PERMUTATION SYMMETRY OF THREE TRIPLETS AND BINDING OF QUARKS

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The idea of permutation symmetry of three triplets in the Han-Nambu model, taken together with the previously proposed saturation mechanism provided by a complex "quark charge", solves the problem of unwanted mesons in this model.

As it is well known the quark model argues that baryons $\frac{1}{2}^+$ and $\frac{3}{2}^+$ belonging to the 56 multiplet of SU(6) group are essentialy three-quark systems in ground states, whereas mesons 0^- and 1^- form the $35+1$ multiplet are two-body systems consisting of one quark and one antiquark in ground states. If three quarks contained in those baryons belong to one SU(3) triplet [1], [2], this triplet must have non-integral electric charges ($2/3$, $-1/3$, $-1/3$) and baryonic charges $1/3$. If they are from two [3] or three [4] different SU(3) triplets, these triplets can have either non-integral or integral [5]–[12] electric charges ($2/3+c_i$, $-1/3+c_i$, $-1/3+c_i$) ($i = 1, 2, 3$), where $c_1+c_2+c_3 = 0$, and baryonic charges $1/3+d_i$ ($i = 1, 2, 3$), where $d_1+d_2+d_3 = 0$.

The three-triplet quark model [4] offers an important theoretical advantage in the discussion of the binding mechanism of quarks. Namely, treating three triplets as different states of the same triplet and imposing on all quarks the over-all Pauli principle, we conclude that three quarks in baryons belonging to the 56 multiplet are not prevented to be bound with each other in some S states. This follows from the fact that the states from the 56 multiplet are symmetric under the exchange of SU(6) quantum numbers and so they are symmetric with respect to orbital degrees of freedom, if they happen to be antisymmetric under the exchange of intertriplet quantum numbers.

If we assume the three-triplet model to be valid, we immediately face two fundamental questions: which quantum numbers infer the difference between three triplets and why only

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three quarks from three different triplets can form comparatively stable baryon states. In a previous paper [13] we have proposed a possible answer to these questions by postulating the existence of a complex "quark charge" Z with the following properties:

1. The Z charge has three "smallest" eigenvalues $z_i (i = 1, 2, 3)$ of the same absolute value

$$|z_1| = |z_2| = |z_3| = 1, \tag{1}$$

if we normalize them to one.

Hence, ascribing the eigenvalues z_i to quarks we get three different triplets of quarks. Antiquarks have then the Z charges equal to $-z_i$, since a quark and its antiparticle should form "neutral" states with respect to the Z charge.

2. The sum of the eigenvalues z_i is equal to zero

$$z_1+z_2+z_3 = 0. \tag{2}$$

Thus, three quarks form "neutral" states with respect to the Z charge if and only if they are from three different triplets. Obviously, three is the smallest number of quarks which can form "neutral" states with respect to Z. Due to (2) we can simply assume that c_i and d_i are linear combinations of Re z_i and Im z_i, *e. g.* $c_i = \lambda \text{ Re } z_i + \mu \text{ Im } z_i$.

3. The Z charge is the source of a neutral vector meson X

$$(\Box - m^2)X_\mu(x) = -ig \sum_{i=1}^{3} z_i \bar{q}_i(x)\gamma_\mu q_i(x), \tag{3}$$

$q_i(x)$ being three quark triplets and g a real coupling constant.

From the properties (1) and (2) we can see that

$$z_i^3 = 1, \tag{4}$$

if one of z_i is equal to one. Putting, *e. g.*, $z_3 = 1$ we get from (4)

$$z_1 = \frac{-1+i\sqrt{3}}{2}, \; z_2 = \frac{-1-i\sqrt{3}}{2}, \; z_3 = 1. \tag{5}$$

The interaction hamiltonian of quarks and an X meson following from the property (3) has the form

$$H = ig \sum_{i=1}^{3} z_i \bar{q}_i(x)\gamma_\mu q_i(x) X_\mu^+(x) + \text{h. c.}$$

$$= ig\sqrt{2} \sum_{i=1}^{3} \bar{q}_i(x)\gamma_\mu q_i(x) \, [\text{Re } z_i X_\mu^R(x) + \text{Im } z_i X_\mu^J(x)], \tag{6}$$

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where

$$X_\mu^R = \frac{X_\mu + X_\mu^+}{\sqrt{2}}, \; X_\mu^J = \frac{X_\mu - X_\mu^+}{i\sqrt{2}}. \tag{7}$$

The formula (6) says that the interaction of X meson is equivalent to the sum of interactions of two different vector mesons X^R and X^J.

