned}$$

ϑ is the angle between the vector \mathbf{s} and the z_1 axis. We denote

$$\begin{aligned} g_1(\cos \theta) &= (1 + \eta \cos^2 \theta)^{5/2} (1 - \cos^2 \theta)^2, \\ g_2(\cos \theta) &= (1 + \eta \cos^2 \theta)^{5/2} (1 - \cos^2 \theta) \cos^2 \theta, \\ g_3(\cos \theta) &= (1 + \eta \cos^2 \theta)^{5/2} \cos^4 \theta, \\ g_4(\cos \theta) &= (1 + \eta \cos^2 \theta)^{5/2} (1 - \cos^2 \theta), \\ g_5(\cos \theta) &= (1 + \eta \cos^2 \theta)^{5/2} \cos^2 \theta, \\ g_6(\cos \theta) &= (1 + \eta \cos^2 \theta)^{3/2} (1 - \cos^2 \theta)^2, \\ g_7(\cos \theta) &= (1 + \eta \cos^2 \theta)^{3/2} (1 - \cos^2 \theta) \cos^2 \theta, \\ g_8(\cos \theta) &= (1 + \eta \cos^2 \theta)^{3/2} \cos^4 \theta, \\ g_9(\cos \theta) &= (1 + \eta \cos^2 \theta)^{3/2} (1 - \cos^2 \theta), \\ g_{10}(\cos \theta) &= (1 + \eta \cos^2 \theta)^{3/2} \cos^2 \theta, \end{aligned} \quad (18c)$$

$$Q[R_a, \Delta(\varrho, \omega, \cos \theta)] = \frac{R_a^4 \exp(-4\pi N_d R_a^3/3)}{[1 - \kappa e^{-2R_a \Delta(\varrho, \omega, \cos \theta)}]^3}, \quad (18d)$$

$$\begin{aligned} T(\varrho, \omega, \cos \theta) &= \varrho \left\{ \frac{\partial R_a[R, \Delta(\varrho, \omega, \cos \theta)]}{\partial R} Q\{R_a[R, \Delta(\varrho, \omega, \cos \theta)], \Delta(\varrho, \omega, \cos \theta)\} - \right. \\ &\quad \left. - \frac{\partial R_a[R, \Delta(\varrho, \omega, \cos \theta)]}{\partial R} Q\{R_a[R, \Delta(\varrho, \omega, \cos \theta)], \Delta(\varrho, \omega, \cos \theta)\} \right\} \Big|_{R=\varrho} \end{aligned} \quad (18e)$$

Evidently

$$\begin{aligned} R_a[R, \Delta(\varrho, \omega, \cos \theta)] &= R/2 + e^2/\kappa \Delta(\varrho, \omega, \cos \theta) - \\ &\quad - [R^2/4 + e^4/\kappa^2 \Delta^2(\varrho, \omega, \cos \theta)]^{1/2}, \\ R_a[R, \Delta(\varrho, \omega, \cos \theta)] &= -R/2 + [R^2/4 + R e^2/\kappa \Delta(\varrho, \omega, \cos \theta)]^{1/2}. \end{aligned} \quad (18f)$$

Introducing the following new variables

$$\begin{aligned} x &= \frac{2\varrho}{a} (1 + \eta \cos^2 \theta)^{1/2}, \\ t &= (1 + \eta \cos^2 \theta)^{-1/2}, \\ y &= \frac{2R_a}{a} \end{aligned} \quad (19)$$

and performing the integration over t we obtain

$$\delta \alpha(H^2) = C \sum_{j=1}^4 a_j(\eta, \vartheta, \theta_H, \varphi_H) J_j, \quad (20)$$

where

$$C = \frac{3^2 a^{10} \pi^4 e^4 K N_d^3 H^2}{5 \times 2^{13} \kappa^{1/2} \eta^{3/2} c^3 \hbar^3}, \quad (20a)$$

$$a_1(\eta, \vartheta, \theta_H, \varphi_H) = \frac{1}{\eta} [(\eta+1)^2 f_6 - (\eta+1) f_7 + f_8] + (\eta+1) f_9 - f_{10},$$

$$a_2(\eta, \vartheta, \theta_H, \varphi_H) = \frac{1}{\eta} [(\eta+1)^2 f_1 - (\eta+1)(f_2 + 2f_6) + (\eta+2)f_7 + f_3 - 2f_8] + (\eta+1)f_4 - f_5 - f_9 + f_{10},$$

