THEORETICAL REMARKS ON CRANE AND HALPERN'S EXPERIMENTAL EVIDENCE FOR THE EXISTENCE OF THE NEUTRINO*

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The relation between the recoil energy of the nucleus in the process $Cl^{38} \xrightarrow{\beta} A^{38}$ and the number of droplets produced by this in a cloud chamber has been calculated on the assumption of Crane and Halpern that the number of droplets is approximately equal to the number of nitrogen and oxygen atoms dissociated in collision chains initiated by the recoil atom. The relation appears to be linear with two constants which have been evaluated.

The occurrence of several ions in clusters of droplets has been explained as caused by X-rays or Auger processes releasing the fraction of the total binding energy of the orbital electrons of the atom connected with the change of the nuclear charge during the beta decay.

Finally, statistical errors in Crane and Halpern's experiments have been discussed. The analysis leads to the conclusion that these experiments give a definitely positive answer in the question of the non-conservation of momentum in the «two-body» beta-decay.

Introduction

In 1944 A. P. Grinberg published in Russian an excellent survey *(1, 1944)* of the experimental evidence for the existence of a neutrino, written chiefly in connection with the important experiments of J. S. Allen *(2, 1942)* which he considered as a definitive discovery of this particle. In this report the former work on this topics by H. R. Crane and J. Halpern (abbreviated henceforth to: C&H) was also discussed and subjected to severe criticism *(3, 4, 1938; 5, 1939)*. Grinberg writes on their work in conclusion: «the distribution of their experimental points quite certainly cannot be regarded as probable». He is not isolated in his critical opinion of C.&H.'s method, although others views are not so extreme. Kan Chang Wang *(6, 1942)* writes: «...owing to the smallness of the

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ionization effect of the recoil atom, it seems worth while to consider a different method of detecting it». Allen *(2, 1942)* states: «Since little is known about the energy range relation of very slow atoms in gases, the momenta of the recoil nuclei could not be measured with any accuracy. Although this method was somewhat refined in another experiment, no very definite evidence regarding the neutrino was discovered» [1]. Even C.&H. recognize *(5, 1939)* that: «The principal unknown in the method is, and always has been, the relation between the number of droplets and the energy of the recoil atom». Provoked by Grinberg's article I undertook an attempt to investigate this «principal unknown» in the method of C.&H. from a theoretical point of view. After Allen's work the analysis seems to be important not only as looking for a confirmation, but also since the C.&H.'s method can give theoretically more information than that of Allen, namely the distribution of angles between the directions of emission of the neutrino and the electron, which is of great importance for the Fermi theory of beta decay.

In the course of the investigation it became apparent to me that the mechanism of ion creation in C.&H.'s experiments is not quite clear even after their reply to the criticism of L. Wertenstein *(9, 1938)* on this subject. This problem will be discussed in the section «The origin of ions». It also seems essential to investigate in detail the statistical errors in the method of C.&H.

The recoil energy as a function of the number of droplets

To obtain theoretically this relation we assume like C.&H. *(5, 1939)* that in their experiments the formation of droplets was initiated in general by a dissociation of nitrogen or oxygen molecules in collisions caused by the recoil of the nucleus. We suppose for the sake of simplicity that every such dissociated atom originates a droplet and that the presence in the chamber of other gases besides air can be neglected in the theoretical treatment of collision processes. Both these assumptions seem to be plausible in the face of the experimental conditions and some results of C.&H.

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[1] E. J. Konopinski in his report on beta decay in the Rev. of Mod. Phys. *(7, 1943)* writes only: «More satisfyingly direct observation of the neutrino was undertaken by Crane and Halpern and Allen. These investigators attempted to observe the recoils of nuclei from neutrino emission. Allen's work seems the most nearly conclusive». Das Gupta and Ghosh *(8, 1946)* qualify the C.&H. experiments as «of limited accuracy», although they estimate their conclusions as «highly probable».

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The recoil energy of a nucleus not exceeding c. 450 eV, we are mainly interested in the order of energy 100 eV for impinging molecules in collisions. Struck gas molecules can be considered as at rest by reason of the relative smallness of their thermal energies (10^{-2} eV). The time of such a collision is c. 10^{-14} sec whereas the periods of quantum processes in nitrogen or oxygen molecules which can be excited by collision to higher energy levels have orders: 10^{-16} sec for the (classical) rotation of the orbital electrons, 10^{-14} sec for the oscillation of the atoms in the molecule, and 10^{-12} sec for the rotation of the molecule as a whole. We see, after C. & H. (*5, 1939*), that a resonance is only possible with the oscillatory energy level, which leads to the dissociation of the molecule, and that other excitations can be neglected. We can therefore dispense from an exact quantum discussion of collision problems, which would involve almost insuperable calculation difficulties.