If we want to have conservation of the charge conjugation C in the interaction (6), we are obliged to assume that X^R and X^J mesons have the same charge parity C (actually equal to -1). Then X and X^+ are also eigenstates of C in spite of the fact that X_μ is not a hermitian operator. This means that the X meson carries in this case no internal charge and its non-hermitian character is in a sense trivial.

From (6) we can calculate the following one-meson-exchange potential for two quarks belonging to the triplets i and j [13]:

$$V_{ij} = a_{ij}V, \text{ where } a_{ij} = 2(\mathrm{Re}\, z_i \,\mathrm{Re}\, z_j + \mathrm{Im}\, z_i \,\mathrm{Im}\, z_j). \tag{8}$$

In the static case V is a positive function equal to $g^2 \exp(-mr)/r$. Making use of (5) we obtain

$$a_{ij} = \begin{cases} 2 & i = j \\ -1 & i \neq j \end{cases}. \tag{9}$$

Thus, in the static case the potential (8) is repulsive for quarks from the same triplet and attractive for quarks belonging to differrent triplets. It may be reasonable to assume that the whole interaction between two quarks has this property. Then, in a three-quark system there appears attraction between all three quarks, if and only if they are from three different triplets. So, in this case we can expect the strongest binding. Moreover, in this case the quark system in "neutral" with respect to the Z charge and, therefore, it is not likely to attach another quark. On the contrary, one-quark and two-quark systems are "charged" with respect to Z. Thus, we can see that the fundamental phenomenon of saturation of quark forces [14] in the three-quark baryons may be related to the Z charge [13].

For a quark from the triplet i and an antiquark from the triplet $\bar{j}$ the one-meson-exchange potential becomes [13]

$$V_{i\bar{j}} = a_{i\bar{j}}V, \tag{10}$$

where

$$a_{i\bar{j}} = \begin{cases} -2 & i = j \\ 1 & i \neq j \end{cases}. \tag{11}$$

Thus, in the static case this potential is attractive for quarks and antiquarks corresponding to the same triplet and repulsive in the case of different triplets. Notice that the attraction is here twice as strong as for the two-quark system. Of course, the system of one quark and one antiquark corresponding to the same triplet is "neutral" with respect to Z and so it is not likely to attach another quark or antiquark.

Notice that according to (10) and (11) the average mass of the 35+1 multiplet should be

$$M_M \approxeq 2M_q - 2\bar{V} = 2(M_q - \bar{V}), \tag{12}$$

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whereas according to (8) and (9) the average mass of the 56 multiplet should be

$$M_B \simeq 3M_q - 3\overline{V} = 3(M_q - \overline{V}). \tag{13}$$

Hence $M_M/M_B \simeq 2/3$ in agreement with the experimental situation.

Notice also that the three-triplet and one-triplet models give the same average ratio of the meson-baryon and baryon-baryon total cross-sections, equal to 2/3.

We should like to add a few remarks on the formal construction of the operator of the Z charge. This can be done by means of the generators of an SU(3) group. This group must commute with the familiar SU(3) group which defines the isospin $I_\alpha = Q_\alpha$ $(\alpha = 1, 2, 3)$, the hypercharge $Y = \frac{2}{\sqrt{3}} Q_8$ and the electric charge

$$Q = Q_3 + \frac{1}{\sqrt{3}} Q_8 + \lambda \operatorname{Re} Z + \mu \operatorname{Im} Z, \tag{14}$$

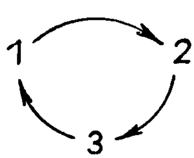
since the inter-triplet quantum number Z is independent of the triplet quantum numbers I_3 and Y. Denoting the new group by SU(3)′ and its generators by Q'_α $(\alpha = 1, 2, \ldots, 8)$ we infer from (5) that

$$Z = \sqrt{3}(-Q'_8 + iQ'_3), \tag{15}$$

because the eigenvalues of Q'_3 and Q'_8 in the fundamental representation are $(1/2, -1/2, 0)$ and $(1/2\sqrt{3}, 1/2\sqrt{3}, -1/\sqrt{3})$, respectively. Notice that the interaction (6) is not invariant under SU(3)′ in a similar way as the electromagnetic interaction is not invariant under SU(3): in both cases there are only gauge invariances generated by the "3" and "8" operators. Due to this, SU(3) ⊗ SU(3)′ does not describe any approximate symmetry of quark interactions, while SU(3) is a considerably good symmetry. On the other hand, however, there is no splitting in mass between three quark triplets caused by the interaction (6), since we have $|z_1| = |z_2| = |z_3|$. If $\lambda = 0$ and $\mu = 0$ there is also no electromagnetic splitting between these triplets. The group SU(3) ⊗ SU(3)′ considered here as an auxiliary structure has been much exploited by Greenberg and collaborators [15] who assume that it is an approximate symmetry group of quark interactions.