$$a_3(\eta, \vartheta, \theta_H, \varphi_H) = \frac{1}{\eta} [-2(\eta+1)f_1 + (\eta+2)f_2 - 2f_3 + f_6 - f_7 + f_8] - f_4 + f_5,$$

$$a_4(\eta, \vartheta, \theta_H, \varphi_H) = \frac{1}{\eta} [f_1 - f_2 + f_3],$$

$$J_1 = -L_1 + M_1 + \frac{1}{8} \left[3 \arccos(\eta+1)^{-1/2} + \frac{\eta^{1/2}(3\eta+5)}{(\eta+1)^2} \right] N + \frac{3}{8} (S_2 - S_1) - \frac{1}{4} (S_4 - S_3) + \frac{5}{8} (S_6 - S_5), \quad (20b)$$

$$J_2 = -L_2 + M_2 + \frac{1}{2} \left[\arccos(\eta+1)^{-1/2} + \frac{\eta^{1/2}}{\eta+1} \right] N + \frac{1}{2} (S_2 - S_1 + S_6 - S_5),$$

$$J_3 = -L_3 + M_3 + N \arccos(\eta+1)^{-1/2} + S_2 - S_1$$

$$J_4 = -L_4 + M_4 + \eta^{1/2} N + S_8 - S_7. \quad (20c)$$

We denote

$$L_k = \int_{x_1(\omega)}^{\infty} dx P_1(x) \int_{y_1[x, t=(\eta+1)^{-1/2}]}^{y_1(x, t=1)} dy Q(x, y) [F_1(x, y)]^{7-2k} \{1 - [F_1(x, y)]^2\}^{-1/2}, \quad k = 1, 2, 3, 4,$$

$$M_k = \int_{x_1(\omega)}^{\infty} dx P_1(x) \int_{y_1[x, t=(\eta+1)^{-1/2}]}^{y_1(x, t=1)} dy Q(x, y) [F_2(x, y)]^{7-2k} \{1 - [F_2(x, y)]^2\}^{-1/2}, \quad k = 1, 2, 3, 4,$$

$$N = \int_{x_1(\omega)}^{\infty} dx P_2(x) \int_{y_1[x, t=(\eta+1)^{-1/2}]}^{y_1[x, t=(\eta+1)^{-1/2}]} dy Q(x, y),$$

$$S_1 = \int_{x_1(\omega)}^{\infty} dx P_2(x) \int_{y_1[x, t=(\eta+1)^{-1/2}]}^{y_1(x, t=1)} dy Q(x, y) \arccos F_1(x, y),$$

$$S_2 = \int_{x_1(\omega)}^{\infty} dx P_2(x) \int_{y_1[x, t=(\eta+1)^{-1/2}]}^{y_1(x, t=1)} dy Q(x, y) \arccos F_2(x, y),$$

$$\begin{aligned}
 S_3 &= \int_{x_1(\omega)}^{\infty} dx P_2(x) \int_{y_1[x, t=(\eta+1)^{-1/2}]}^{y_1(x, t=1)} dy Q(x, y) F_1(x, y) \{1 - [F_1(x, y)]^2\}^{3/2}, \\
 S_4 &= \int_{x_1(\omega)}^{\infty} dx P_2(x) \int_{y_2[x, t=(\eta+1)^{-1/2}]}^{y_2(x, t=1)} dy Q(x, y) F_2(x, y) \{1 - [F_2(x, y)]^2\}^{3/2}, \\
 S_5 &= \int_{x_1(\omega)}^{\infty} dx P_2(x) \int_{y_1[x, t=(\eta+1)^{-1/2}]}^{y_1(x, t=1)} dy Q(x, y) F_1(x, y) \{1 - [F_1(x, y)]^2\}^{1/2}, \\
 S_6 &= \int_{x_1(\omega)}^{\infty} dx P_2(x) \int_{y_2[x, t=(\eta+1)^{-1/2}]}^{y_2(x, t=1)} dy Q(x, y) F_2(x, y) \{1 - [F_2(x, y)]^2\}^{1/2}, \\
 S_7 &= \int_{x_1(\omega)}^{\infty} dx P_2(x) \int_{y_1[x, t=(\eta+1)^{-1/2}]}^{y_1(x, t=1)} dy Q(x, y) \{1 - [F_1(x, y)]^2\}^{1/2} [F_1(x, y)]^{-1}, \\
 S_8 &= \int_{x_1(\omega)}^{\infty} dx P_2(x) \int_{y_2[x, t=(\eta+1)^{-1/2}]}^{y_2(x, t=1)} dy Q(x, y) \{1 - [F_2(x, y)]^2\}^{1/2} [F_2(x, y)]^{-1}, \tag{20d}
 \end{aligned}$$