The argon atom as well the homopolar molecules of nitrogen and oxygen can be considered in the first approximation as spheres. The familiar classical theory of inelastic collision of two spheres with absorption of the energy of dissociation shows that this absorption is only then possible when the angle ϑ between the direction of the impinging molecule and the line of centers of the molecules in the moment of impact does not exceed a certain maximum value i. e. when

$$\cos\vartheta \geqslant \sqrt{\frac{M+m}{m}\frac{D}{E}},$$

where M and m denote respectively the masses of the impinging and the struck molecules, E kinetic energy of the impinging molecule before the collision and D dissociation energy. We see that the maximum value of ϑ (or the minimum one of $\cos\vartheta$) depends upon E and this fact creates the «classical» dependence of the pissociation cross-section upon E, which must be taken into consideration even in our narrow range of energy (where the above mentioned «quantum» dependence of this cross-section upon E may be neglected).

It should be stressed that the part of a dissociating agent is played not only by the recoil atom itself but also by all gas molecules, dissociated or not, which receive in collisions sufficient kinetic energy to dissociate in turn other modecules of air. Every recoil atom produces chains of collisions branching forth like a genealogical tree (or, e. g. a cosmic ray shower). These collision chains can be regarded as terminated when the kinetic energy of the colliding molecules drops to thermal level, but we are not interested beyond the point where further dissociation processes are not more possible. There are

10 types of collisions which may lead to dissociation: $A - N_2$, $A - O_2$, $N - N_2$, $N - O_2$, $O - N_2$, $O - O_2$, $N_2 - N_2$, $N_2 - O_2$, $O_2 - N_2$, $O_2 - O_2$. As calculation shows, the last four types of collisions may be neglected since — in the energy range considered — they produce no dissociation at all.

The following items were computed succesively:

(1) the dissociation cross-section for the above mentioned 10 types of collisions as a function of E,

(2) the average energies of all products of these collisions, also as functions of E, and

(3) on the basis of (1) and (2) the average shape of the «collision genealogical tree» for recoil atoms with energies[2]: 25, 100, 200, 300 and 400 eV. Finally, the numbers of dissociated atoms of nitrogen and oxygen were summed up for every «tree».

As dissociation energies were assumed: 9.5 eV for nitrogen, and 6.2 eV for oxygen.

Table I gives the results of these somewhat tedious and lengthy (over 100 pages) but straightforward calculations.

Table I

Energy of A^{38} (eV) (E)	Average Number of Collisions in a «Tree»	Average Number of Dissociated Atoms			Average Energy Spent (E/N) (eV/diss. atom)
		of Nitrogen	of Oxygen	Total (N)	
1	2	3	4	5	6
25	1	0.3	0.2	0.5	50
100	10	4.0	1.6	5.6	18
200	23	9.3	3.6	12.9	15.5
300	41	14.4	5.8	20.2	14.8
400	47	20.8	7.8	28.6	14.0

The numbers of columns 3, 4 and 5 are presented graphically in Fig. 1, those of column 6 in Fig. 2. We see that the points in Fig. 1 lay on straight lines not through the origin of coordinates (there-

[2] The calculation for 25 eV was made chiefly for the purpose of checking. The results are in this case not so certain as for other value of E since then the time of collision is 10^{-13} sec and an excitation of the rotatory spectrum of the struck gas molecule it is possible in some degree. It is clear therefore that in reality the dissociation effect will be on an average still smaller than indicated in the table, and the energy spent per diss. atom larger. But since, in any case, the mean number of dissociated atoms is under 1, the loss in accuracy is of little importance for our problem.