Now we go over to a serious difficulty of the three-triplet model connected with the appearance in it of too many meson states built up of one quark and one antiquark. In general, for each admissible SU(6) ⊗ O(3) state there may exist nine such mesons $\varphi_{ij} = (q_i^{(1)+} q_j^{(2)})_{\text{bound}}$ $(i, j = 1, 2, 3)$ in place of one expected in the one-triplet model. In our theory of Z charge there appears an attraction between quark and antiquark only for $i = j$ (see (10) and (11)), thus the number nine is here reduced *a priori* to three.

In order to reduce further this number three (actually to one) we introduce into the three-triplet model a new exact symmetry. Namely, we will postulate that *real particle states of zero Z charge must be invariant (except for possible phase factors) under all permutations of three triplets.* In particular, they must be invariant under the cyclic permutation



of three triplets. Here the numbers 1,2, and 3 are values of the index i labelling three triplets $q_i = (q_{i\alpha})(i = 1, 2, 3; \alpha = 1, 2, 3)$. This postulate taken together with the theory of Z charge gives the following admissible SU(3)$\otimes$SU(3)$'$ states:

1. for quark+antiquark system: only one SU(3) $3^* \times 3$ multiplet

$$\frac{1}{\sqrt{3}}(q_{1\alpha}^{(1)+} q_{1\beta}^{(2)} + q_{2\alpha}^{(1)+} q_{2\beta}^{(2)} + q_{3\alpha}^{(1)+} q_{3\beta}^{(2)}),$$

2. for three-quark system: two SU(3) $3\times 3\times 3$ multiplets

$$\frac{1}{\sqrt{6}}(q_{1\alpha}^{(1)} q_{2\beta}^{(2)} q_{3\gamma}^{(3)} + q_{2\alpha}^{(1)} q_{3\beta}^{(2)} q_{1\gamma}^{(3)} + q_{3\alpha}^{(1)} q_{1\beta}^{(2)} q_{2\gamma}^{(3)} \pm$$

$$\pm q_{1\alpha}^{(1)} q_{3\beta}^{(2)} q_{2\gamma}^{(3)} \pm q_{2\alpha}^{(1)} q_{1\beta}^{(2)} q_{3\gamma}^{(3)} \pm q_{3\alpha}^{(1)} q_{2\beta}^{(2)} q_{1\gamma}^{(3)}).$$

Here we have written down explicitly only the SU(3)$\otimes$SU(3)$'$ parts of the wave functions.

We can see that in the case of the three-quark system the new principle follows from the ordinary symmetrization postulate for the wave function of a system of identical objects (with respect to the intertriplet quantum number i). However, in the case of other quark systems it is a new postulate. Notice that the potentials (8) and (10) exhibit the permutation symmetry of three-triplets ($V_{11} = V_{22} = V_{33}$, $V_{12} = V_{23} = V_{31} = V_{21} = V_{32} = V_{13}$). So, the configurational hamiltonians of quark systems are consistent with the permutation symmetry of three triplets (*e.g.* $H_{11} = H_{22} = H_{33}$, $H_{12} = H_{23} = H_{31} = H_{21} = H_{32} = H_{13}$, where $H_{ij} = T^{(1)} + T^{(2)} + V_{ij}$). This enables us to assume our postulate on the invariance of particle states under permutations of thre triplets. The states under consideration must have zero Z charge to avoid the mixing of different eigenvalues of Z.

Concluding we can say that in the case of mesons built up of one quark and one anti-quark, the theory of Z charge and the postulate of permutation symmetry of three triplets reduce the number of admissible meson states to the number given by the one-triplet model. More specifically, the postulate of permutation symmetry of three triplets admits only the state

$$\frac{1}{\sqrt{N_1}}(\varphi_{\bar{1}1} + \varphi_{\bar{2}2} + \varphi_{\bar{3}3}),$$

whereas two other states allowed by the theory of Z charge (see Appendix),

$$\frac{1}{\sqrt{N_2}}(\varphi_{\bar{1}1} - \varphi_{\bar{2}2}) \quad \text{and} \quad \frac{1}{\sqrt{N_2}}(\varphi_{\bar{1}1} - \varphi_{\bar{3}3}),$$

are excluded (these two states correspond to one degenerated mass, if we have $\lambda = 0$ and $\mu = 0$ or if the electromagnetic interactions are neglected). Here $\varphi_{\bar{i}j} = (q_i^{(1)+} q_j^{(2)})_{\text{bound}} = \delta_{ij}\varphi_{\bar{i}i}$.