$$\begin{aligned}
 P_1(x) &= x^8 \exp(-x) / [D(x, \omega)]^{1/2} (2-x), \\
 P_2(x) &= P_1(x) [7 + (x/2 - x)], \tag{20e}
 \end{aligned}$$

$$\begin{aligned}
 y_1(x, t) &= \frac{1}{2} xt + 3/[D(x, \omega)]^{1/2} - [x^2 t^2/4 + 9/D(x, \omega)]^{1/2}, \\
 y_2(x, t) &= -\frac{1}{2} xt + \{x^2 t^2/4 + 3xt/[D(x, \omega)]^{1/2}\}^{1/2}, \tag{20f}
 \end{aligned}$$

$$\begin{aligned}
 F_1(x, y) &= y \{6 - y[D(x, \omega)]^{1/2}\} / x \{3 - [D(x, \omega)]^{1/2}\}, \\
 F_2(x, y) &= y^2 [D(x, \omega)]^{1/2} / x \{3 - y[D(x, \omega)]^{1/2}\}, \tag{20g}
 \end{aligned}$$

$$Q(x, y) = y^4 \exp(-\beta y^3) / \{3 - y[D(x, \omega)]^{1/2}\}^3, \tag{20h}$$

$$D(x, \omega) = \gamma^2 - x^2 \exp(-x), \tag{20i}$$

$$\begin{aligned}
 \gamma &= 3 \hbar \omega \kappa a / 2e^2, \\
 \beta &= \pi a^3 N_d / 6. \tag{20j}
 \end{aligned}$$

$x_1(\omega)$ is defined by the solution with the largest value of the equation

$$D(x, \omega) = 0. \tag{20k}$$

According to the remarks given in the Sect. 4, the case of $\mathbf{H} \parallel [111]$ is of particular interest. For this case we obtain

$$\delta \alpha(H^2) = \delta \alpha_{\perp}(H^2) \sin^2 \vartheta + \delta \alpha_{\parallel}(H^2) \cos^2 \vartheta, \tag{21}$$

where

$$\begin{aligned} \delta\alpha_{\perp}(H^2) = & \frac{Ca^2}{\eta} \left\{ 3(\eta+1)^2(M_1-L_1) + \frac{1}{3}(11\eta^2+4\eta-7)(M_2-L_2) - \right. \\ & - \frac{1}{3}(22\eta+13)(M_3-L_3) + \frac{11}{3}(M_4-L_4) + \left[\frac{1}{24}(71\eta^2-106\eta-105) \operatorname{acr} \cos(\eta+1)^{-1/2} + \right. \\ & + \left. \frac{\eta^{1/2}}{24(\eta+1)}(71\eta^2+176\eta+105) \right] N + \frac{1}{24}(71\eta^2-106\eta-105)(S_2-S_1) - \\ & - \left. \frac{3}{4}(\eta+1)^2(S_4-S_3) + \frac{1}{24}(89\eta^2+106\eta+17)(S_6-S_5) + \frac{11}{3}(S_8-S_7) \right\}, \quad (21a) \end{aligned}$$

$$\begin{aligned} \delta\alpha_{\parallel}(H^2) = & \frac{Ca^2}{\eta} \left\{ -6(\eta+1)(M_1-L_1) + \frac{1}{3}(-4\eta+14)(M_2-L_2) + \right. \\ & + \frac{1}{3}(22\eta+26)(M_3-L_3) - \frac{22}{3}(M_4-L_4) + \left[\frac{1}{12}(53\eta+105) \operatorname{arc} \cos(\eta+1)^{-1/2} - \right. \\ & - \left. \frac{\eta^{1/2}}{12(\eta+1)}(123\eta+105) \right] N + \frac{1}{12}(53\eta+105)(S_2-S_1) + \frac{3}{2}(\eta+1)(S_4-S_3) - \\ & - \left. \frac{1}{12}(53\eta+17)(S_6-S_5) - \frac{22}{3}(S_8-S_7) \right\}. \quad (21b) \end{aligned}$$

$\delta\alpha_{\perp}(H^2)$ and $\delta\alpha_{\parallel}(H^2)$ are the magnetic field-dependent correction to the absorption coefficients of the electro-magnetic waves with the electric vector perpendicular and parallel to the [111] direction, respectively.