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fore, the points in Fig. 2, which indicate the ratio E/N, lay on a hyperbola). Thus we have obtained the solution of our problem in the form

$$E = aN + b,$$

where a and b are constants. Evaluating them from the graph we get

$$E = 13.3\ N + 22, \tag{1}$$

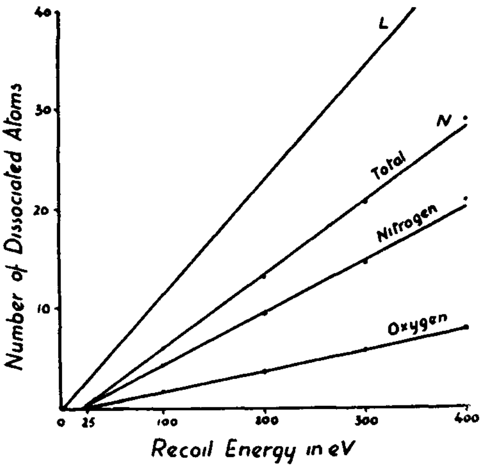

Fig. 1

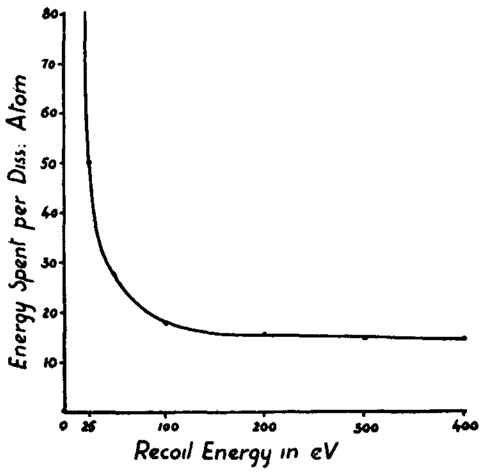

Fig. 2

where E is measured in eV. Hence, the energy spent per droplet is

$$\frac{E}{N} = 13.3 + \frac{22}{N} = 13.3 + \frac{292}{E - 22}.$$

This ratio is a function of E (or of N), but for large values of E it approaches the constant 13.3 eV/droplet. In the interval 300—500 eV the ratio is nearly 14 eV/droplet.

C. & H. assumed as a working hypothesis the linear proportionality of the relation. This includes the supposition that the free term $b = 0$. We see that for E of order 10^2 eV (or N of order 10^1) this approximation is not so bad in spite of Grinberg's criticism.

It is clear to-day — due to L. Wertenstein — that the first estimation of a by C. & H., namely 15 eV/droplet (*3, 1938*), was based on a false assumption. Nevertheless, we see that this value is very near ours, much nearer than that which was admitted hy C. & H. in their second paper (*5, 1938*), c. 8 eV/droplet, although in the last case the principle of determination seems quite correct. The question of this discrepancy will be discussed below in the section «Statistical errors».

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The line L in Fig. 1 represents the number of dissociated atoms of N and O under the assumption that all the energy of the recoil is spent in dissociation and no fraction of it remains for elastic collisions and other losses. The mean value of D for air is c. 8.8 eV and thus the function to compute is equal directly to $E/8.8$ (E in eV). The differences of the ordinates of the lines L and T corresponding to the same abscissas represent the values of energy transformed ultimately into heat. For $E=400$ eV the energy loss so computed equals c. 70 per cent of E, for $E=50$ eV to c. 80 per cent. It seems as certain that in reality these losses will be still greater, but probably not very much so.

The relation (1) enables us to make an «absolute calibration» of the ordinate scale in the C. & H. graphs. This will be done in the last section of this paper.

The origin of ions

C. & H. have observed — by the method of switching on and off the clearing field in the chamber — that «in some cases as many as 40 droplets are due to neutral molecules, whereas the total number of droplets (due to ions and molecules) extends up to only about 60» *(5, 1939)*. This shows the possibility [3] of creation of several (at least 8) ion pairs by one recoil atom, in spite of the recognised argument of Wertenstein that this atom can produce only one ion pair [4].

To remove this difficulty I put here forward the idea that ions are produced as a result of the orbital radiation connected with the nuclear decay. By nuclear disintegration the charge of the nucleus is changed and, hence, the total binding energy of the electrons with the nucleus of the atom must be changed too. The electrons must pass in some way from the normal state of the original atom to the normal state of the final one. This phenomenon, so far as I know, has not yet been investigated, probably because of its weakness in comparison with the nuclear radiation.

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[3] From the shape of the curves on Fig. 3 and Fig. 4 (see below) we can infer that the difference is not caused by casual fluctuations in either set of observations.

[4] Grinberg *(1, 1944)* supposes that ions can result from Auger processes of gas atoms excited in collisions. But in the Auger processes electrons of the inner atom shells must be excited and in the case in question the energy required for so doing is not at our disposal (the binding energy of inner electrons of O or N is of the order 10^3—10^4 eV whereas the maximum energy of the recoil atom is c. 400 eV).