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APPENDIX

Three states $\varphi_{\bar{i}i} = (q_i^{(1)+} q_i^{(2)})_{\text{bound}}$ $(i = 1, 2, 3,)$ correspond to the same mass M, if $\lambda = 0$ and $\mu = 0$ or if the electromagnetic interaction is neglected. They are not stable with respect to the mutual transitions $\bar{i}i \rightleftarrows \bar{j}j$. These transitions are described in the lowest order by the one-meson-annihilation diagram. The corresponding one-meson-annihilation potential turns out to be $V_{ij} = a_{ij}V$, *i.e.* it is equal to the one-meson-exchange potential for two quarks given by (8) and (9).

We will assume that the states $\varphi_i = \varphi_{\bar{i}i}$ satisfy the equation

$$H_i\varphi_i + \sum_{j=1}^{3} V_{ij}\varphi_j = M\varphi_i \qquad (i = 1, 2, 3), \tag{1A}$$

where $H_i = T^{(1)} + T^{(2)} + V_{\bar{i}i}$. Making use of the symmetry $H_1 = H_2 = H_3 \equiv H$, $V_{11} = V_{22} = V_{33} \equiv 2V$ and $V_{12} = V_{23} = V_{31} = V_{21} = V_{32} = V_{13} \equiv -V$ we obtain the following eigensolutions of (1A):

$$\begin{array}{lll} 1. \;\; \varphi_{1M_1} = \varphi_{2M_1} = \varphi_{3M_1} & \text{where} & H\varphi_{1M_1} = M_1\varphi_{1M_1}, \\ \left.\begin{array}{l} 2. \;\; \varphi_{1M_2} = -\varphi_{2M_2}, \varphi_{3M_2} = 0 \text{ or} \\ 3. \;\; \varphi_{1M_2} = -\varphi_{3M_2}, \varphi_{2M_2} = 0 \end{array}\right\} & \text{where} & (H+3V)\varphi_{1M_2} = M_2\varphi_{1M_2}. \end{array} \tag{2A}$$

Then the eigenvectors and eigenvalues of the matrix $(\delta_{ij}H + V_{ij})$ are respectively,

$$1. \begin{pmatrix} \varphi_{1M_1} \\ 0 \\ 0 \end{pmatrix}, \quad 2. \begin{pmatrix} 0 \\ -\varphi_{1M_2} \\ 0 \end{pmatrix}, \quad 3. \begin{pmatrix} 0 \\ 0 \\ -\varphi_{1M_2} \end{pmatrix} \tag{3A}$$

and

$$M_1 = \mathring{M}_1, \qquad M_2 = \mathring{M}_2 + 3\overline{V}_2, \tag{4A}$$

where $\mathring{M}_s = \langle\varphi_{1M_s}|H|\varphi_{1M_s}\rangle$ and $\overline{V}_s = \langle\varphi_{1M_s}|V|\varphi_{1M_s}\rangle$ $(s = 1, 2)$. In the lowest order of the perturbation theory we have $\mathring{M}_1 = \mathring{M}_2$ and $\overline{V}_1 = \overline{V}_2$. In the static limit $V > 0$, then $\overline{V}_s > 0$. Since $M_M \simeq 2(M_q - \overline{V})$ and $M_B \simeq 3(M_q - \overline{V})$, we get an estimate $\overline{V} \simeq M_q - (M_B - M_M)$, so the numbers $\overline{V}_s \simeq \overline{V}$ may be quite large. Then M_q is much larger than M_1. Using (2A) we can write

$$1. \;\; \varphi_{1M_1} = \frac{1}{3}(\varphi_{1M_1} + \varphi_{2M_1} + \varphi_{3M_1}),$$

$$2. \;\; \varphi_{1M_2} = \frac{1}{2}(\varphi_{1M_2} - \varphi_{2M_2}) = \frac{1}{3}(\varphi_{1M_2} - 2\varphi_{2M_2} + \varphi_{3M_2}) \qquad \text{or}$$

$$3. \;\; \varphi_{1M_2} = \frac{1}{2}(\varphi_{1M_2} - \varphi_{3M_2}) = \frac{1}{3}(\varphi_{1M_2} + \varphi_{2M_2} - \varphi_{3M_2}).$$

If our postulate of permutation symmetry of three triplets is true, then only the first of these states is allowed. Notice, however, that $\overline{H} + 3\overline{V} = \overline{T}^{(1)} + \overline{T}^{(2)} + \overline{V}$, whereas $\overline{H} = \overline{T}^{(1)} + \overline{T}^{(2)} - 2\overline{V}$, so two excited states corresponding to mass M_2 simply do not exist as bound states. Thus, in this estimate no postulate of permutation symmetry od three triplets is needed to exclude these excited states.

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