6. Discussion of the assumptions

Performing our calculations we have assumed — following Blinowski [4] — that in the high uniaxial [111] stress limit it is necessary to take into account only the contribution to the wave function of the donor ground state from the minimum [111]. We have recognized the fact that the contributions from other minima are negligible as the essential effect of the deformation induced by high [111] compression. Considering the influence of this deformation on the envelope function we have confined ourselves only to the change of the electron radii a and b , according to the formulae (5). Very recently Trylski [14] had investigated the simultaneous effect of uniaxial stress and weak, magnetic field on donor states by the method applied previously by Nguyen Van Huong [5] in the case of magnetic field alone. Trylski was able to find a more exact envelope function than one used in the present paper. It is possible to show however that for compressions no higher than 10^9 dyne cm^{-2} and a magnetic field no lower than 7800 gauss the corrections to the absorption coefficient due to the magnetic field are the same for both types of the envelope function.

The weakness of the external magnetic field was an essential condition for expanding the phase factor in (10d) and also for the construction of the two-donor ground state wave functions with an exactness to the term proportional to H^2 . We have also used this assumption

to calculate W_R and, therefore, to calculate the correction $\delta\alpha(H^2)$ to the absorption coefficient dependent on the magnetic field. The maximum field which allows us to use the approximate equality (10d) in our calculation (taking a rather extreme case when three vectors \mathbf{H} , \mathbf{R} and \mathbf{r}_a are mutually perpendicular, $r_a = 3a$ and $R = 542.4 \text{ \AA}$, and taking into account the fact that the terms proportional to the H and H^3 vanish in the calculation of W_R and, therefore, of $\delta\alpha(H^2)$) is equal to about 11.4 kgauss. In fact, the use of the approximate equation (10d) following from the above estimation is even more justified for the majority of pairs because, in general, the vectors \mathbf{H} , \mathbf{R} and \mathbf{r}_a are not mutually perpendicular. Moreover, because in our wavelength region the function

$$R^3 W_R^2(\mathbf{H}, \mathbf{R}, \theta, \varphi) / \Delta(\mathbf{H}, \mathbf{R}, \omega)$$

in (15) rapidly decreases with the increase of R , the main contribution to the absorption coefficient is given by R 's not far from $R_1(\mathbf{H}, \omega, \theta, \varphi)$ and, therefore, by ϱ 's not far from ϱ_1 (because in a weak magnetic field R is approximately equal to ϱ). Depending on the wavelength in the studied region, the values ϱ_1 lie in the range from about $3a$ to about $7.6a = 542.4 \text{ \AA}$. On the short-wavelength side the values of $R_1 \approx \varrho_1$ are, therefore, much smaller than the value 542.4 \AA accepted in the estimation of the maximum H . We can, therefore, increase the magnetic field on the short-wavelength side of our region without affecting the validity of (10d).

The essential assumption which enables us to use the variation procedure to obtain the two-center electron states and to make all the following calculations is that $|S| \ll 1$. In the studied wavelengths region this assumption is satisfied because $|S| < 0.15$.

We have assumed previously that we can neglect the dipole potentials of the ionized acceptor-ionized donor pairs. The dipoles have various positions and orientations with respect to the given two-donor pair and their potentials cancel themselves partially. We can show, as it was done in the paper [2] that if

$$K < [\kappa \hbar \omega / e^2 \varrho_1(\omega)] N_d^{-2/3}$$

we can neglect the dipole potentials in our calculations.