The approximate formula of E. A. Milne and A. B. Baker (sėe *10, 1933*, where further references are given) for the total binding energy W (in eV) of electrons in an atom with atomic number Z, namely

$$W = -20.8\, Z^{7/3},$$

leads in the case of the process $Cl^{38} \xrightarrow{\beta} A^{38}$ to a value of the change ΔW equal to c. 2200 eV. This energy must be given away by the atom. It can either be carried off by the emitted beta particle in the form of its increased kinetic energy, or it can excite the final atom (A^{38}) and then be emitted as a radiation quantum (in the last case an Auger process is also possible). The average distribution of the released energy over these two processes has been calculated (in connection with the present work) by Prof. J. Blaton[5] in the first approximation and for a one-electron atom on the basis of Boltzmann's Virial Theorem. If ΔW is the energy change mentioned above and Z the atomic number of the initial atom in the beta process, then the mean excitation of the final atoms is

$$\Delta W' = \frac{1}{2Z+1} \Delta W\,, \tag{2}$$

whereas the emitted beta particle gets on the average the energy

$$\Delta W'' = \frac{2Z}{2Z+1} \Delta W\,.$$

For an atom with 17 electrons, such as that of chlorine, Blaton's result can give only a very rough evaluation (when we choose in a suitable way a «screen number» to diminish the value of Z in the formula (2)). Probably, for large atoms some methods of approximation (of Thomas-Fermi or of Hartree) with the aid of machine integration might givė a second approximation for the problem. But, since relatively large statistical fluctuations (see next section) lay in the very nature of the experiment discussed, it is not worth while making a precise evaluation of $\Delta W'$, and for our purpose a half-quantitative estimation on the basis of Blaton's formula suffices. We diminish Z by the average of the screen numbers of all electrons of the chlorine atom, namely by c. 5, and from (2) we obtain for $\Delta W'$ c. 1×10^2 eV. It is the mean for a large number of disintegration processes.

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[5] These calculations were kindly given to me in manuscript.

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Table II

Limits of Spectral Series of Argon

Series	Energy		Wave length (Å)	For normal air	
	(Ry)	(eV)		number of ion pairs produced	mean range (mm)
1	2	3	4	5	6
K	237	3200	3.84	106	5×10^{1}
L	18	244	50.6	8	2×10^{-2}
M	0.6	8	1545	—	—

Table II gives the data of possible excitations of the X-ray spectrum of an argon atom and of the effects of respective radiations in normal air (the figures of column 2 are interpolated from *(11, 1926)*, the others are calculated on their basis). The ionization of air by an X-ray photon is composed in general of one photo-effect (with absorption of the ionization energy amounting to c. 16.7 eV in the average, i. e. 16.7 eV/ion pair) and many ionizations by secondary electrons with average energy spent of 30 eV/ion pair. Therefore we may adopt rather the latter value for the calculation of the figures in column 5.

We see that in our phenomenon an excitation of the K-series is impossible (since the total ΔW is only 2200 eV), but the following series can be excited. The excitation of the L-series can produce 8 ion pairs in air. It seems that agreement of this number with the experimental result of C. & H. mentioned at the beginning of this section is not accidental and that it supports our hypothesis. The range in air of the respective X-ray photon (2×10^{-2} mm) lies quite within the limits of the dimensions of the clusters of droplets observed by C. & H. (radii of 1—2 mm after diffusion of the ions).

As mentioned above, on the average over a great number of beta decays the excitation energy of the final atom is c. 100 eV and after releasing it gives probably in air about 3 ion pairs or 6 droplets. This figure we take as the first approximation of the average number of ions produced by the phenomenon discussed, which we may call e. g., the *nucleo-electronic* effect. It is obvious that this effect has no connection with the recoil of the nucleus after disintegration and that the observed number of droplets in a cluster should be reduced by the obtained figure, i. e. 6, to get the value of N which is connected with the recoil energy.