We have also made the assumption that the field of the acceptor is homogeneous enough in the region of the localized donor pair so that we can neglect Z in (10a). This, as well as the whole procedure of obtaining the two-center electronic states, cannot be justified in the case of jumps "through" the acceptor (Z is then of the order of W_R). The values of R_a close to R_{a_1} correspond to such jumps. Because of the fact that the integral N is the same character as the integral in the formula for the absorption coefficient $\alpha(\mathbf{H} = 0)$ in paper [2], one can show — similarly as in paper [2] — that the assumption of the homogeneous electrostatic field is valid for the integral N if the following condition is fulfilled

$$N_d < \frac{8}{\pi \varrho_1^3(\omega)}.$$

The integrals M_k , $k = 1, 2, 3, 4, S_2, S_4, S_6$ and S_8 appear because of the derivation over R of the limit R_{a_2} , so the jumps through the acceptor are neglected here. However, the integrals L_k , $k = 1, 2, 3, 4, S_1, S_3, S_5$ and S_7 appear because of the derivation over R of the limit R_{a_1} .

corresponding to jumps through the acceptor. Results of numerical computations show that — depending on the wavelength of the radiation — these integrals at low concentrations ($N_d = 6.0 \times 10^{14} \text{ cm}^{-3}$ and $N_d = 1.9 \times 10^{15} \text{ cm}^{-3}$) contribute only a few per cent to the whole effect $\delta\alpha(H^2)$. The contributions from these integrals increase with increasing N and the wavelength λ of the radiation. On the whole these contributions are equal to near $\frac{15}{100} \delta\alpha_{\perp}(H^2)$ if $N_d = 6.0 \times 10^{15} \text{ cm}^{-3}$ and $\lambda = 1 \text{ cm}$, and are equal to about $\frac{23}{100} \delta\alpha_{\perp}(H^2)$, if at the same concentration $N_d \lambda$ is equal to 2 cm. At a given N_d and λ the integrals L_k , $k = 1, 2, 3, 4, S_1, S_3, S_5$ and S_7 always yield a larger contribution to $\delta\alpha_{\perp}(H^2)$ than $\delta\alpha_{\parallel}(H^2)$.

7. Numerical results

In numerical computations we have used the following values of the material constants for *n*-Ge:

$$\eta = 18.8, \quad \kappa = 16, \quad a = 71.5 \text{ \AA}, \quad b = 16.1 \text{ \AA}.$$

We are interested in samples with donor concentration up to $6 \times 10^{15} \text{ cm}^{-3}$ and in photon energies of the incident radiation smaller than one fourth of the effective mass value of the ionization energy of the donors. The effect of the excited states is probably not very important in this region of energies. On the long wavelength side we have confined ourselves to microwaves for which $\lambda \leq 2 \text{ cm}$. We have done this because at low concentrations the absorption effect is very small, but at higher concentrations — yielding larger absorption — and for wavelength $\lambda > 1.5 \text{ cm}$ the localized pair model is not (as it was stated in [4]) well justified. Moreover, the contribution to $\delta\alpha(H^2)$ coming from jumps through the acceptor is — as we have stated above — no longer small.

The assumption $kT \ll \hbar\omega$ means numerically $T \ll 1.4^\circ\text{K} \times \text{cm} \lambda^{-1}$ and restricts sharply the long wavelength side of the wavelength region. Nevertheless, even for $\lambda = 0.5 \text{ cm}$ temperatures satisfying this condition are experimentally available [6].

Fig. 1 presents the values of $\frac{\alpha_{\perp}(\mathbf{H} = 0)}{K}$ and $\frac{\alpha_{\parallel}(\mathbf{H} = 0)}{K}$ as the functions of λ for given donor concentrations, calculated on the basis of Blinowski's formula [4]. The wavelength dependence of the dichroic ratio $\frac{\alpha_{\perp}(\mathbf{H} = 0)}{\alpha_{\parallel}(\mathbf{H} = 0)}$ for the same donor concentrations is presented in Fig. 2. Because of the different values of the material constants the dichroic ratio is here about 2 to 2.5 times larger than that of Blinowski [4]. The wavelength dependence of the dichroic ratio is similar to that obtained by Blinowski.

Fig. 3 presents the wavelength dependence of the corrections $-\frac{1}{H^2} \frac{\delta\alpha_{\perp}(H^2)}{K}$ and $-\frac{1}{H^2} \frac{\delta\alpha_{\parallel}(H^2)}{K}$, calculated on the basis of our formulae (21a) and (21b). The relative corrections $-\frac{1}{H^2} \frac{\delta\alpha_{\perp}(H^2)}{\alpha_{\perp}(\mathbf{H} = 0)}$ and $-\frac{1}{H^2} \frac{\delta\alpha_{\parallel}(H^2)}{\alpha_{\parallel}(\mathbf{H} = 0)}$ as functions of λ are presented in

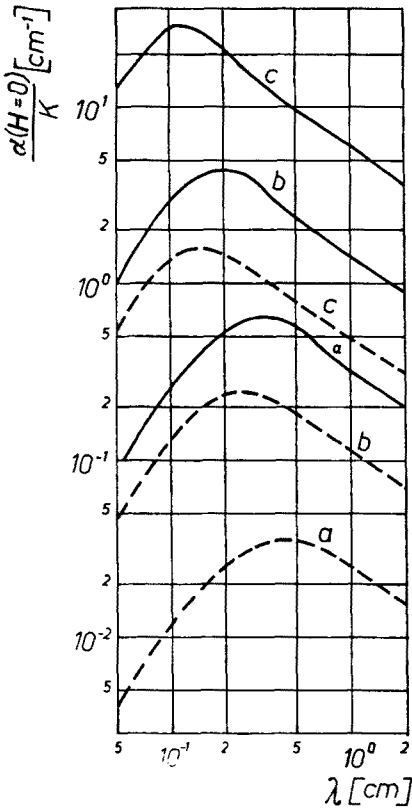


Fig. 1

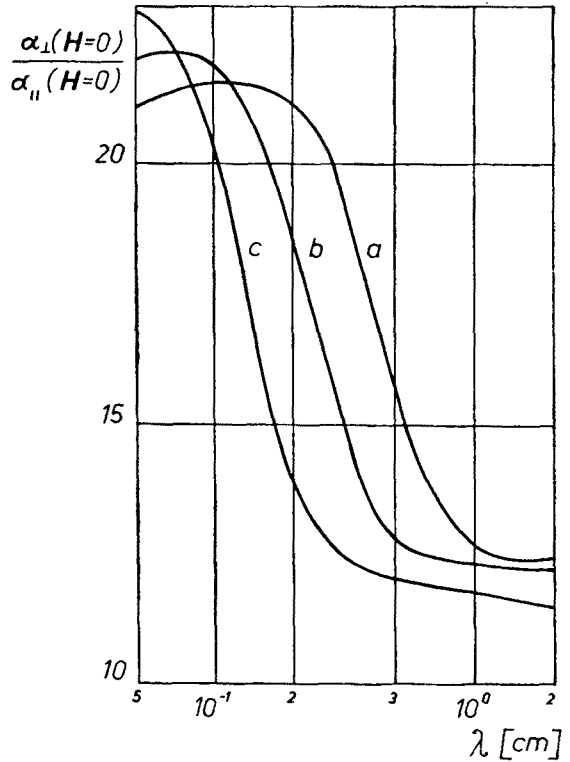


Fig. 2

Fig. 1. The wavelength dependence of the ratios $\frac{\alpha_{\perp}(H=0)}{K}$ (solid curves) and $\frac{\alpha_{\parallel}(H=0)}{K}$ (dashed curves) for various concentration of donors. The curves a, b, and c correspond to the concentrations $N_d = 6 \times 10^{14} \text{ cm}^{-3}$, $1.9 \times 10^{15} \text{ cm}^{-3}$, and $6 \times 10^{15} \text{ cm}^{-3}$, respectively

Fig. 2. The wavelength dependence of the dichroic ratio $\frac{\alpha_{\perp}(H=0)}{\alpha_{\parallel}(H=0)}$ for the donor concentrations $N_d = 6 \times 10^{14} \text{ cm}^{-3}$, $1.9 \times 10^{15} \text{ cm}^{-3}$, and $6 \times 10^{15} \text{ cm}^{-3}$ (curves a, b, and c, respectively)

Fig. 4. Let us note that for a given λ and N_d the relative correction $-\frac{1}{H^2} \frac{\delta\alpha_{\perp}(H^2)}{\alpha_{\perp}(H=0)}$ is larger than $-\frac{1}{H^2} \frac{\delta\alpha_{\parallel}(H^2)}{\alpha_{\parallel}(H=0)}$. At magnetic field equal to 10^4 gauss $-\frac{\delta\alpha_{\perp}(H^2)}{\alpha_{\perp}(H=0)} < 9.7 \times 10^{-2}$ and $-\frac{\delta\alpha_{\parallel}(H^2)}{\alpha_{\parallel}(H=0)} < 6.3 \times 10^{-2}$.

Fig. 5 gives the ratio $\frac{\delta\alpha_{\perp}(H^2)}{\delta\alpha_{\parallel}(H^2)}$ as a function of the wavelength λ . In all wavelength regions studied the values of this ratio are much larger than unity. Therefore, the magnetic field diminishes the dichroic ratio.