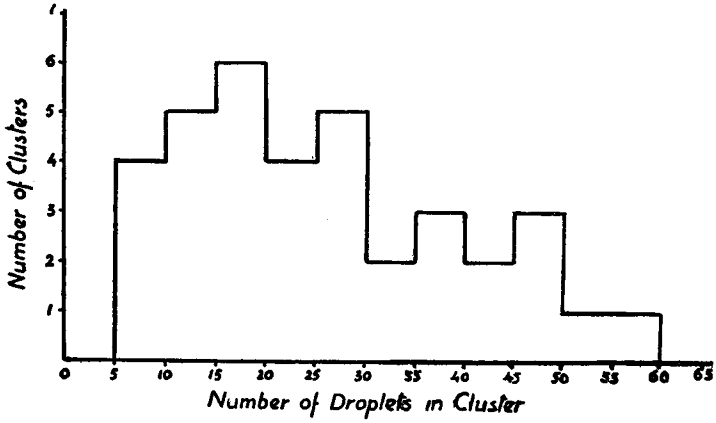

Fig. 3

It we compare the statistics of the droplet numbers in the clusters observed by C. & H. containing ions with that for clusters without ions (Fig. 3 and Fig. 4), we see that the displacement of the maxima of the respective curves corresponds to about 5 droplets. This fact seems also to support our aforesaid result. It must, however, be stressed that Fig. 4 is probably distorted by the existence of the lower energy component in the chlorine beta-ray spectrum and this circumstance considerably diminishes the strength of the last evidence.

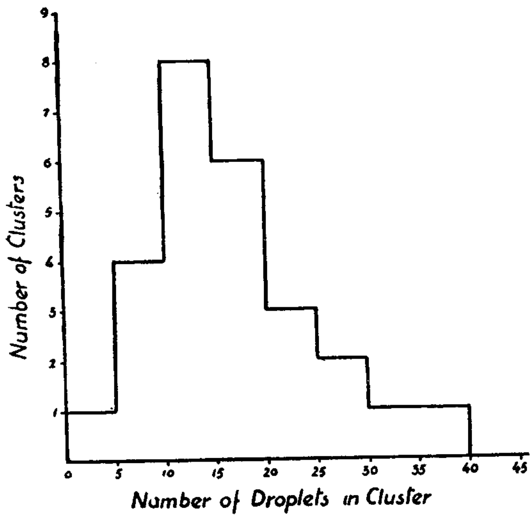

Fig. 4

Statistical errors

The preceding considerations show the existence of two sources of possible deviations in the result of the experiment:

(1) fluctuations in the number of dissociated nitrogen and oxygen atoms, and

(2) fluctuations in the number of ion pairs produced by the nucleo-electronic effect.

The first deviations are of a purely «classical» type known from kinetic theories (they are caused by fluctuations in number and types of collisions) and can therefore well be estimated with

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the aid of Poisson's $\sqrt{n}$-law. Since the number of dissociated atoms does not exceed 30, the standard deviation will not be greater than 6, and with a probability 0.9 the deviation will not exceed $1.7 \times 6 \gtrsim 10$ droplets.

The theoretical treatment of the fluctuations (2) would be very difficult because of the quantum character of the whole phenomenon, but we can avoid this discussion altogether by remarking that the relevant deviation could not be greater than $16 - 6 = 10$ droplets (see the preceding section).

A third source of statistical errors was pointed out by C. & H. *(3, 1938)*: fluctuation in the number of ions produced by beta-ray electrons per 1 cm of their path within a sphere over which the droplets due to the nucleus extend. This error is equal to about $\sqrt{\rho R}$, where ρ denotes the linear density of droplets along the electron track and R the radius of the sphere. From C. & H.'s graph (3, *1938*) we see that this standard error does not surpass 5 droplets. With a probability 0.9 the deviation will not exceed $1.7 \times 5 \gtrsim 9$ droplets.

Finally, C. & H. write *(3, 1938)*: «It is more difficult to estimate the uncertainty in the actual counting of the droplets, because this depends largely upon the quality of a given track. We estimate that this error ranges from 2 to 5 droplets». Let us take 5.

As a resulting maximum standard error we can write $\sqrt{10^2 + 10^2 + 9^2 + 5^2} \gtrsim 18$ droplets, since we may suppose that all the sources of errors mentioned are independent of each other. This figure seems very essential to the whole problem under discussion. Indeed, if the observational points in C. & H.'s plot (see Fig. 5) would deviate from curve 2 by not more than 18 droplets, the nonconservation of momentum could not be considered as proved. Actually in many cases this deviation reaches 40 droplets [6].