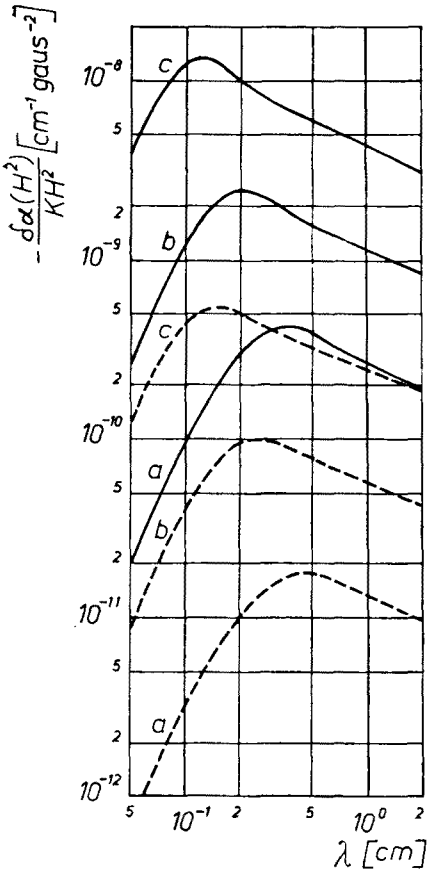


Fig. 3

Fig. 3. The values of $-\frac{1}{H^2} \frac{\delta\alpha_{\perp}(H^2)}{K}$ (solid curves) and $-\frac{1}{H^2} \frac{\delta\alpha_{\parallel}(H^2)}{K}$ (dashed curves) as the functions of λ for the donor concentrations $N_d = 6 \times 10^{14} \text{ cm}^{-3}$, $1.9 \times 10^{15} \text{ cm}^{-3}$, and $6 \times 10^{15} \text{ cm}^{-3}$ (curves a, b, and c, respectively)

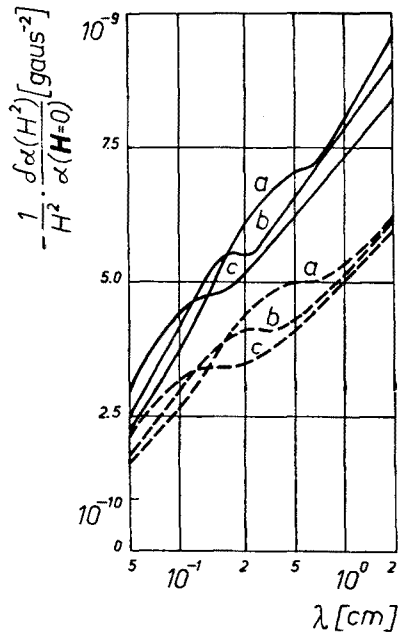


Fig. 4

Fig. 4. The values of $-\frac{1}{H^2} \frac{\delta\alpha_{\perp}(H^2)}{\alpha_{\perp}(H=0)}$ (solid curves) and $-\frac{1}{H^2} \frac{\delta\alpha_{\parallel}(H^2)}{\alpha_{\parallel}(H=0)}$ (dashed curves) as the functions of λ for the donor concentrations $N_d = 6 \times 10^{14} \text{ cm}^{-3}$, $1.9 \times 10^{15} \text{ cm}^{-3}$, and $6 \times 10^{15} \text{ cm}^{-3}$ (curves a, b, and c, respectively)

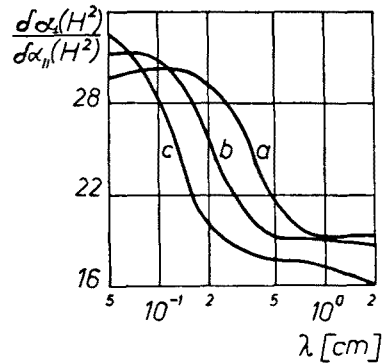


Fig. 5

Fig. 5. The wavelength dependence of the ratio $\frac{\delta\alpha_{\perp}(H^2)}{\delta\alpha_{\parallel}(H^2)}$ for the donor concentrations $N_d = 6 \times 10^{14} \text{ cm}^{-3}$, $1.9 \times 10^{15} \text{ cm}^{-3}$, and $6 \times 10^{15} \text{ cm}^{-3}$ (curves a, b, and c, respectively)

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