It seems that our analysis of errors gives a sufficient explanation for the discrepancy between the value of a obtained theoretically by us and that determined experimentally by C. & H. The latter

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[6] To exclude the possibility of an accidental addition of errors of the types named, we can multiply the deviation of 18 droplets once again by 1.7, and obtain so the number of c. 30 droplets. The actual deviation will not exceed this value with a probability of 0.9 at least. 30 is still smaller than 40. It must be pointed out, however, that so large a deviation occurs in very exceptional cases only, and that the mean fluctuation (not the maximum possible mean fluctuation) can be estimated as

$$\sqrt{6^2 + 8^2 + 4^2 + 3^2} = c.\,11 \quad \text{for } N = 30 \text{ droplets, and as}$$

$$\sqrt{2^2 + 6^2 + 4^2 + 2^2} = c.\,8 \quad \text{for } N = 5 \text{ droplets.}$$

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was namely deduced on the basis of one event of beta particle carrying the entire energy of disintegration. It is possible that in this case an L-series excitation of an argon atom occurred, and thus N was increased by 16. When we subtract this number from the experimental value 56 we obtain $a = E/N = 430/40 \approx 11$ and this figure is already much nearer to our result 14. The standard deviation of a can be evaluated as

$$\Delta a = \frac{1}{N}\Delta E + \frac{E}{N^2}\Delta N = \frac{30}{30} + \frac{430}{900} \cdot 11 \approx 6.5,$$

where we have taken as ΔE 30 eV and as ΔN (see footnote [6]) 11 droplets. C. & H's value 8 lies within the limits of this deviation from our result 14.

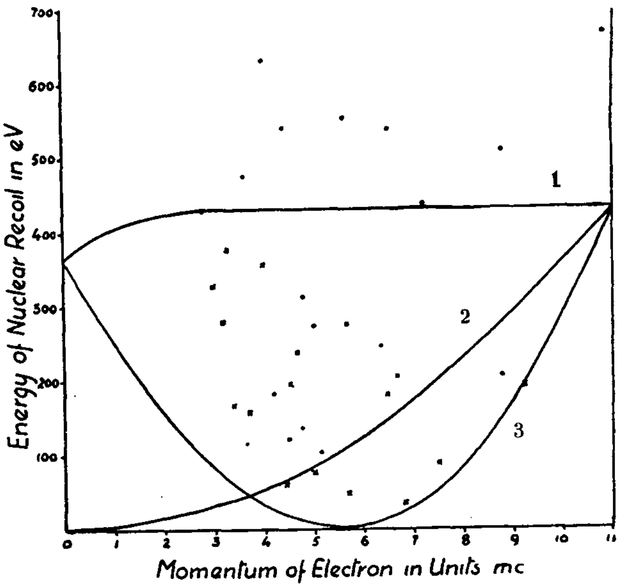

Fig. 5

Conclusions

In Fig. 5 are presented the experimental results of C. & H. as interpreted by our analysis. The numbers of droplets given by C. & H. *(3, 1938), (5, 1939)* have been diminished by 6 (the mean result of the nucleo-electronic effect) and from the values so obtained the relative recoil energies E have been calculated by the aid of formula (1).

We see that only 7 points lie above the upper curve and that of these all are contained within the limits of a deviation of 18 droplets (260 eV) from this curve.

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We thus come to the conclusion that, if our preceding considerations are correct, C. & H's deductions from their experiments of the non-conservation of momentum in the «two-body» beta disintegration seem to be well founded in spite of criticisms mentioned at the beginning of this paper.

However, the above quantitatively determined possibility of large statistical fluctuations in the experiments shows that the drawing of conclusions about the statistics of angles between the directions of emission of the neutrino and the electron from so small a number of experimental points (35) is very uncertain. In this respect C. & H. are wholly correct when they write *(5, 1939)*: «It is not safe to attach much significance to the results obtained on this aspect of the problem, because of the possibility of rather large experimental errors». We have seen above (footnote [6]) that these errors are on an average equal to about 30 per cent for large N, and to over 100 per cent (150% or so) for small N ($=5$, it is on the limit of the possibility distinguish a cluster from the beta ray track). By increasing n times the number of measurements, this error would be theoretically diminished $\sqrt{n}$ times. In practice it would not probably be rational to go further than $n = 16$. In any case, it seems very interesting to investigate further the beta tracks with entire energy of disintegration, since it is the only method which can be imagined at present to measure the ratio a and to verify thus our calculation.

The author expresses his thanks and indebtedness to Prof. J. Weyssenhoff, Prof. J. Blaton and Prof. A. Sołtan for their interest in the present work and their valuable advice and opinions expressed during the discussion on it held in Cracow, June 6, 1945.

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